

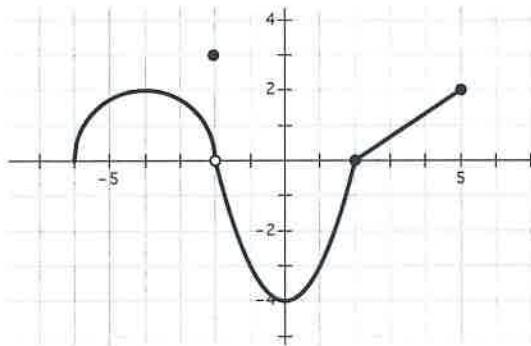
1. The graph of the function f is shown in the figure above. For how many values of x in the open interval $(-5, 5)$ is f not differentiable?

- a) One b) Two c) Three d) Four e) Five
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2. Let f be defined by $f(x) = \begin{cases} x^2 - kx & \text{for } x < 4 \\ 4\sin\left(\frac{\pi}{2}x\right) & \text{for } x \geq 4 \end{cases}$. Determine the value of k for which f is continuous for all real x .

- a) -6 b) -2 c) -4 d) 4 e) -3
- $\cancel{-4k = 40}$
- $\cancel{-4k = 28}$
- $\cancel{k = }$
-

3. The function f is shown below. Which of the following statements about the function f , shown below, is **false**?



- a) $\lim_{x \rightarrow 0} f(x)$ does not exist b) $\lim_{x \rightarrow 2} f(x)$ exists
- c) f is ~~dis~~ continuous at $x = -2$ d) $\lim_{h \rightarrow 0} \frac{f(1-h)+3}{h}$ exists
-

~~4~~ The function f defined on all the Reals such that $f(x) = \begin{cases} 2 - mx, & \text{if } x \leq 3 \\ k\sqrt{x^2 + 7}, & \text{if } x > 3 \end{cases}$

. For which of the following values of k and m will the function f be both continuous and differentiable on its entire domain?

- a) $m = \frac{2}{13}$, $k = \frac{8}{13}$
- b) $m = \frac{2}{13}$, $k = -\frac{8}{13}$
- c) $m = -\frac{2}{13}$, $k = \frac{8}{13}$
- d) $m = -\frac{2}{13}$, $k = -\frac{8}{13}$
-

5. A function $f(x)$ has a vertical asymptote at $x = 2$. The derivative of $f(x)$ is negative for all $x \neq 2$. Which of the following statements are false?

I. $\lim_{x \rightarrow 2} f(x) = +\infty$

II. $\lim_{x \rightarrow 2^-} f(x) = -\infty$

III. $\lim_{x \rightarrow 2^+} f(x) = +\infty$

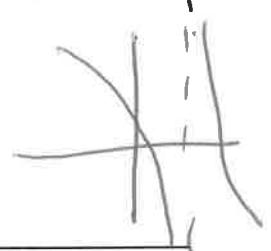
(a) I only

(b) II only

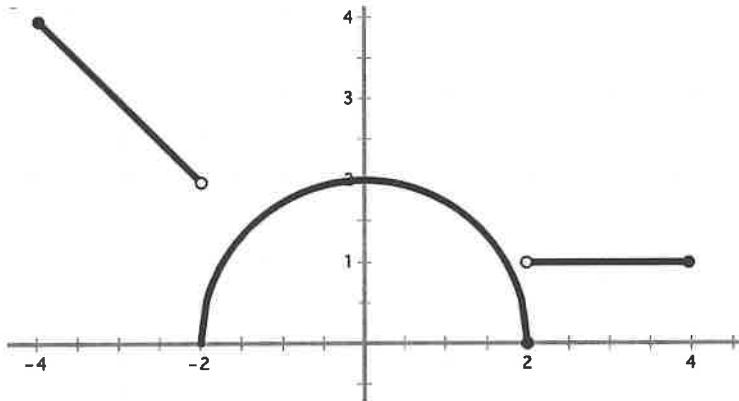
(c) III only

(d) I and II only

(e) I, II and III



6. Given this graph of $f(x)$, which of the following might be the equation?



a)

$$f(x) = \begin{cases} -x, & \text{if } -4 \leq x < -2 \\ \sqrt{4-x^2}, & \text{if } -2 \leq x \leq 2 \\ 1, & \text{if } 2 < x \leq 4 \end{cases}$$

b) $f(x) = \begin{cases} -x-2, & \text{if } -4 \leq x < -2 \\ 2 - \frac{1}{2}x^2, & \text{if } -2 \leq x \leq 2 \\ 1, & \text{if } 2 < x \leq 4 \end{cases}$

