

Round to 3 decimal places. Show all work.

1. If $y = \frac{x}{\sqrt{x^2+6}}$, then $\frac{dy}{dx} =$

$$\frac{(x^2+6)^{1/2}(1) - x(\frac{1}{2}(x^2+6)^{-1/2}(2x))}{(x^2+6)^1}$$

$$= \frac{(x^2+6)^{1/2} - \cancel{\frac{x^2}{2}}}{x^2+6}$$

a) $\frac{6}{(x^2+6)^{3/2}}$ b) $\frac{-x}{(x^2+6)^{3/2}}$ c) $\frac{-x^2}{x^2+6}$
 d) $\frac{x}{(x^2+6)^{3/2}}$ e) $\frac{-x^2-6x}{x^2+6}$

2. Let $f(x)$ be the function given by $f(x) = \sqrt{x+3}$. What is the y-intercept of the line tangent to $f(x)$ at $(1, 2)$?

- a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) $\frac{5}{4}$ e) $\frac{7}{4}$
-

$$f'(x) = \frac{1}{2}(x+3)^{-1/2}(1) \quad m = f'(1) = \frac{1}{4}$$

$$y - 2 = \frac{1}{4}(x - 1)$$

$$y = \frac{1}{4}x + \frac{7}{4}$$

3. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-3	1	-2	5	6
1	5	7	-3	-5
5	-3	-4	1	2

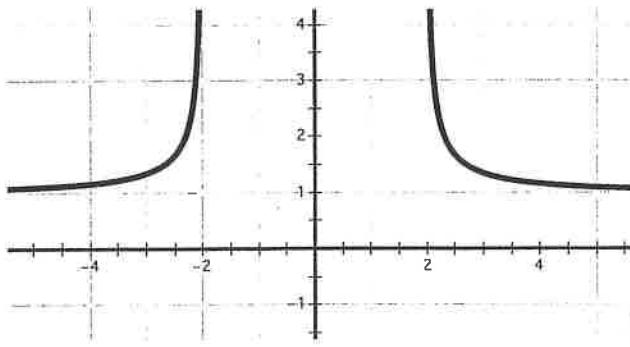
Given that $h(x) = f(g(x))$, $h'(-3) = f'(g(-3)) \cdot g'(-3) = f'(5) \cdot 6 = -4 \cdot 6$

-
- a) -24 b) -12 c) -4 d) -5 e) -2

3. $3x^2 - 2y \frac{dy}{dx} + 6 \frac{dy}{dx} = 0$
 4. If $x^3 - y^2 + 6y - 5 = 0$, then $\frac{dy}{dx} = \frac{-3x^2}{6 - 2y}$

- a) $3x^2 - 2y$ b) $\frac{3x^2 + 6}{-2y}$ c) $\frac{-2y + 6}{3x^2}$

d) $\frac{3x^2}{2y - 6}$ e) $3x^2 - 2y + 6$



VAS @ $x = \pm 2$

DOMAIN $\times E(-\infty, -2) \cup (2, \infty)$

a) $y = \sqrt{\frac{x^2}{4-x^2}}$

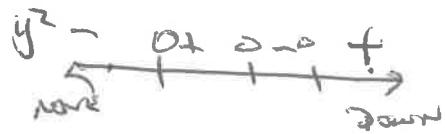
b) $y = \sqrt{\frac{4-4x^2}{x^2+4}}$

c) $y = \sqrt{\frac{4x^2-4}{x^2+4}}$

d) $y = \sqrt{\frac{x^2}{x^2-4}}$

6. What is the end behavior of $y = \sqrt[3]{x^3 - 2x^2 - 5x + 6}$?

- a) None on the left and down on the right
- b) None on the left and none on the right
- c) Up on the left and none on the right
- d) Down on both ends
- e) Down on the left and none on the right



7. The x -value(s) of the relative maximum(s) of $y = \sqrt{27x - x^3}$ is/are

- a) 3 b) $3\sqrt{6}$ c) -3 d) $0, \pm 3\sqrt{3}$ e) 0

$$\frac{dy}{dx} = \frac{1}{2}(27x - x^3)^{\frac{1}{2}} (27 - 3x^2)$$



$$d) x = \pm 3$$

$$e) x = 0, \pm 3\sqrt{3}$$

8. The volume of a sphere is increasing at a rate of 20 cc/sec. How fast, in square centimeters per second, is the surface area changing at the instant when the radius is 10 cm long?

