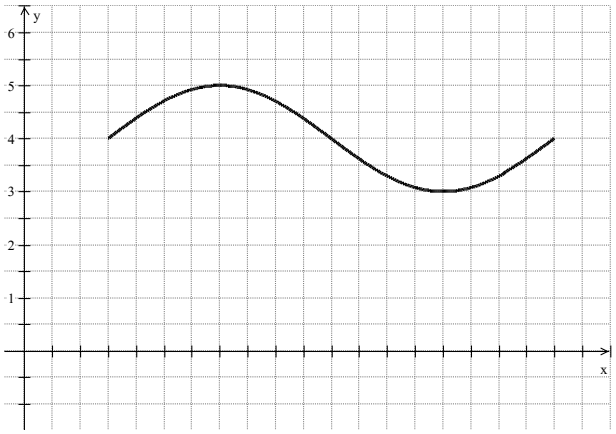
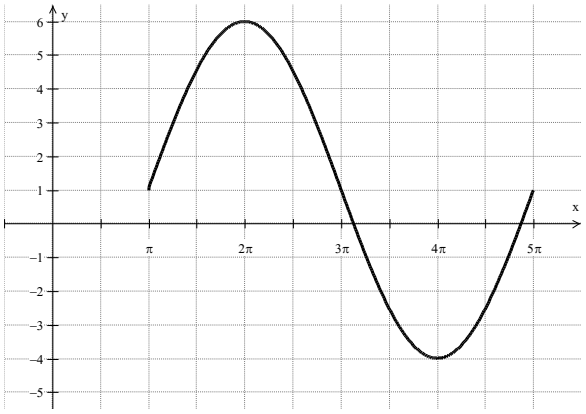
General Trigonometric Functions Answer Key

11-1 Free Response Homework

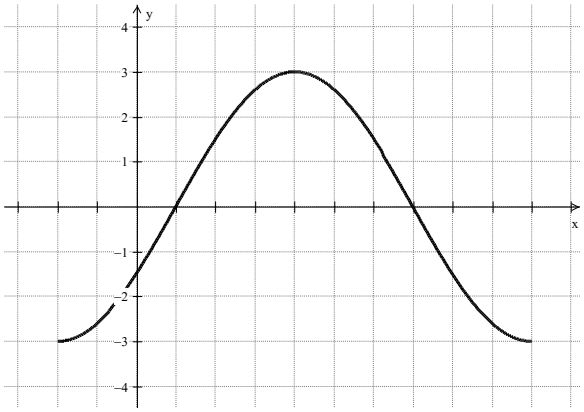
1. $A = 1$ Period = $\frac{2\pi}{\frac{\pi}{8}} = 16$ $k = 4$ $h = 3$



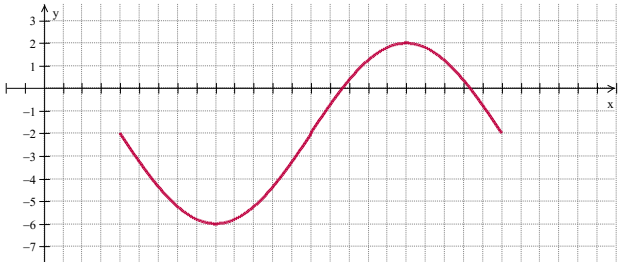
3. $A = 5$ Period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$ $k = 1$ $h = \pi$



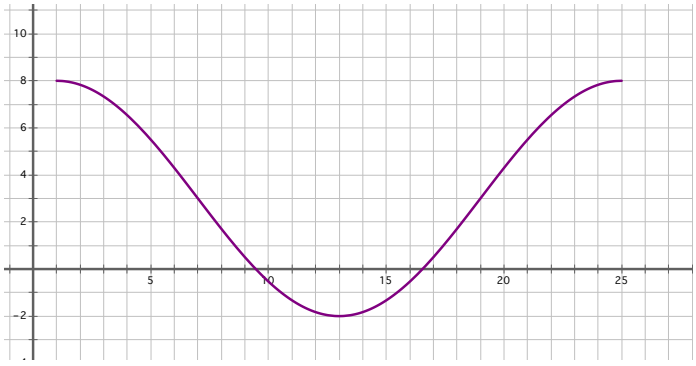
5. $A = 3$ Period $= \dfrac{2\pi}{\pi/6} = 12$ $k = 0$ $h = -2$



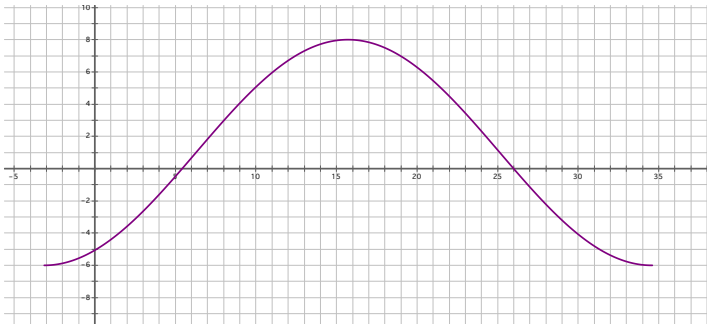
7. $A = 4$ Period $= \dfrac{2\pi}{\pi/5} = 10$ $k = -2$ $h = 2$



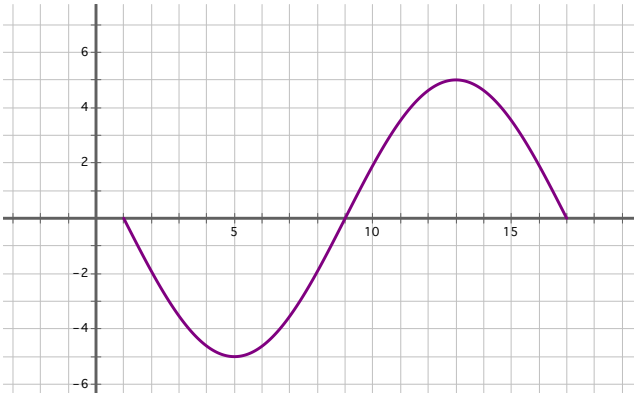
9. $A = 3$ Period $= \dfrac{2\pi}{\pi/12} = 24$ $k = 3$ $h = 1$



