Honors PreCalculus '22-23 Name: Solution Key
Dr. Quattrin
Rational Functions Test v1
CALCULATOR ALLOWED Score _____
Round to 3 decimal places. Show all work.

1. The equation of the line tangent to the graph of $y = \dfrac{x}{2x-3}$ at the point $(1, f(1))$ is

$f(1) = -1$

(a) $3x + y = 4$ (b) $3x + y = 2$ (c) $x - 3y = -2$

(d) $x - 3y = 4$ (e) $x + 3y = 2$

$\dfrac{dy}{dx} = \dfrac{(2(1)-3)(1) - (1)(2)}{(2(1)-3)^2}$

$= -3$

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2. $\lim\limits_{x \to \infty} \dfrac{4x^5 + 3x^4 + 2x^3 + x^2 + 1}{3x^5 - 9x^4 + 4x^3 + 15} =$

a) 0 b) $\dfrac{3}{4}$ c) $\dfrac{4}{3}$ d) 3 e) DNE

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3. Let $f(x)$ and $g(x)$ be differentiable functions. The table below gives the values of $f(x)$ and $g(x)$, and their derivatives, at several values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	-5	-1	6	1
1	-5	1	6	-1
2	-3	3	4	-3

Find $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$, when $x=1$.

$$= \frac{g(1)\cdot f'(1) - f(1)\,g'(1)}{(g(1))^2} = \frac{6(1)-(-5)(-1)}{6^2}$$

a) $\frac{7}{36}$ b) $\frac{5}{36}$ c) $\frac{5}{36}$ d) $\frac{1}{36}$ e) $-\frac{1}{36}$

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4. Which of the following functions has a slant asymptote with a negative slope?

a) $y=\frac{x^3-5x^2-2x+24}{2x^2+7x-15}$

b) $y=\frac{2x^3-9x^2+7x+6}{6-x-x^2}$

c) $y=\frac{x^3-8}{x^4-13x^2+36}$

d) $y=\frac{x^2-1}{x^2+2x-3}$

e) $y=\frac{1-x^2}{x^2+2x-3}$

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5. Suppose $f'(x)=\dfrac{(x+4)^3(x-2)^2}{(x^4+1)}$. Which of the following statements must be **true**?

$f'(1) = \dfrac{5^3(-1)^2}{2} = \dfrac{125}{2}$

I. The slope of the line tangent to $y=f(x)$ at $x=1$ is 62.5. T

II. $f(x)$ is increasing on $x\in(-4, 2)$ F

III. $f(x)$ has a maximum at $x=-4$ F

f' $-$ 0 $+$ 0 $+$

x -4 2

a) I only b) II only c) III only

d) II and III only e) I, II and III

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6. If $y=\dfrac{3-2x}{3x+2}$, then $\dfrac{dy}{dx}=$ $\dfrac{(3x+2)(-2)-(3-2x)(3)}{(3x+2)^2} = \dfrac{-6x-4+6x-9}{(3x+2)^2}$

a) $\dfrac{12x+2}{(3x+2)^2}$ b) $\dfrac{12x-2}{(3x+2)^2}$ c) $\dfrac{13}{(3x+2)^2}$

d) $\dfrac{-13}{(3x+2)^2}$ e) $-\dfrac{2}{3}$

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7. The VA(s) of $y=\dfrac{x^2+2x-3}{x^2+5x+6}$ is/are at $= \dfrac{(x+3)(x-1)}{(x+3)(x+2)}$

a) $x=1$ b) $x=-3$ c) $x=1\ \&\ -3$

d) $x=-2$ e) $x=-2\ \&\ -3$

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8. A function is defined as $g(x) = \dfrac{(x-3)^2}{x-7}$ and $g(x)$ has the sign pattern

$g'(x)$	+	0	−	VA	−	0	+
x		5		7		9	

Which of the following is **false**?

a) $g(x)$ is increasing for $x > 9$. T
b) $g(x)$ is decreasing on $[5, 9]$. T
c) $g(x)$ has a local maximum at $x = 5$. T
d) $g(x)$ has a horizontal asymptote at $y = 0$. F
e) $g(x)$ has a vertical asymptote at $x = 7$.