- a) 8 b) $\frac{1}{25\pi}$ c) $\frac{4}{3}$ d) 4 e) 2

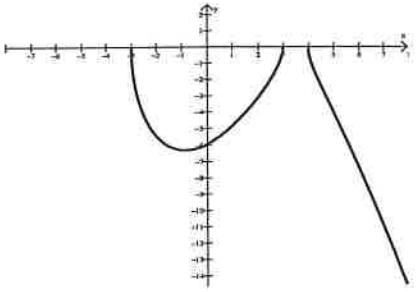
$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 20$$

$$\frac{dr}{dt} = \frac{1}{20\pi}$$

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(10)\left(\frac{1}{20\pi}\right) = 4$$



9. Given the graph above, which of the following might be the sign pattern of $f(x)$?

a)
$$\begin{array}{c} f(x) \\ \hline x \end{array} \quad \begin{matrix} + & 0 & - & 0 & + & 0 & - \end{matrix}$$

b)
$$\begin{array}{c} f(x) \\ \hline x \end{array} \quad \begin{matrix} - & 0 & + & 0 & - & 0 & + \end{matrix}$$

c)
$$\begin{array}{c} f(x) \\ \hline x \end{array} \quad \begin{matrix} + & 0 & 0 & + & 0 \end{matrix}$$

d)
$$\begin{array}{c} f(x) \\ \hline x \end{array} \quad \begin{matrix} - & 0 & 0 & - & 0 \end{matrix}$$

e)
$$\begin{array}{c} f(x) \\ \hline x \end{array} \quad \begin{matrix} 0 & - & 0 & 0 & - \end{matrix}$$

(e)

PreCalculus Honors '22-23

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Radical Functions Test

Round to 3 decimal places.

Show all work.

Name: Solution Key

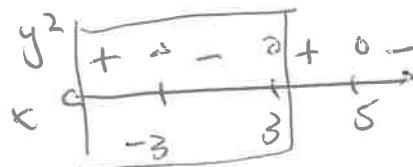
Score _____

1. Find the zeros and Domain of $y = \sqrt{-x^3 + 5x^2 + 9x - 45}$ on $x \in [-5, 3]$.
Show the algebraic work to support the zeros.

Zeros: $(\pm 3, 0)$

Domain: $\{x \in [-5, 3] \cup \{3\}\}$

$$-x^2(x+5) + 9(x-5)$$



2. Find the Extreme Points of $y = \sqrt{-x^3 + 5x^2 + 9x - 45}$ on $x \in [-5, 3]$.
Show the algebraic work to support the critical values.

Extreme Points:

$(\pm 3, 0)$

$(-5, 4\sqrt{10})$

$$\frac{dy}{dx} = \frac{-3x^2 + 10x + 9}{2(-x^3 + 5x^2 + 9x - 45)^{1/2}}$$

$$i) x = \frac{-10 \pm \sqrt{100 + 108}}{-6} = \frac{-10 \pm \sqrt{208}}{-6} = \frac{-10 \pm 14.42}{-6} = \frac{4.42}{-6} = -0.737$$

$$ii) x = \pm 3, 5 \quad 4, 070$$

$$iii) x = -5$$

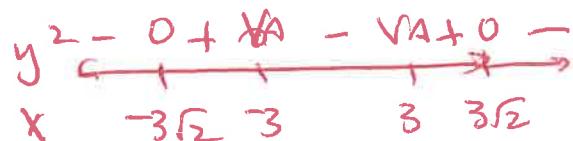
3. Find the zeros, VAs, and domain of $y = -\sqrt{\frac{18-x^2}{x^2-9}}$. Show the Algebra that supports your answer.

Zeros: $(\pm 3\sqrt{2}, 0)$

VAs: $x = \pm 3$

Domain:

$$x \in [-3\sqrt{2}, -3) \cup (3, 3\sqrt{2}]$$



4. Find the Extreme Points of $y = -\sqrt{\frac{18-x^2}{x^2-9}}$. Show the Algebra and derivative that supports your answer.

Extreme Points: $(\pm \sqrt{18}, 0)$ or $(\pm 3\sqrt{2}, 0)$

$$\frac{dy}{dx} = -\frac{1}{2} \left(\frac{18-x^2}{x^2-9} \right)^{-1/2} \left[\frac{(x^2-9)(-2x) - (18-x^2)(2x)}{(x^2-9)^2} \right]$$

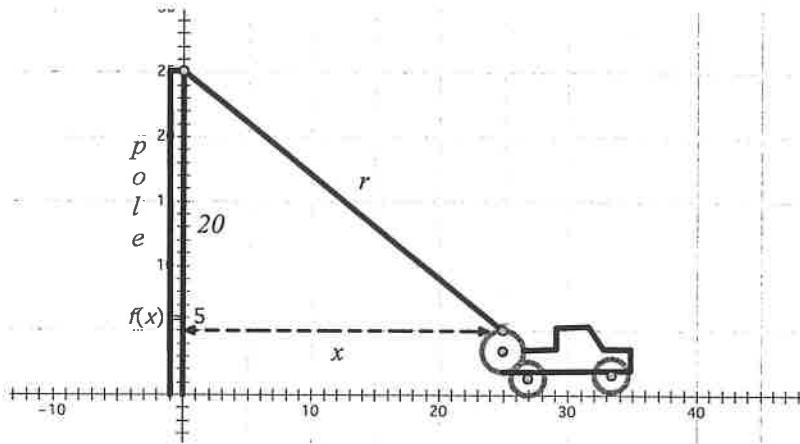
$$= \frac{-9x}{(x^2-9)^{1/2} (18-x^2)^{1/2}}$$

i) $x = 0 \rightarrow$ NOT IN DOMAIN

ii) $x = \pm 3, \pm 3\sqrt{2}$

iii) NONE

A telephone crew is replacing a phone line from one telephone pole to the next. The line is on a spool on the back of a truck, and one end is attached to the top of a 25' pole. The vertical distance from the top of the pole to the level of the spool is 20'.