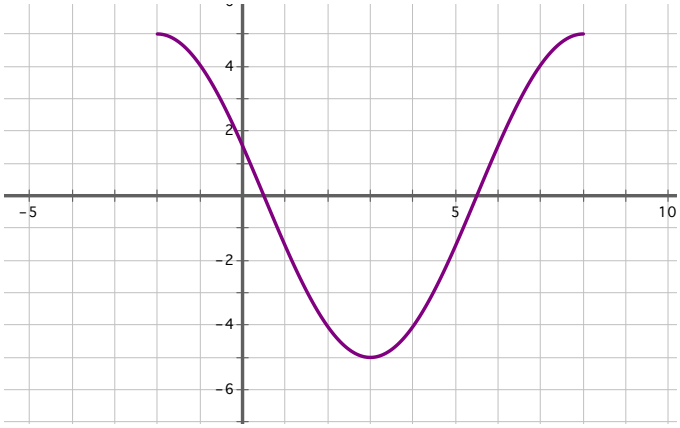
11. $A = 1$ Period $= \dfrac{2\pi}{1/6} = 12\pi$ $k = 1$ $h = -\pi$



13. $A = 2$ Period $= \dfrac{2\pi}{\pi/8} = 16$ $k = 0$ $h = 1$



15. $A = 5$ Period $= \dfrac{2\pi}{\pi/5} = 10$ $k = 0$ $h = -2$



17. $k = 2;\ A = 3;.\ \ period = 4 - (-4) = 8 \rightarrow N = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$

Cosine waves start at the top or bottom, so $h = -4 \pm 8n$ with a positive amplitude, or $h = 0 \pm 8n$ with a negative amplitude. Sine waves start at the sinusoidal axis and go up if the amplitude is positive, so $h = 2 \pm 8n$

$y = 2 + 3\cos\frac{\pi}{4}(x-4)$ $\qquad$ $y = 2 + 3\sin\frac{\pi}{4}(x-2)$

19. $k = 0;\ A = 4;\ period = 7-3 = 4 \rightarrow B = \frac{2\pi}{4} = \frac{\pi}{2}.$

Cosine waves start at the top or bottom, so $h = -1 \pm 4n$ with a positive amplitude, or $h = 1 \pm 4n$ with a negative amplitude. Sine waves start at the sinusoidal axis and go up if the amplitude is positive, so $h = -2 \pm 4n$

$y = 4\cos\frac{\pi}{2}(x-3)$ $\qquad$ $y = 4\sin\frac{\pi}{2}(x-2)$

<u>11-1 Multiple Choice Homework</u>

1. $g(x) = 2 + 3\tan\left[\frac{\pi}{8}(x+1)\right]$

The amplitude is 3, so I is false.

The period is $\frac{\pi}{\pi/8} = \pi \cdot \frac{8}{\pi} = 8$, so II is true.

The Phase shift is -1 , so III is false.

The correct answer is B.

3. $y = 3 - 2\sin\left[\frac{\pi}{8}(x+3)\right]$ has a period of $\frac{2\pi}{\pi/8} = 2\pi \cdot \frac{8}{\pi} = 16$, so the key points are every four units. $h = -3$ and the amplitude is negative, so the first minimum is the second key value, namely $-3 + 4 = 1.$

The correct answer is A.

5. $y = 4 + 2\csc\left(\frac{2\pi}{7}(x+3)\right)$ has $h = -3$, which has a vertical asymptote, so only C or D could be correct. $y = 4 + 2\csc\left(\frac{2\pi}{7}(x+3)\right)$ has a period $\frac{2\pi}{2\pi/7} = 2\pi \cdot \frac{7}{2\pi} = 7$, so the correct answer is D.

11-2 Free Response Homework

1. $\frac{d}{dx}\left(\cos^4 x\right) = \frac{d}{dx}\left(\cos x\right)^4 = 4\left(\cos x\right)^3\left(-\sin x\right) = -4\cos^3 x \sin x$

3. $\frac{d}{dx}\left(\sin^2 x + \cos^2 x\right) = \frac{d}{dx}(1) = 0$

5. $\frac{d}{dx}\left(\cos^4 2x\right) = \frac{d}{dx}\left(\cos(2x)\right)^4 = 4\left(\cos(2x)\right)^3\left(-\sin(2x)\right)(2) = -8\cos^3(2x)\sin(2x)$

7. $\frac{d}{dx}\left(\sec\frac{x^2}{3}\right) = \left(\sec\frac{x^2}{3}\tan\frac{x^2}{3}\right)\left(\frac{2x}{3}\right) = \frac{2}{3}x\sec\frac{x^2}{3}\tan\frac{x^2}{3}$

9. $\frac{d}{dx}\left(\csc\sqrt[3]{x^4}\right) = \frac{d}{dx}\left(\csc x^{4/3}\right) = \left(-\csc x^{4/3}\cot x^{4/3}\right)\left(\frac{4}{3}x^{1/3}\right) = -\frac{4}{3}x^{1/3}\csc x^{4/3}\cot x^{4/3}$

11. $f'(x) = \sec^2 x + \sec x \tan x \rightarrow$

$f'(x) = \sec^2 x + \sec x \tan x \rightarrow$

13. $f'(x)=\frac{\sqrt{3}}{2}+\cos x=0\rightarrow\cos x=-\frac{\sqrt{3}}{2}\rightarrow x=\pm\frac{5\pi}{6}\pm\pi n$

15. $f'(x)=\frac{\sqrt{3}}{2}-\sin x$

i) $f'(x)=\frac{\sqrt{3}}{2}-\sin x=0\rightarrow\sin x=\frac{\sqrt{3}}{2}\rightarrow x=\frac{\pi}{3},\frac{2\pi}{3}$

ii) $f'(x)=dne\rightarrow$ no solution

iii) End points on a domain restriction: $x=0,2\pi$

17. $v(t)=2\sin t\cos t=\sin 2t$

19.