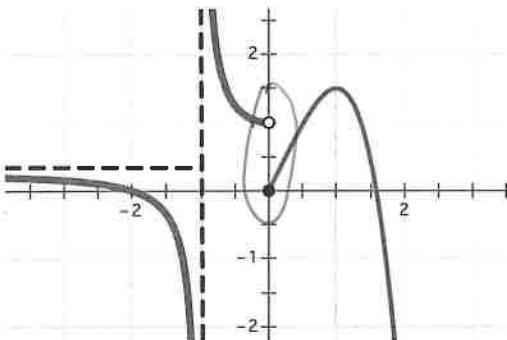
c)

$$f(x) = \begin{cases} x+2, & \text{if } -4 \leq x < -2 \\ \sqrt{4-x^2}, & \text{if } -2 \leq x \leq 2 \\ 1, & \text{if } 2 < x \leq 4 \end{cases}$$

d) $f(x) = \begin{cases} -x-2, & \text{if } -4 \leq x < -2 \\ 2 - \frac{1}{2}x^2, & \text{if } -2 \leq x < 2 \\ 1, & \text{if } 2 \leq x \leq 4 \end{cases}$

7. At $x = 0$, the function given by $f(x) = \begin{cases} x^2 + 1, & \text{if } x \leq 0 \\ -\sqrt{4x - x^2}, & \text{if } 0 < x \end{cases}$ is

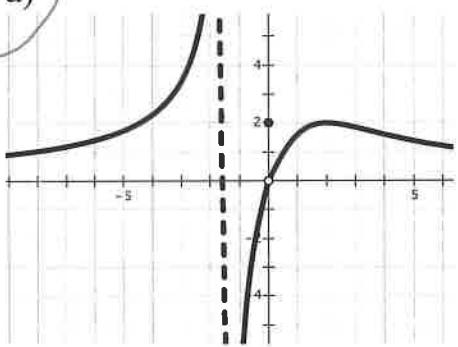
- a) Undefined
 - b) Continuous but not differentiable
 - c) Differentiable but not continuous
 - d) Neither continuous nor differentiable
 - e) Both continuous and differentiable
-



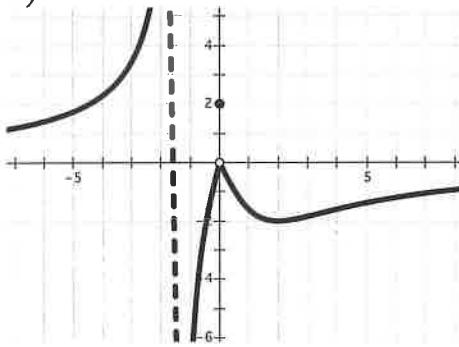
8. Given the graph of $f(x)$ above, the reason $x = 0$ is a critical value is because
- a) $f(x)$ is not continuous at $x = 0$
 - b) $f'(x) = 0$ at $x = 0$
 - c) $f'(x)$ does not exist at $x = 0$
 - d) $x = 0$ is an endpoint of the domain restriction
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9. Which of the following is the graph of $f(x) = \begin{cases} \frac{8x}{4+x^2}, & \text{if } 0 < x \\ 2, & \text{if } x = 0 \\ \frac{-8x}{x^2-2}, & \text{if } x \leq 0 \end{cases}$?

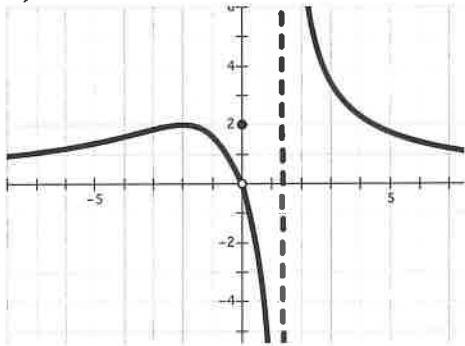
a)



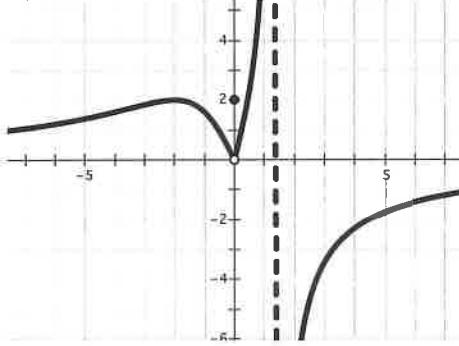
b)



c)



d)



Honors PreCalculus '22-23
Piece-Wise Defined Functions Test
Dr. Quattrin
Calculator allowed

Name: Solution Key
score _____

$$1. \quad f(x) = \begin{cases} \frac{1}{x+2}, & \text{if } x < -2 \\ 3, & \text{if } x = -2 \\ 4-x^2, & \text{if } -2 < x < 2 \\ \frac{x-2}{x-1}, & \text{if } 2 \leq x \end{cases}$$

i) Is $f(x)$ continuous at $x = -2$? Why or why not?

i) $f(-2)$ exists

ii) $\lim_{x \rightarrow -2} f(x)$ DNE because $\lim_{x \rightarrow -2^-} f(x)$ DNE

$\therefore f(x)$ is not continuous at $x = -2$

ii) Is it differentiable at $x = -2$? Why or why not?

$f(x)$ is not differentiable because $f(x)$ is not continuous

$$2. \quad f(x) = \begin{cases} \frac{1}{x+2}, & \text{if } x < -2 \\ 3, & \text{if } x = -2 \\ 4-x^2, & \text{if } -2 < x < 2 \\ \frac{x-2}{x-1}, & \text{if } 2 \leq x \end{cases}$$

i) Is $f(x)$ continuous at $x = 2$? Why or why not?