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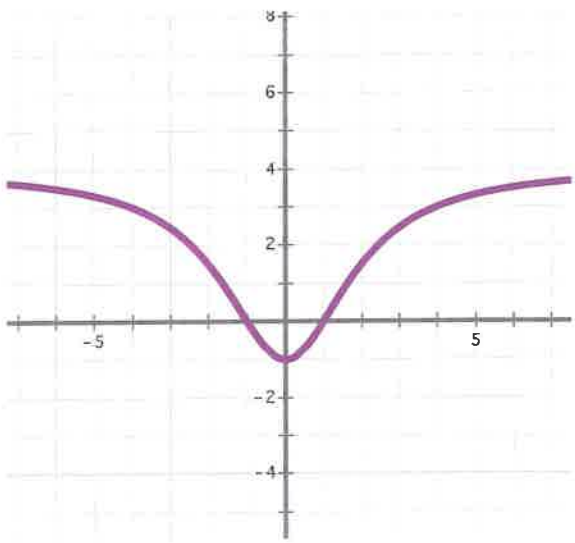
9. Which of the following equations match this graph?



$\dfrac{x^2-1}{\text{NO VAs}}$

a) $y = \dfrac{4x}{x^2+4}$

b) $y = \dfrac{x^2-3x-4}{x^2+4}$

c) $y = \dfrac{4x^2-4}{x^2+4}$

d) $y = \dfrac{4-4x^2}{x^2-4}$

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1. Find the zeros, VAs, and POEs of $y = \dfrac{2x^2 - 5x + 2}{4 - x^2}$ on $x \in (-\infty,\ 5]$. Show the supporting algebraic work.

$$= \frac{(2x-1)(x-2)}{(2-x)(2+x)} \approx \frac{1-2x}{x+2}$$

Zeros: $(1/2,\ 0)$

VAs: $x = -2$

POE: $(2,\ -3/4)$

2. Find the extreme points of $y = \dfrac{2x^2 - 5x + 2}{4 - x^2}$ on $x \in (-\infty,\ 5]$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = \frac{(x+2)(-2) - (1-2x)(1)}{(x+2)^2} = \frac{-5}{(x+2)^2}$$

i) $\frac{dy}{dx} = 0 \rightarrow$ NONE

ii) $\frac{dy}{dx}$ DNE $\Rightarrow x = -2$ BUT NOT IN DOMAIN

iii) EP: $(5,\ \frac{-9}{7})$

3. Find the zeros, VAs, POEs and EB of $y = \dfrac{x^3 - 4x^2 - x + 4}{4x^2 - 16x}$. Show the supporting algebraic work.

$$\frac{(x^2-1)(x-4)}{4x(x-4)}$$

Zeros: $(\pm 1, 0)$

VAs: $x=0$

POE: $\left(4, \frac{15}{16}\right)$

EB: $y = \frac{1}{4}x$

4. Find the extreme points of $y = \dfrac{x^3 - 4x^2 - x + 4}{4x^2 - 16x}$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = \frac{4x(2x) - (x^2-4)(4)}{(4x)^2} = \frac{4x^2+4}{16x^2}$$

i) NONE

ii) $x=0$, NOT IN DOMAIN

iii) NONE

NO EXTREMES

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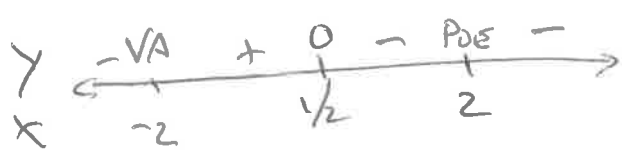
5. Find the equations of the lines tangent to and normal to $y = \dfrac{2x^2 - 5x + 2}{4 - x^2}$ at $x = -1$?

