

Honors Precalc '23 (Quattrin)
Spring Final – Part I; 30 Minutes
Calculator Allowed

Name: Solution Key
score 15

1. Which of the following statements must be **false**?

(A) $\frac{d}{dx} \left(x^3 + 4x^2 - \sqrt[3]{x^2} - \frac{1}{7x} \right) = 3x^2 + 8x - \frac{3}{2}x^{1/2} + \frac{1}{7x^2}$ **T**

(B) $\frac{d}{dx} \ln(1 - x^3) = \frac{-3x^2}{1 - x^3}$ **T**

(C) $\frac{d}{dx} (x \tan x) = \tan x + x \sec^2 x$ **T**

(D) $\frac{d}{dx} e^{\csc x} = e^{\csc x} \csc^2 x$ **F**

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

2. A particle is moving along the x -axis in such a way that its velocity at time $t > 0$ is given by $v(t) = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

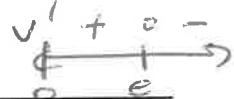
(A) 1 (B) $e^{1/2}$

(C) e

(D) $e^{3/2}$

(E) There is no maximum value of v .

$$v' = \frac{t(\frac{1}{t}) - \ln t}{t^2}$$



3. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below, find $h'(4)$, given that $h(x) = f\left(\frac{1}{2}x\right) \cdot g(2x)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

(A) 12

(B) 30

(C) -8 (D) 62 (E) 76

$$\begin{aligned} h'(4) &= f(2) \cdot g'(8)(2) + g(8) \cdot f'(2)\left(\frac{1}{2}\right) \\ &= 4(4)(2) + 8(-2)(\cancel{4}) \end{aligned}$$

= 30

4. The equation of the line tangent to the graph of $y = \frac{x-4}{1-2x}$ at the point $(0, f(0))$ is $y(0) = -4$
- $\frac{dy}{dx} = \frac{(1-2x) - (x-4)(-2)}{(1-2x)^2}$
- (A) $7x + y = -4$ (B) $7x - y = 4$ (C) $x - 7y = 28$
- (D) $x - 7y = 12$ (E) $x + 7y = -28$
- $M = \frac{1-8}{1} = -7$
-

5. Which of these functions has a point of exclusion at $(1,3)$ and a vertical asymptote at $x = \frac{1}{2}$?

- (A) $g(x) = \frac{x-1}{4x^2-1}$
- (B) $h(x) = \frac{x-1}{2x^2-3x+1}$ $h(0) \approx 1$
- (C) $f(x) = \frac{2x+1}{4x^2-1}$
- (D) $k(x) = \frac{3x-3}{2x^2-3x+1}$
-

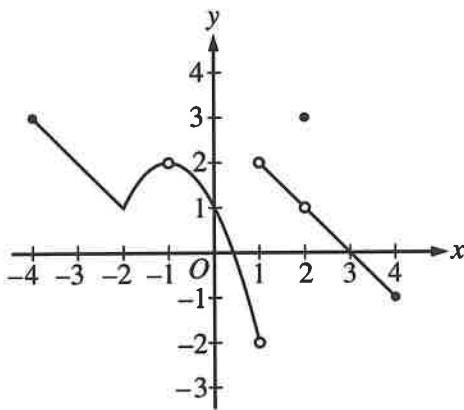
6. What is the slope of the line tangent to the curve $y^2 + x = -2xy - 5$ at the point $(2,1)$?

- (A) $-\frac{4}{3}$ (B) $-\frac{3}{4}$ (C) $-\frac{1}{2}$ (D) $-\frac{1}{4}$ (E) 0

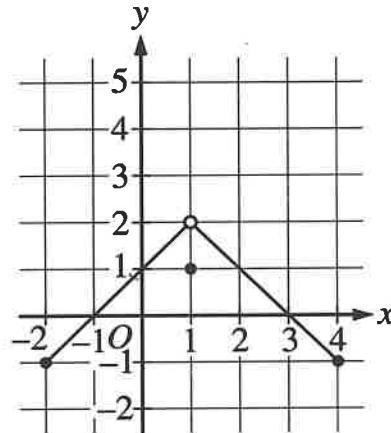
$$2y \frac{dy}{dx} + 1 = -2x \frac{dy}{dx} - 2y$$

$$2 \frac{dy}{dx} + 1 = -4 \frac{dy}{dx} - 2$$

$$3 = -6x$$



Graph of f



Graph of g

7. The graphs of the functions f and g are shown above. If $h(x) = g(f(x))$,

then $h'(3) =$

- (A) -1 (B) 0 (C) 1 (D) 3 (E) dne

$$\begin{aligned} h'(3) &= g'(f(3)) \cdot f'(3) \\ &= g'(0) \cdot f'(3) \\ &= 1 \cdot (-1) \end{aligned}$$