(i) $f(2)$ exists

(ii) $\lim_{x \rightarrow 2^-} f(x)$ exists because $\lim_{x \rightarrow 2^-} f(x) = 0$ and

$$\lim_{x \rightarrow 2^+} f(x) = 0$$

(iii) $\lim_{x \rightarrow 2} f(x) = f(2)$

$\therefore f(x)$ is continuous at $x = 2$

ii) Is it differentiable at $x = 2$? Why or why not?

(i) $f(x)$ is continuous

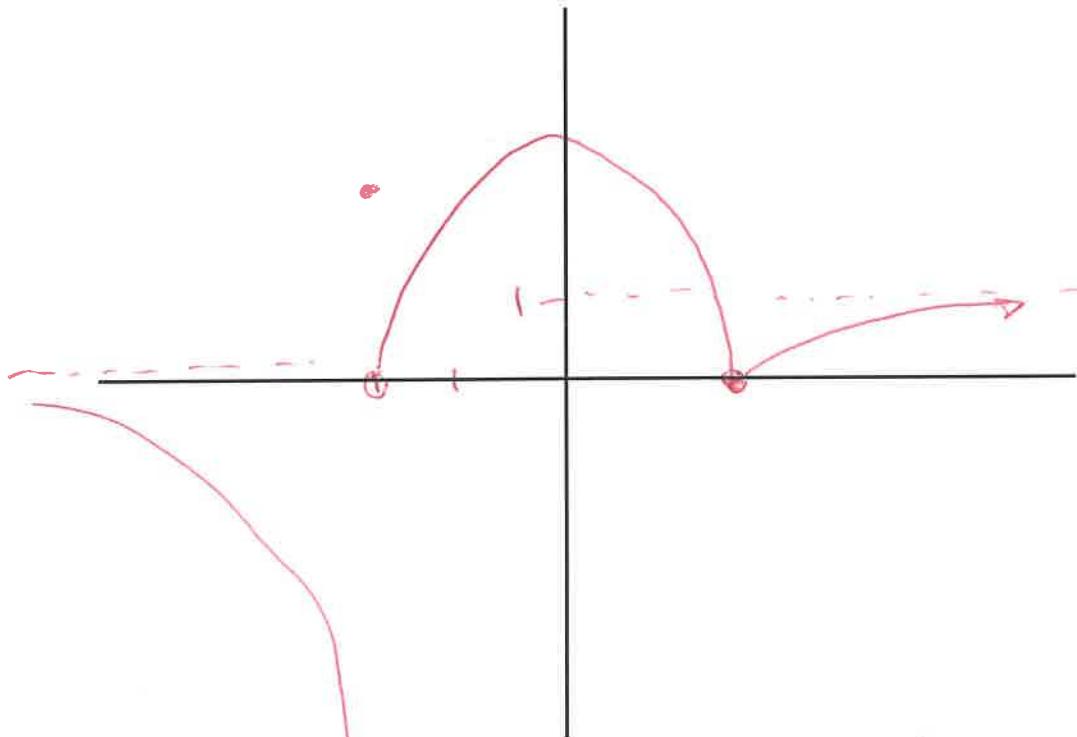
(ii) $\lim_{x \rightarrow 2^-} f(x)$ exists & $\lim_{x \rightarrow 2^+} f(x)$ exists.

(iii) $\lim_{x \rightarrow 2^-} f'(x) = 2 \neq \lim_{x \rightarrow 2^+} f'(x) = 1$

$$f'(x) = \begin{cases} -2x & \\ \frac{(x-1)-(x-2)}{(x-1)^2} = \frac{1}{(x-1)^2} & \end{cases}$$

$f(x)$ is not differentiable at $x = 2$

3. Sketch $f(x) = \begin{cases} \frac{1}{x+2}, & \text{if } x < -2 \\ 3, & \text{if } x = -2 \\ 4-x^2, & \text{if } -2 < x < 2 \\ \frac{x-2}{x-1}, & \text{if } 2 \leq x \end{cases}$. State the Traits listed.



Domain: $x \in \mathbb{R} \setminus \{-2\}$

Zeros: $(2, 0)$

VAs: NONE

EB (Left): $y = 0$

Range: $y \in (-\infty, 4]$

Y-int: $(0, 4)$

EB (Right): $y = 1$

x -values of discontinuities: $x = -2,$

x -values of non-differentiability: $x = -2, 2$

Extreme Points (provide non-graphical evidence):

$$f'(x) = \begin{cases} \frac{-1}{(x+2)^2} & \text{IF } x < -2 \\ -2x & \text{IF } -2 < x < 2 \\ \frac{1}{(x-1)^2} & \text{IF } 2 < x \end{cases}$$

- i) $f'(x)=0 \rightarrow x=0 \rightarrow (0, e)$
- ii) $f'(x) \text{ DNE} \rightarrow x = -2, 2 \quad (-2, 3), (2, 0)$
- iii) END POINTS : NONE GIVEN