$H'(t)=4\cdot-\sin\left[\frac{\pi}{8}(t-5)\right]\cdot\frac{\pi}{8}$

$=-\frac{\pi}{2}\sin\left[\frac{\pi}{8}(t-5)\right]=0,$ DNE, EoaASD

$\sin\left[\frac{\pi}{8}(t-5)\right]=0$

$\frac{\pi}{8}(t-5)=\begin{cases}0\pm2\pi n\\ \pi\pm2\pi n\end{cases}=\{0\pm\pi n$

$t-5=\begin{cases}0\pm16n\\ 8\pm16n\end{cases}=\{0\pm8n$

$t=\begin{cases}5\pm16n\\ 13\pm16n\end{cases}=\{5\pm8n$

c.v.: $t=5,\ 13,\ 0,\ 20$

v	END	+	0	−	0	+	END
t	0		5		13		20

The weight moves up during the first 5 seconds, then back down until 13 seconds have passed, the back up again until time reaches 20 seconds.

11-2 Multiple Choice Homework

1. $\lim_{x\to 0}\dfrac{1-\cos x}{x^2}=\dfrac{0}{0}$

$\lim_{x\to 0}\dfrac{1-\cos x}{x^2}\overset{L'H}{=}\lim_{x\to 0}\dfrac{\sin x}{2x}\overset{L'H}{=}\lim_{x\to 0}\dfrac{\cos x}{2}=\dfrac{1}{2}$

So, the correct answer is B.

3. $y=\cos^2 x-\sin^2 x\to\dfrac{dy}{dx}=2\cos x(-\sin x)-2\sin x(\cos x)=-4\sin x\cos x$

So, the correct answer is E

5. D

For f to be continuous at $x=\pi$:

1. $f(\pi)=?$

2. $\lim_{x\to\pi} f(x)=\lim_{x\to\pi} f(x)=\lim_{x\to\pi}\dfrac{\tan x}{\sin x}=\dfrac{0}{0}$

$\overset{LH}{=}\lim_{x\to\pi}\dfrac{\sec^2 x}{\cos x}=\dfrac{1}{-1}=-1$

3. $f(\pi)=\lim_{x\to\pi} f(x)$

$\therefore f(\pi)=-1$

7. $x(t)=\cos^3 t\to v(t)=3\cos^2 t(-\sin t)$

$3\cos^2 t\geq 0$, so the sign of $v(t)$ is the sign of $(-\sin t)$. The first time $(-\sin t)$ is positive in in Quadrant III.

So, the correct answer is C

9. $x=0\to\sec(0^2)+(0)y^3=2-y\to y=1$

$\frac{d}{dx}\left[\sec\left(x^2\right)+xy^3=2-y\right]\rightarrow \sec^2\left(x^2\right)\left(2x\right)+x\left(3y^2\frac{dy}{dx}\right)+y^3\left(1\right)=-\frac{dy}{dx}$. The slope of the tangent line is $\left.\frac{dy}{dx}\right|_{(0,1)}\rightarrow \sec^2\left(0^2\right)\left(2\left(0\right)\right)+\left(0\right)\left(3\left(1\right)^2\frac{dy}{dx}\right)+\left(1\right)^3\left(1\right)=-\frac{dy}{dx}\rightarrow\frac{dy}{dx}=1$.

So, the correct answer is D

11-3 Free Response Homework

1.

$$\frac{d}{dx}\left(x^3\cdot\sec x\right)=x^3\cdot\sec x\tan x+\sec x\cdot 3x^2$$
$$=x^2\sec x\left(x\tan x+3\right)$$

3.

$$D_x\left(x^2\cdot\sin x+2x\cdot\cos x\right)$$
$$=x^2\cdot\cos x+\sin x\cdot 2x+2x\cdot-\sin x+\cos x\cdot 2$$
$$=x^2\cos x+2x\sin x-2x\sin x+2\cos x$$
$$=x^2\cos x+\cancel{2x\sin x}\ \cancel{-2x\sin x}+2\cos x$$
$$=x^2\cos x+2\cos x$$
$$=\cos x\left(x^2+2\right)$$

5.

$$\begin{aligned}
f'(x) &= 2\left[\sin^2 x \cdot 2\cos x \cdot -\sin x + \cos^2 x \cdot 2\sin x \cdot \cos x\right] \\
&= 2\left[-2\sin^3 x\cos x + 2\cos^3 x \sin x\right] \\
&= 4\sin x\cos x\left[-\sin^2 x + \cos^2 x\right] \\
&= 4\sin x\cos x\left[\cos^2 x - \sin^2 x\right] \\
&= 2\cdot 2\sin x\cos x\left[\cos 2x\right] \\
&= 2\sin 2x \cdot \cos 2x \\
&= \sin 4x
\end{aligned}$$

7.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{(\cos x - 3)\sec^2 x - \tan x \cdot -\sin x}{(\cos x - 3)^2} \\
&= \frac{(\cos x - 3)\cdot \dfrac{1}{\cos^2 x} + \dfrac{\sin^2 x}{\cos x}}{(\cos x - 3)^2} \\
&= \frac{\dfrac{\cos x - 3}{\cos^2 x} + \dfrac{\sin^2 x \cdot \cos x}{\cos^2 x}}{(\cos x - 3)^2} \\
&= \frac{\cos x - 3 + \sin^2 x\cos x}{\cos^2 x(\cos x - 3)^2} \\
&= \frac{\cos x(1 - 3\sec x + \sin^2 x)}{\cos^2 x(\cos x - 3)^2} \\
&= \frac{\cancel{\cos x}\,(1 - 3\sec x + \sin^2 x)}{\cos^{\cancel{2}} x(\cos x - 3)^2} \\
&= \frac{1 - 3\sec x + \sin^2 x}{\cos x(\cos x - 3)^2}
\end{aligned}$$