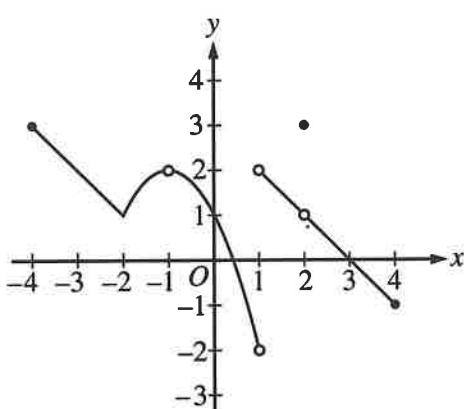
8. At what approximate rate (in cubic meters per minute) is the volume of a cube changing at the instant when the surface area is 54 square meters and each edge is increasing at the rate of 3 meters per minute?

(A) 9 (B) 27 (C) 54 (D) 81 (E) d162ne

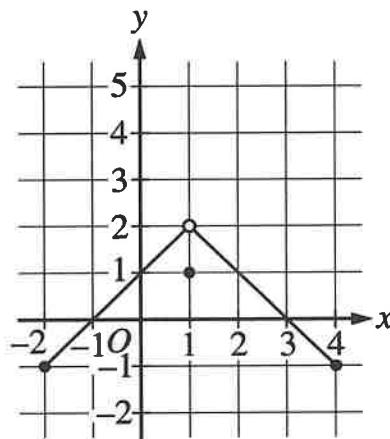
$$V = s^3 \quad \frac{dV}{dt} = 3s^2 \frac{ds}{dt} = 3(3)^2(3)$$

9. $\lim_{x \rightarrow \infty} \frac{2 + \ln(3x)}{5 + \ln(2x^2)} = \frac{\cancel{L^1}_u}{\cancel{L^1}_m} = \frac{1/x}{2/x} = \frac{1}{2}$

(A) 0 (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) ∞



Graph of f

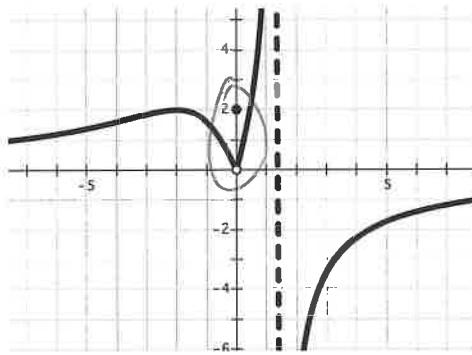


Graph of g

10. The graphs of the functions f and g are shown above. The value of

$$\lim_{x \rightarrow 2} g(f(x)) = g(\lim_{x \rightarrow 2} f(x)) = g(1) = 1$$

-
- (A) 0 (B) 1 (C) 2 (D) -2 (E) dne



11. Given the graph of $f(x)$ above, the reason that $f(x)$ is not continuous at $x = 0$ is because

- (A) $f(0)$ does not exist
 - (B) $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$
 - (C) $\lim_{x \rightarrow 0} f(x) \neq f(0)$
 - (D) $\lim_{x \rightarrow 0} f(x)$ does not exist
-

12. If $y = x^2 e^{2x}$, then $\frac{dy}{dx} =$ $e^{2x}(2x) + e^{2x}(2x)$

- a) $2xe^{-2x}$
- b) $4xe^{-2x}$
- c) $xe^{-2x}(x+1)$
- d) $2xe^{-2x}(x+1)$
- e) $xe^{-2x}(x+2)$

13. Let f be the function defined below, where a and b are constants. If f is differentiable at $x = 1$, what are the values of a and b ?

$$f(x) = \begin{cases} ax^3 - x, & \text{if } x \leq 1 \\ bx^2 + 5, & \text{if } x > 1 \end{cases}$$

$$\begin{aligned} a - 1 &= b + 5 \rightarrow a = b + 6 \\ f'(x) &= \frac{3ax^2 - 1}{2bx} \end{aligned}$$