$y(-1) = 3 \quad m = -5$

Tangent: $y - 3 = -5(x+1)$

Normal: $y - 3 = \frac{1}{5}(x+1)$

6. Show the sign pattern and solve $\dfrac{2x^2 - 5x + 2}{4 - x^2} \geq 0$.



$x \in (-2, 1/2]$

7. Find the traits and **sketch** $y = \dfrac{x^3 - 4x^2 - x + 4}{4x^2 - 16x}$.

Domain: $x \neq 0, 4$

Y – Intercept: NONE

Zeros: $(\pm 1, 0)$

Range:

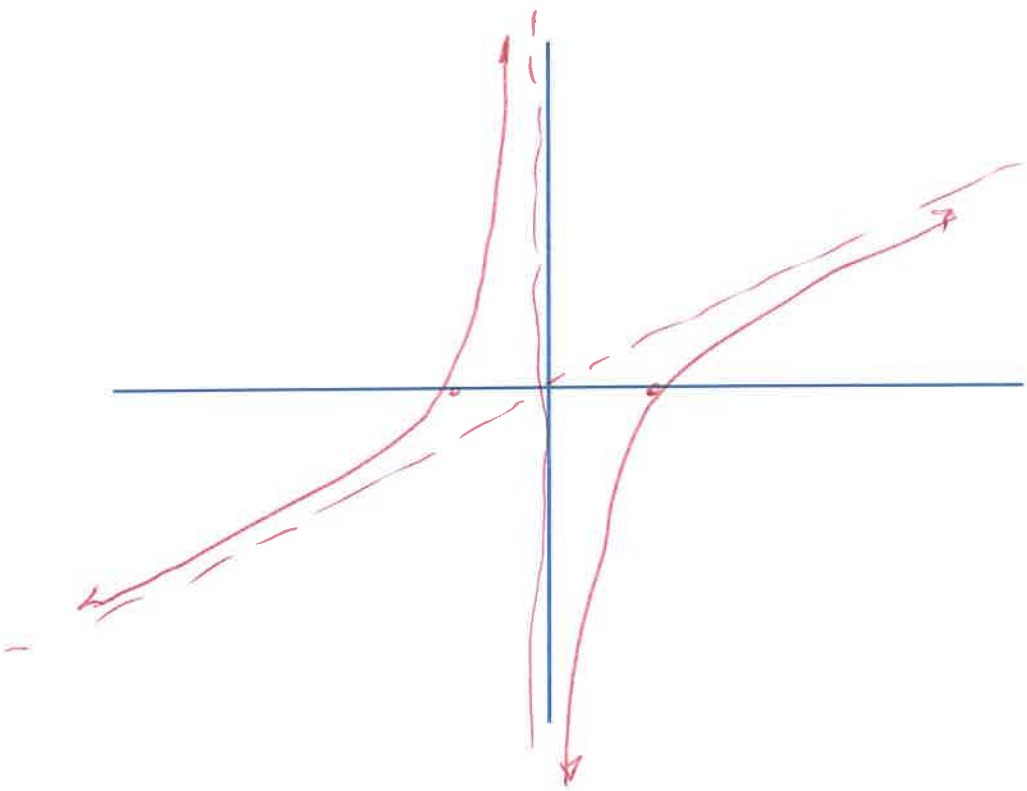
VAs: $x = 0$

POEs: $\left(4, \frac{15}{16}\right)$

End Behavior (left): $y = \frac{1}{4}x$

End Behavior (right): $y = \frac{1}{4}x$

Extreme Points: NONE



8. Find the traits and **sketch** of $y = \dfrac{2x^2 - 5x + 2}{4 - x^2}$ on $x \in (-\infty,\ 5]$.

Domain: $x \neq \pm 2$

Y – Intercept: $(0, 1/2)$

Zeros: $(1/2, 0)$

Range: $y \in (-\infty, -2) \cup [-9/7, -3/4) \cup (-\frac{3}{4}, \infty)$

VAs: $x = -2$ &

POEs: $(2, -3/4)$

End Behavior (left): $y = -2$

End Behavior (right): NONE

Extreme Points: $(5, -9/7)$