9.

$$\frac{d}{dx}\left(\frac{\sin x}{1-\cos x}\right)=\frac{(1-\cos x)\cos x-\sin x\cdot\sin x}{(1-\cos x)^2}$$

$$=\frac{\cos x-\cos^2 x-\sin^2 x}{(1-\cos x)^2}$$

$$=\frac{\cos x-\left(\cos^2 x+\sin^2 x\right)}{(1-\cos x)^2}$$

$$=\frac{\cos x-1}{(1-\cos x)^2}$$

$$=\frac{-(1-\cos x)}{(1-\cos x)^2}$$

$$=\frac{-\cancel{(1-\cos x)}}{(1-\cos x)^{\cancel{2}}}$$

$$=\frac{-1}{1-\cos x}$$

$$=-\frac{1}{2\left(\frac{1-\cos x}{2}\right)}$$

$$=-\frac{1}{2\sin^2\frac{1}{2}x}$$

$$=-\frac{1}{2}\csc^2\frac{1}{2}x$$

11. $\dfrac{d}{dx}\left(\sin^4 3x^2\right)=\left(4\sin^3 3x^2\right)\left(\cos 3x^2\right)\left(6x\right)=24x\sin^3 3x^2\cos 3x^2$

13. $\dfrac{d}{dx}\left(e^x\sec x\right)=e^x\left(\sec x\tan x\right)+\sec x\left(e^x\right)=e^x\sec x\left(\tan x+1\right)$

15. $\dfrac{d}{dx}\left(e^{\sin x^4}\right)=e^{\sin x^4}\left(\sin x^4\right)\left(4x^3\right)=e^{\sin x^4}\left(4x^3\cos x^4\right)$

17. $$\frac{d}{dx}\left(\frac{x^2+1}{e^{3x}}\right)=\frac{\left(e^{3x}\right)\left(2x\right)-\left(x^2+1\right)\left(e^{3x}(3)\right)}{\left(e^{3x}\right)^2}=\frac{\left(e^{3x}\right)\left(2x-3x^2-3\right)}{\left(e^{3x}\right)^2}=\frac{-3x^2+2x-3}{e^{3x}}$$

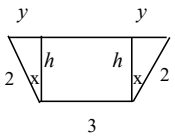
19. $$f'(x)=\sec x\left(\sec^2 x\right)+\tan x\left(\sec x\tan x\right)$$

$$f'\left(\frac{\pi}{4}\right)=\sec\left(\frac{\pi}{4}\right)\left(\sec^2\left(\frac{\pi}{4}\right)\right)+\tan\left(\frac{\pi}{4}\right)\left(\sec\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right)\right)$$
$$=\sqrt{2}\left(\left(\sqrt{2}\right)^2\right)+(1)\left(\left(\sqrt{2}\right)(1)\right)=2\sqrt{2}+\sqrt{2}=3\sqrt{2}$$

21.

$$f'(x)=x\cdot-\sin x+\cos x+x\cdot\cos x+\sin x$$
$$f'\left(\frac{\pi}{4}\right)=\frac{\pi}{4}\cdot-\sin\frac{\pi}{4}+\cos\frac{\pi}{4}+\frac{\pi}{4}\cdot\cos\frac{\pi}{4}+\sin\frac{\pi}{4}$$
$$=\frac{\pi}{4}\cdot-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{\pi}{4}\cdot\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}$$
$$=\frac{\pi}{\cancel{4}}\cdot-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{\pi}{\cancel{4}}\cdot\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}$$
$$=\frac{2}{\sqrt{2}}=\sqrt{2}$$

23.



Note:

$$\sin x=\frac{y}{2}$$

$$y^2+h^2=4\Rightarrow h=\sqrt{4-y^2}$$

$$\begin{aligned}
\text{Volume} &= l\left(\text{Area}_{\text{trapezoid}}\right) \\
&= l\left[\frac{1}{2}(b_1+b_2)h\right] \\
&= 12\left[\frac{1}{2}(3+3+2y)\sqrt{4-y^2}\right] \\
&= 12\left[\frac{1}{2}(6+4\sin x)\sqrt{4-4\sin^2 x}\right] \\
&= 12\left[\frac{1}{2}(6+4\sin x)\cdot 2\cdot\sqrt{1-\sin^2 x}\right] \\
&= 12\left[\frac{1}{\cancel{2}}(6+4\sin x)\cdot \cancel{2}\cdot\sqrt{1-\sin^2 x}\right] \\
&= 12\left[(6+4\sin x)\sqrt{\cos^2 x}\right] \\
&= 12\left[(6+4\sin x)\cos x\right] \\
&= 72\cos x + 48\sin x\cos x
\end{aligned}$$

$$\begin{aligned}
\text{Volume}' &= -72\sin x + 48\left[\sin x\cdot -\sin x + \cos x\cdot\cos x\right] \\
&= -72\sin x - 48\sin^2 x + 48\cos^2 x
\end{aligned}$$

$$c.v.: x = 0.439 \text{ radians} \equiv 25.175^\circ$$

25 a.

$$x'(t) = 5\cos t,\ y'(t) = 2t$$

$$\vec{v} = \langle 5\cos t,\ 2t\rangle$$

b.

$$x''(t) = -5\sin t,\ y''(t) = 2$$

$$\vec{a} = \langle -5\sin t,\ 2\rangle$$

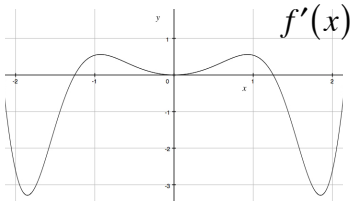
c. $\text{Speed}\big|_{t=\pi} = \sqrt{(5\cos\pi)^2 + (2\pi)^2} = 8.030$