(A) $a = -7, b = -11$

$$3a - 1 = 2b$$

(B) $a = -11, b = -17$

$$3(b+6) - 1 = 2b$$

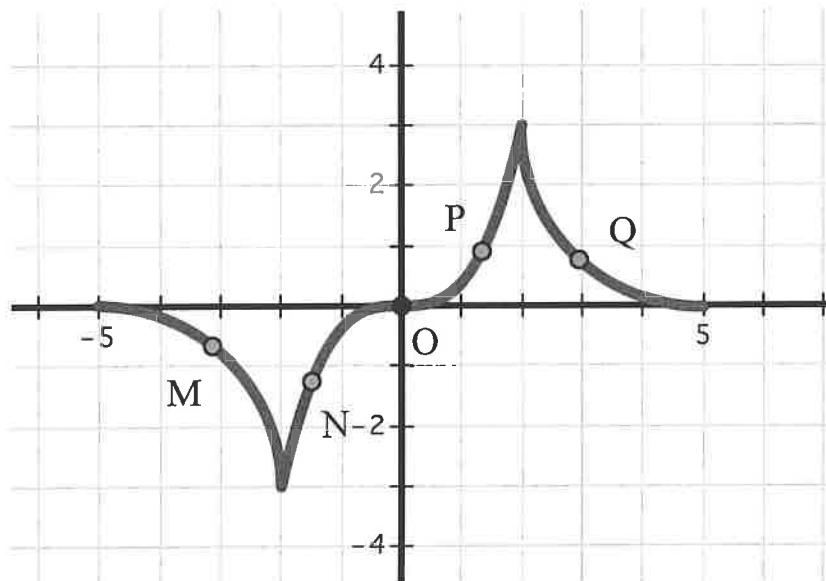
(C) $a = -17, b = -11$

$$3b + 17 = 2b$$

(D) $a = -11, b = -7$

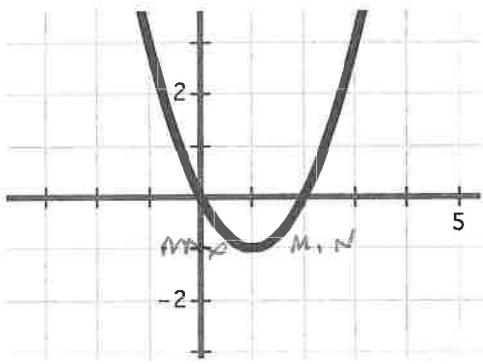
$$b = -17$$

$$a = -11$$

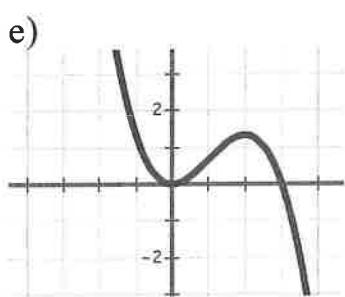
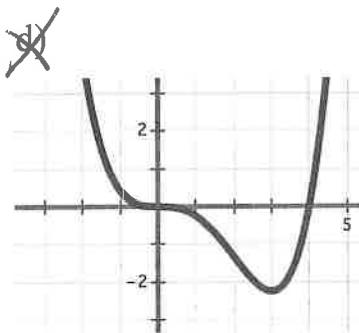
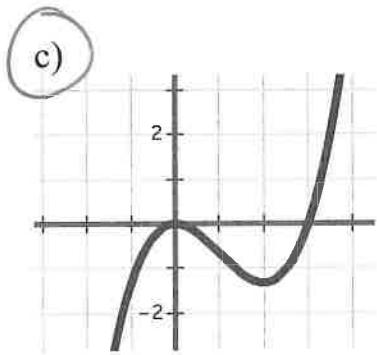
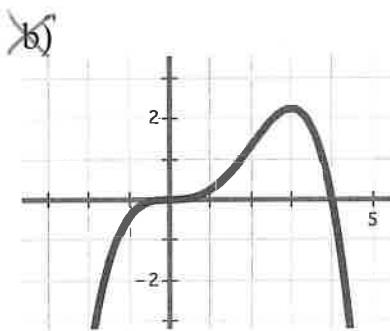
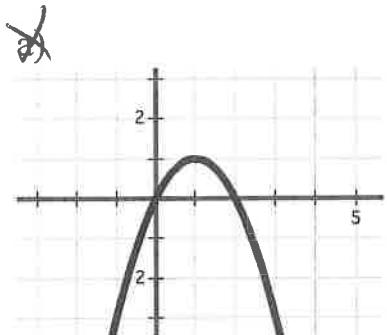