The truck moves down the street at 20 ft/sec.

- a) Find the length of line that has been rolled out when $t = 15$ sec

$$\text{Find } r \text{ if } t = 15$$

3

$$x = 15(20) = 300$$

$$r = \sqrt{20^2 + 300^2} = 300.666'$$

b) Find the rate at which the telephone line is coming off the spool when the truck is 50 feet from the pole.

$$\begin{aligned} \frac{d}{dt} [x^2 + 20^2 = r^2] \\ 2x \frac{dx}{dt} = 2r \frac{dr}{dt} \\ 5 \quad \frac{dr}{dt} = \frac{50(20)}{\sqrt{2900}} = 18.570 \frac{\text{ft}}{\text{sec}} \end{aligned}$$
$$r = \sqrt{50^2 + 20^2}$$

c) What is the relationship between the angle θ and the truck's distance from the pole? Find θ , in radians, when the truck is 40 feet from the pole.

$$\begin{aligned} 2 \quad \tan \theta &= \frac{20}{40} = \frac{1}{2} \\ \theta &= 0.464 \text{ RAD} \end{aligned}$$

~~XX~~ Find the rate, in radians per second, at which the angle the line forms with horizontal is changing when the truck is 40 feet from the pole.

PreCalculus Honors '22-23

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Radical Functions Test – NO CALCULATOR ALLOWED

Show all work.

6. Find the traits and sketch $y = \sqrt{-x^3 + 5x^2 + 9x - 45}$ on $x \in [-5, 3]$.

Domain: $x \in [-5, -3] \cup \{3\}$

Range: $y \in [0, 4\sqrt{10}]$

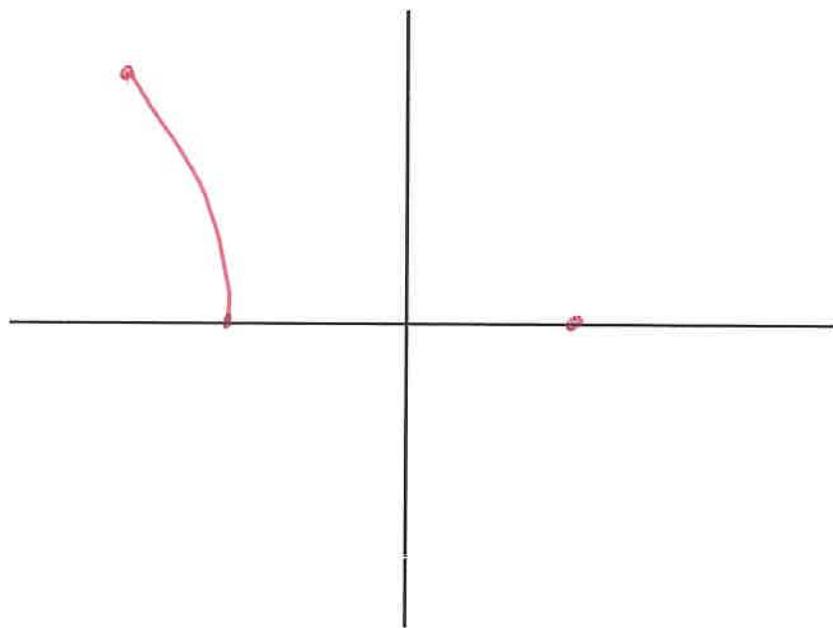
Zeros: $(\pm 3, 0)$

Y-Int: NONE

End Behavior (Left): NONE

End Behavior (Right): NONE

Extreme Points: $(\pm 3, 0), (-5, 4\sqrt{10})$



7. List the traits and sketch of $y = -\sqrt{\frac{18-x^2}{x^2-9}}$ on $x \in [-\infty, 3]$.

Domain: $x \in [-3\sqrt{2}, -3] \cup (3, 3\sqrt{2}]$

Zeros: $(-3\sqrt{2}, 0)$

End Behavior (Left): ~~yz none~~

VAs: $\pm 3 = x$

Extreme Points: $(3\sqrt{2}, 0)$

Range: $y \in (-\infty, 0]$

Y-Int: ~~NONE~~

End Behavior (Right): ~~NONE~~

POEs: ~~NONE~~