27.

$$x'(t) = e^t \cdot \cos t + \sin t \cdot e^t$$
$$y'(t) = e^t \cdot -\sin t + \cos t \cdot e^t$$
$$x'(1) = e\cos 1 + e\sin 1$$
$$y'(1) = -e\sin 1 + e\cos 1$$
$$\text{Speed}\big|_{t=1} = \sqrt{(e\cos 1 + e\sin 1)^2 + (-e\sin 1 + e\cos 1)^2}$$
$$= \sqrt{e^2\cos^2 1 + e^2\sin 1\cos 1 + e^2\sin^2 1 + e^2\sin^2 1 - e^2\sin 1\cos 1 + e^2\cos^2 1}$$
$$= \sqrt{e^2\cos^2 1 + \cancel{e^2\sin 1\cos 1} + e^2\sin^2 1 + e^2\sin^2 1 \cancel{- e^2\sin 1\cos 1} + e^2\cos^2 1}$$
$$= \sqrt{e^2\cos^2 1 + e^2\sin^2 1 + e^2\sin^2 1 + e^2\cos^2 1}$$
$$= \sqrt{e^2\left(\cos^2 1 + \sin^2 1 + \sin^2 1 + \cos^2 1\right)}$$
$$= \sqrt{e^2(1+1)}$$
$$= e\sqrt{2}$$

11-3 Multiple Choice Homework

1. B



Min @ $x \approx -1.253$, Max @ $x \approx 1.253$
No sign change @ $x = 0$

3. B

$$y' = x \cdot -\sin x + \cos x = 0$$
$$\Rightarrow x\sin x = \cos x$$
$$\Rightarrow x = \cot x$$
$$\Rightarrow \tan x = \frac{1}{x}$$

5. $y = x^2 \cos 2x \rightarrow \frac{dy}{dx} = x^2(-\sin 2x)(2) + (\cos 2x)(2x) = 2x(-x\sin 2x + 2x\cos 2x)$

So, the correct answer is D

7. If $f(x) = x\tan x \rightarrow f'(x) = x\left(\sec^2 x\right) + (\tan x)(1)$, then

$$f'\left(\frac{\pi}{4}\right) = \left(\frac{\pi}{4}\right)\left(\sec^2\left(\frac{\pi}{4}\right)\right) + \left(\tan\left(\frac{\pi}{4}\right)\right) = \left(\frac{\pi}{4}\right)\left(\sqrt{2}\right)^2 + 1 = \frac{\pi}{2} + 1$$

So, the correct answer is B.

11- 4 Free Response Homework

1.

$$y' = \sin 3x \cdot -\sin 3x \cdot 3 + \cos 3x \cdot \cos 3x \cdot 3$$
$$= 3\left(\cos^2 3x \cdot -\sin^2 3x\right)$$
$$= 3\cos 6x = 0,\ \cancel{\text{DNE}},\ \cancel{\text{EoaASD}}$$

$$3\cos 6x = 0$$
$$\cos 6x = 0$$
$$6x = \begin{cases} \frac{\pi}{2} \pm 2\pi n \\ -\frac{\pi}{2} \pm 2\pi n \end{cases} = \left\{ \pm\frac{\pi}{2} \pm 2\pi n \right.$$
$$x = \begin{cases} \frac{\pi}{12} \pm \frac{\pi}{3} n \\ -\frac{\pi}{12} \pm \frac{\pi}{3} n \end{cases} = \left\{ \pm\frac{\pi}{12} \pm \frac{\pi}{3} n \right.$$

3.

$y' = 2\cot 5x \cdot -\csc^2 5x \cdot 5$

$= -10\csc^2 5x \cot 5x = 0,\ \text{DNE},\ \cancel{\text{EoaASD}}$

$\csc^2 5x = 0$ $\quad$ $\cot 5x = 0$

$\csc 5x = 0$ $\quad$ $\tan 5x = \text{DNE}$

$\sin 5x = \text{DNE}$

$\Rightarrow$ no solution

or

$$5x = \begin{cases} \frac{\pi}{2} \pm 2\pi n \\ -\frac{\pi}{2} \pm 2\pi n \end{cases} = \left\{ \frac{\pi}{2} \pm \pi n \right.$$

$$x = \begin{cases} \frac{\pi}{10} \pm \frac{2\pi}{5} n \\ -\frac{\pi}{10} \pm \frac{2\pi}{5} n \end{cases} = \left\{ \frac{\pi}{10} \pm \frac{\pi}{5} n \right.$$

$\csc^2 5x = \text{DNE}$ $\quad$ $\cot 5x = \text{DNE}$

$\csc 5x = \text{DNE}$ $\quad$ $\tan 5x = 0$

$\sin 5x = 0$

$$5x = \begin{cases} 0 \pm 2\pi n \\ \pi \pm 2\pi n \end{cases} = \{ 0 \pm \pi n \quad \text{or} \quad 5x = \begin{cases} 0 \pm 2\pi n \\ \pi \pm 2\pi n \end{cases} = \{ 0 \pm \pi n$$

$$x = \begin{cases} 0 \pm \frac{2\pi}{5} n \\ \frac{\pi}{5} \pm \frac{2\pi}{5} n \end{cases} = \left\{ 0 \pm \frac{2\pi}{5} n \right. \qquad x = \begin{cases} 0 \pm \frac{2\pi}{5} n \\ \frac{\pi}{5} \pm \frac{2\pi}{5} n \end{cases} = \left\{ 0 \pm \frac{2\pi}{5} n \right.$$

$$x = \pm\frac{\pi}{10} \pm \frac{2\pi}{5} n,\ 0 \pm \frac{2\pi}{5} n$$

5.