14. At what point on the above curve is $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$
 INC CON UP

- a) M b) N c) P d) Q
-



15. The function $h'(x)$ is graphed above. Which of these functions represents $h(x)$?



Honors Precalc '23

Spring Final – Part IIA; 60 Minutes

Dr. Quattrin

Calculator Allowed

Name: Samantha Kay

score _____

1a. $\frac{d}{dx}(\cos(x^2+4))$

$$= -\sin(x^2+4) (2x)$$

$$= -2x \sin(x^2+4)$$

1b. $\frac{d}{dx}(\sqrt{3x} \cot 6x) = (3x)^{1/2} (-\csc^2(6x))(6) + \cos(6x) \frac{1}{2}(3x)^{-1/2}(3)$

$$= -6\sqrt{3x} \csc^2(6x) + \frac{3 \cot(6x)}{2\sqrt{3x}}$$

$$= \frac{-36x \csc^2(6x) + 3\cot(6x)}{2\sqrt{3x}}$$

1c. $\frac{d}{dx}(\sqrt{\tan(1-x^2)}) =$

$$= \frac{1}{2} \tan^{-1/2}(1-x^2) \sec^2(1-x^2) (-2x)$$

$$= \frac{-x \sec^2(1-x^2)}{\tan^{1/2}(1-x^2)}$$

1d. $\frac{d}{dx}(\sec^{-1}(e^{5x})) =$

$$\frac{1}{e^{5x}\sqrt{(e^{5x})^2-1}} e^{5x}(5) = \frac{5}{\sqrt{e^{10x}-1}}$$

2. Find the domain and Zeros of $f(x) = \frac{x^2+2x-3}{3x^3+16x^2+21x}$. Show the supporting algebraic work.

Domain: $x \neq 0, -3, -\frac{7}{3}$

VAs: $x = -9, -\frac{7}{3}$

Zeros: $(1, 0)$

3. Find the extreme points of $f(x) = \frac{x^2+2x-3}{3x^3+16x^2+21x}$. Show the algebraic work to support the critical values.

Extreme Points: $(-0.826, 0.489), (2.826, 0.042)$

$$f(x) \approx \frac{x^2+2x-3}{3x^3+7x}$$

$$f'(x) = \frac{(3x^2+7x)(1) - (x^2+2x-3)(6x)}{(3x^2+7x)^2} = \frac{-3x^2+6x+7}{(3x^2+7x)^2}$$

$$i) \frac{dy}{dx} = 0 \rightarrow x = \frac{-6 \pm \sqrt{36-4(-3)(7)}}{2(-3)} = \begin{cases} -0.826 \\ +2.826 \end{cases}$$

$$ii) \frac{dy}{dx} \text{ DNE} \rightarrow x = 0, -\frac{7}{3}$$

iii) None

4. Find the domain and Zeros of $g(x) = (4-x)\sqrt{6x-x^2}$. Show the supporting algebraic work.

Domain: $x \in [0, 6]$

Zeros: $(4, 0) (0, 0) (6, 0)$

5. Find the extreme points of $g(x) = (4-x)\sqrt{6x-x^2}$. Show the algebraic work to support the critical values.

Extreme Points: $(0, 0) (6, 0)$

$$\begin{aligned} \frac{dy}{dx} &= (4-x) \left(\frac{1}{2}(6x-x^2)^{-\frac{1}{2}} (6-2x) + (6x-x^2)^{\frac{1}{2}} (-1) \right) \\ &= \frac{(4-x)(3-x)}{(6x-x^2)^{\frac{1}{2}}} - \frac{(6x-x^2)^{\frac{1}{2}}}{1} = \frac{2x^2-13x+12}{(6x-x^2)^{\frac{1}{2}}} \end{aligned}$$

i) $\frac{dy}{dx} = 0 \Rightarrow \frac{2x^2-13x+12}{(6x-x^2)^{\frac{1}{2}}} = 0 \Rightarrow \{$

ii) $\frac{dy}{dx} \text{ DNE} \Rightarrow x = 0, 6$

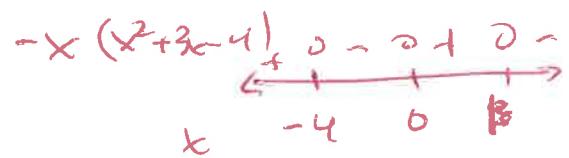
iii) None

6. Find the domain, VAs, and Zeros of $f(x) = \ln(-x^3 - 3x^2 + 4x)$ on $x \in [-6, 2]$. Show the sign pattern to support the domain.

Domain: $\underline{x \in (-\infty, -4) \cup (0, 1)}$

VAs: $\underline{x=0, -4, 1}$

Zeros: $\underline{(3.87, 0), (6.92, 0), (-4.149, 0)}$



7. Find the extreme points of $f(x) = \ln(-x^3 - 3x^2 + 4x)$ on $x \in [-6, 2]$. Show the algebraic work to support the critical values.

Extreme Points: $\underline{(0.5, 28), (-6, 4.43)}$

$$\frac{dy}{dx} = \frac{-3x^2 + 6x + 4}{-x^3 - 3x^2 + 4x}$$

i) $\frac{dy}{dx} = 0 \Rightarrow x = \frac{-2.528}{0.528}$

ii) $x = 0, 1, -4$

iii) $x = 6, 2$

8. Using the functions from #1 and #2, find the traits and sketch

$$K(x) = \begin{cases} f(x) & \text{if } x < 0 \\ g(x) & \text{if } x \geq 0 \end{cases}$$

Domain:

Domain: $x \in (-\infty, -3) \cup (-3, -2/3) \cup (-2/3, 6]$

Zeros: $(0, 0), (4, 0)$

VAs: $x = -2/3$

EB (Left): $y = 0$

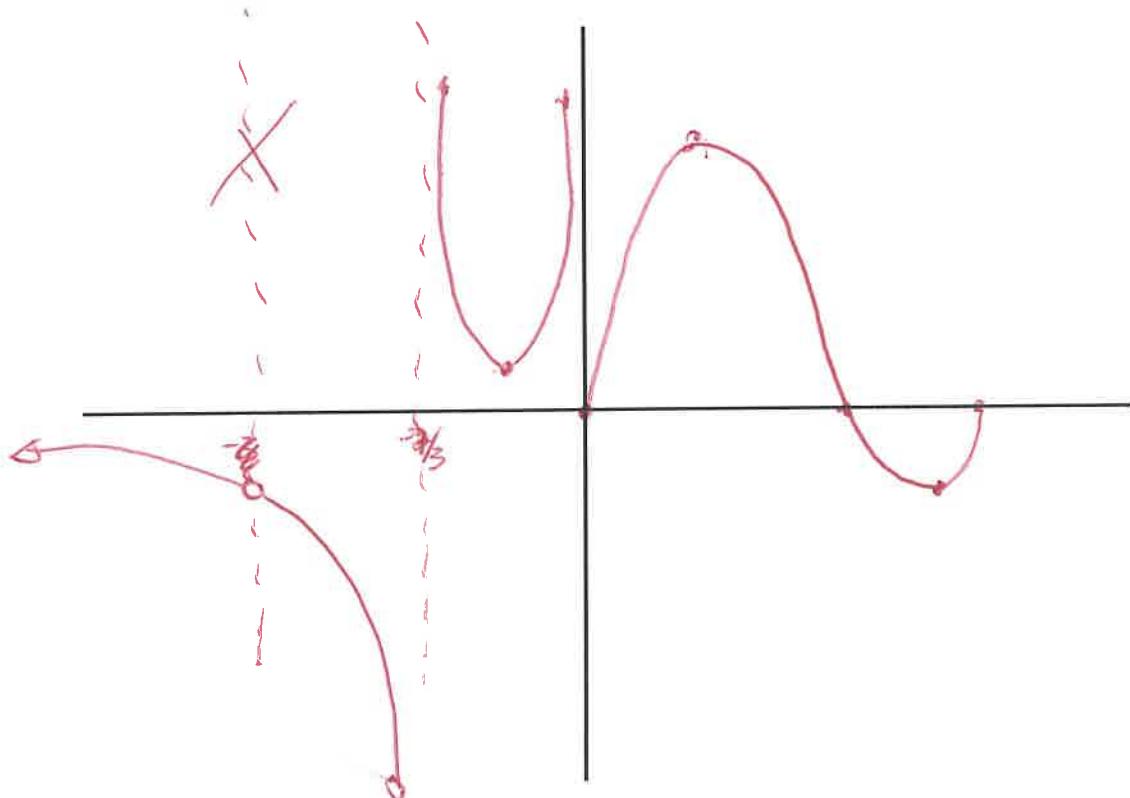
Extremes: $(0, 0), (6, 0), (5.386, -2.520), (1.114, 6.733), (-.826, 0.489)$

Range: $y \in \mathbb{R}$

y -intercept: $(0, 0)$

POEs: $(-3, -2/3)$

EB (Right): NONE



9. Find the traits and sketch of $f(x) = \ln(-x^3 - 3x^2 + 4x)$ on $x \in [-6, 2]$.

Domain: $x \in [-6, -4) \cup (0, 1)$ Range: $y \in (-\infty, 4.43]$

Zeros: $(-3.57, 0), (0.692, 0), (-4.045, 0)$ - intercept: **NONE**

VAs: $x = -4, 0, 1$

POEs: **NONE**

EB (Left): **NONE**

EB (Right): **NONE**

Extremes: ~~$(-3.57, 0.021), (-6, 4.43)$~~