$$y' = \frac{\sqrt{3}}{2} - \sin x = 0, \ \cancel{\text{DNE}}, \ \cancel{\text{EoaASD}}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = \begin{cases} \frac{\pi}{3} \pm 2\pi n \\ \frac{2\pi}{3} \pm 2\pi n \end{cases}$$

7. No fractions with a denominator of zero, no radicals, and no logarithms

$\therefore$ Domain: $x \in [0, \ 2\pi]$

Axis Points:

$$-\sin x = 0$$

$$x = \begin{cases} 0 \pm 2\pi n \\ \pi \pm 2\pi n \end{cases} = \{0 \pm \pi n$$

$$\Rightarrow x = 0, \ \pi, \ 2\pi$$

$$(0, \ 0), \left(\pi, \ \frac{\pi}{2}\right), \ (2\pi, \ \pi)$$

$h = 0$

EB: None

POEs: None

VAs: None

Extreme Points:

$y' = \frac{1}{2} - \cos x = 0,\ \cancel{\text{DNE}}$, EoaASD

$\frac{1}{2} - \cos x = 0$

$\cos x = \frac{1}{2}$

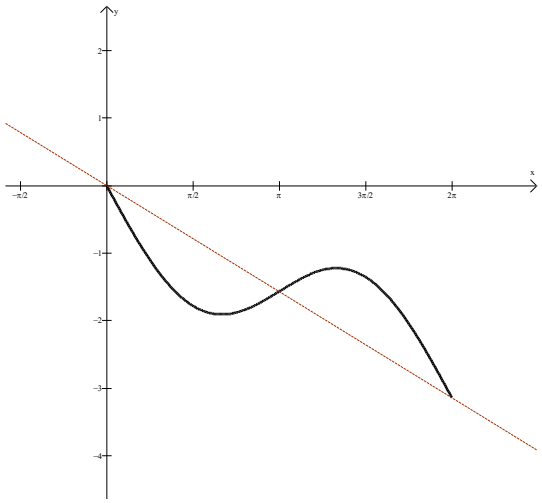
$x = \begin{cases} \frac{\pi}{3} \pm 2\pi n \\ -\frac{\pi}{3} \pm 2\pi n \end{cases} = \left\{ \frac{\pi}{3} \pm \pi n \right.$

$c.v.: x = \frac{\pi}{3},\ \frac{2\pi}{3},\ 0,\ 2\pi$

$\left(\frac{\pi}{3},\ -0.342\right), \left(\frac{5\pi}{3},\ 3.484\right),$

$(0,\ 0), (2\pi,\ \pi)$

Range: $y \in [-0.342,\ 3.484]$



9. No fractions with a denominator of zero, no radicals, and no logarithms
$\therefore$ Domain: $x \in [0,\ 2\pi]$

Axis Points:

$$\sin x = 0$$
$$x = \begin{cases} 0 \pm 2\pi n \\ \pi \pm 2\pi n \end{cases} = \{0 \pm \pi n$$
$$\Rightarrow x = 0,\ \pi,\ 2\pi$$
$$(0,\ 0),\ (\pi,\ 0),\ (2\pi,\ 0)$$

$h = 0$

EB: None

POEs: None

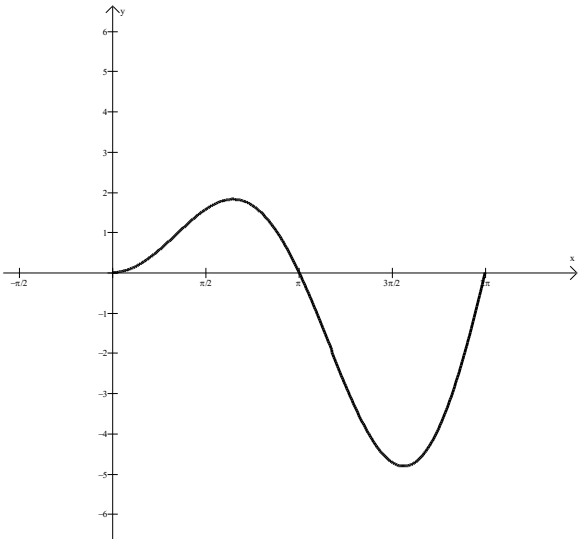
VAs: None

Extreme Points:

$$y' = x \cdot \cos x + \sin x = 0,\ \cancel{\text{DNE}},\ \text{EoaASD}$$
$$x\cos x + \sin x = 0$$
$$x\cos x = -\sin x$$
$$\frac{x\cos x}{\cos x} = \frac{-\sin x}{\cos x}$$
$$x = -\tan x$$
$$c.v.:\ x = 2.029,\ 4.914,\ 0,\ 2\pi$$

$(0,\ 0),\ (2\pi,\ 0)\ (4.913,\ -4.814),\ (2.029,\ 1.820)$

Range: $y \in [-4.814,\ 1.820]$



11. No fractions with a denominator of zero, no radicals, and no logarithms
$\therefore$ Domain: $x \in [-\pi, \pi]$

h=0 EB: None POEs: None VAs: None

Axis Points:

$\sin x = 0$

$x = \begin{cases} 0 \pm 2\pi n \\ \pi \pm 2\pi n \end{cases} = \{0 \pm \pi n$

$\Rightarrow x = -\pi, 0, \pi$

$(-\pi, 0), (0, 0), (\pi, 0)$

Extreme Points:

$y' = e^x \cdot \cos x + \sin x \cdot e^x \cdot = 0,$ ~~DNE~~ , EoaASD

$e^x(\cos x + \sin x) = 0$

$e^x = 0$ $\quad$ $\cos x + \sin x = 0$

$\ln(e^x) = \ln 0$ $\quad$ $\sin x = -\cos x$

DNE $\quad$ or $\quad$ $\dfrac{\sin x}{\cos x} = \dfrac{-\cos x}{\cos x}$

$\tan x = -1$

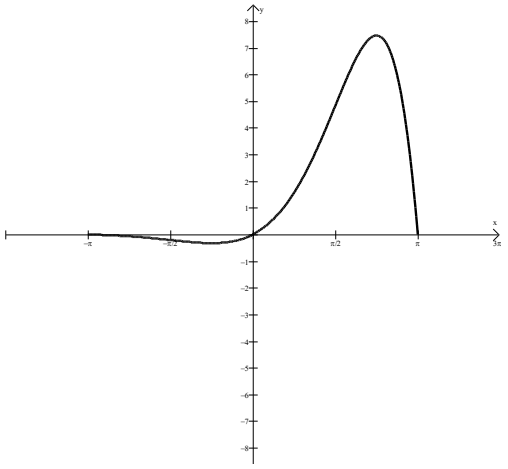
$$x = \begin{cases} -\dfrac{\pi}{4} \pm 2\pi n \\ \dfrac{3\pi}{4} \pm 2\pi n \end{cases} = \left\{ \frac{\pi}{4} \pm \pi n \right.$$

$c.v.: x = \dfrac{\pi}{4},\ \dfrac{3\pi}{4},\ -\pi,\ \pi$

$(-\pi,\ 0),\ (\pi,\ 0),$

$\left(-\dfrac{\pi}{4},\ -0.322\right),\ \left(\dfrac{3\pi}{4},\ 7.460\right)$

Range: $y \in [0,\ 1]$



11-4 Multiple Choice Homework

1. E

I. $\lim_{x\to 0}\frac{\sin x}{x}=\frac{0}{0}\to\lim_{x\to 0}\frac{\sin x}{x}\overset{LH}{=}\lim_{x\to 0}\frac{\cos x}{1}=1\neq\infty$

Since the limit is finite (not infinite) we have a POE rather than a VA at $x=0$.

II. $\lim_{x\to\pm\infty}\frac{\sin x}{x}=0$

Since the limit is 0 (which is finite) we have an HA of $y=0$.

III. $\frac{\sin x}{x}=0\Rightarrow\sin x=0\Rightarrow x=\begin{cases}0\pm 2\pi n\\ \pi\pm 2\pi n\end{cases}=\{0\pm\pi n$

We have an infinite number of zeros. Statements II and III are true.

3. Graph $y=e^{\tan x}-2$ on $x\in[0,1]$. 2nd Trace #2 shows the zero at $x=0.606$. Math 8 or 2nd Trace 6 gives $\frac{dy}{dx}=2.966$

5. The sinusoidal axis is $+\frac{1}{2}x$, so only A or E can be correct. A shows a sine wave and D shows a cosine wave. Therefore, there correct answer is E.

7. According to the graph, $y(0)=-1$. Since $\sin(0)=0$ and $\cos(0)=1$, only D or E can be correct. Since $\sin\left(\frac{\pi}{2}\right)=1$ and $\cos\left(\frac{\pi}{2}\right)=0$, $y\left(\frac{\pi}{2}\right)=-2$ for D and $y\left(\frac{\pi}{2}\right)=-1$ for E. The correct answer is D.

11-5 Free Response Homework

1.

$$y' = \frac{1}{\sqrt{1-\left(x\sqrt{2}\right)^2}} \cdot \sqrt{2}$$
$$= \frac{\sqrt{2}}{\sqrt{1-2x^2}}$$

3.

$$y' = -\frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot -x^{-2} - \frac{1}{1+x^2}$$
$$= -\frac{1}{1+\frac{1}{x^2}} \cdot -\frac{1}{x^2} - \frac{1}{1+x^2}$$
$$= \frac{1}{x^2+1} - \frac{1}{1+x^2} = 0$$

5.

$$y' = \frac{x \cdot \frac{1}{|x|\sqrt{x^2-1}} - \sec^{-1} x}{x^2}$$
$$= \frac{\frac{\cancel{x}}{\cancel{|x|}\sqrt{x^2-1}} - \frac{\sec^{-1} x \cdot \sqrt{x^2-1}}{\sqrt{x^2-1}}}{x^2}$$
$$= \frac{\frac{x - \sec^{-1} x\sqrt{x^2-1}}{\sqrt{x^2-1}}}{x^2}$$
$$= \frac{x - \sqrt{x^2-1}\sec^{-1} x}{x^2\sqrt{x^2-1}}$$

7.

$$y' = \frac{-1}{\left|2e^{3x}\right|\sqrt{\left(2e^{3x}\right)^2 - 1}} \cdot 2e^{3x} \cdot 3$$
$$= \frac{-1}{\cancel{\left|2e^{3x}\right|}\sqrt{\left(2e^{3x}\right)^2 - 1}} \cdot \cancel{2e^{3x}} \cdot 3$$
$$= \frac{-3}{\sqrt{4e^{6x} - 1}}$$

9.

$$y' = \frac{1}{\left(\frac{2x}{1-x^2}\right)^2 + 1} \cdot \frac{\left(1-x^2\right)\cdot 2 - 2x \cdot -2x}{\left(1-x^2\right)^2}$$
$$= \frac{1}{\frac{4x^2}{\left(1-x^2\right)^2} + \frac{\left(1-x^2\right)^2}{\left(1-x^2\right)^2}} \cdot \frac{2 - 2x^2 + 4x^2}{\left(1-x^2\right)^2}$$
$$= \frac{1}{\frac{4x^2 + \left(1-x^2\right)^2}{\left(1-x^2\right)^2}} \cdot \frac{2 + 2x^2}{\left(1-x^2\right)^2}$$
$$= \frac{\left(1-x^2\right)^2}{4x^2 + \left(1-x^2\right)^2} \cdot \frac{2\left(1+x^2\right)}{\left(1-x^2\right)^2}$$
$$= \frac{\cancel{\left(1-x^2\right)^2}}{4x^2 + 1 - 2x^2 + x^4} \cdot \frac{2\left(1+x^2\right)}{\cancel{\left(1-x^2\right)^2}}$$
$$= \frac{2\left(1+x^2\right)}{x^4 + 2x^2 + 1}$$
$$= \frac{2\left(x^2+1\right)}{\left(x^2+1\right)\left(x^2+1\right)}$$
$$= \frac{\cancel{2\left(x^2+1\right)}}{\cancel{\left(x^2+1\right)}\left(x^2+1\right)}$$
$$= \frac{2}{x^2+1}$$

11.

$$\text{Speed}\big|_{t=4} = \sqrt{\left(\tan^{-1}\frac{4}{5}\right)^2 + (\ln 17)^2} = 2.912$$

$$x''(t) = \frac{1}{\left(\frac{t}{t+1}\right)^2+1}\cdot\frac{(t+1)\cdot 1 - t\cdot 1}{(t+1)^2},\ y''(t) = \frac{1}{t^2+1}\cdot 2t$$

$$x''(t) = \frac{1}{\frac{t^2}{(t+1)^2}+1}\cdot\frac{1}{(t+1)^2},\ y''(t) = \frac{2t}{t^2+1}$$

$$x''(t) = \frac{1}{\frac{t^2}{(t+1)^2}+\frac{(t+1)^2}{(t+1)^2}}\cdot\frac{1}{(t+1)^2},\ y''(t) = \frac{2t}{t^2+1}$$

$$x''(t) = \frac{1}{\frac{t^2+(t+1)^2}{(t+1)^2}}\cdot\frac{1}{(t+1)^2},\ y''(t) = \frac{2t}{t^2+1}$$

$$x''(t) = \frac{(t+1)^2}{t^2+(t+1)^2}\cdot\frac{1}{(t+1)^2},\ y''(t) = \frac{2t}{t^2+1}$$

$$x''(t) = \frac{\cancel{(t+1)^2}}{t^2+(t+1)^2}\cdot\frac{1}{\cancel{(t+1)^2}},\ y''(t) = \frac{2t}{t^2+1}$$

$$x''(t) = \frac{1}{t^2+(t+1)^2},\ y''(t) = \frac{2t}{t^2+1}$$

$$x''(4) = \frac{1}{4^2+(4+1)^2},\ y''(t) = \frac{2\cdot 4}{4^2+1}$$

$$a(t) = \left\langle \frac{8}{41}, \frac{8}{17} \right\rangle$$

11-5 Multiple Choice Homework

1. Graph $y = \arcsin x$ and $y = 2\arccos x$, and find the intersection with 2nd Trace 5. $x = 0.9$, so the correct answer is B.

3. $g(x) = \arcsin 2x = \sin^{-1}(2x)$. $g'(x) = \dfrac{1}{\sqrt{1-(2x)^2}}(2) = \dfrac{2}{\sqrt{1-4x^2}}$

So, the correct answer is E

5. If $\sin^{-1} x = \ln y \to e^{\sin^{-1}x} = y$, then $\dfrac{dy}{dx} = e^{\sin^{-1}x}\left(\dfrac{1}{\sqrt{1-x^2}}\right) = \dfrac{y}{\sqrt{1-x^2}}$

So, the correct answer is A

7. $f'(t) = \dfrac{d}{dt}\left[t\left(1-t^2\right)^{\frac{1}{2}} + \cos^{-1} t\right]$

$= t\left[\dfrac{1}{2}\left(1-t^2\right)^{\frac{1}{2}}(-2t)\right] + \left(1-t^2\right)^{-\frac{1}{2}}(1) + \dfrac{-1}{\sqrt{1-t^2}}$

$= \dfrac{-t^2}{\sqrt{1-t^2}} + \dfrac{1}{\sqrt{1-t^2}} + \dfrac{-1}{\sqrt{1-t^2}} = \dfrac{-t^2}{\sqrt{1-t^2}}$

So, the correct answer is E

General Trigonometric Functions Practice Test Answer Key

Multiple Choice – Calculator Allowed

1. E

$$f'(x) = \cos\left(e^{-x}\right) \cdot e^{-x} \cdot (-1)$$
$$= -e^{-x} \cos\left(e^{-x}\right)$$

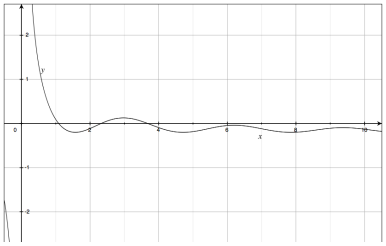
2. B

$$y' = 1 - \sin x$$
$$y'\big|_{x=0} = 1 - \sin 0 = 1$$

$$y - 1 = 1(x - 0)$$
$$\Rightarrow y = x + 1$$

3. B

Graph $f'(x) = \dfrac{\cos^2 x}{x} - \dfrac{1}{5}$.



The sign of the derivative changes 3 times – i.e. from + to –, then – to +, and finally + to – again.

4. D

$$E'(B) = 2(B+1)\cdot\frac{dB}{dt}$$
$$= 2(B+1)\cdot 20\cos\left(\frac{t}{10}\right)\cdot\frac{1}{10}$$
$$= 2\left[20\sin\left(\frac{t}{10}\right)+50+1\right]\cdot 20\cos\left(\frac{t}{10}\right)\cdot\frac{1}{10}$$
$$E'(100) = 4\left(20\sin\left(\frac{100}{10}\right)+51\right)\cos\left(\frac{100}{10}\right) = -134.396 \text{ dollars/day}$$

<u>Free Response – Calculator Allowed</u>

1.

No fractions with a denominator of zero, no radicals, and no logarithms
$\therefore$ Domain: $x \in [-2\pi,\ 2\pi]$

$$y' = \frac{\sqrt{3}}{2} - \sin x = 0,\ \cancel{\text{DNE}},\ \text{EoaASD}$$
$$\sin x = \frac{\sqrt{3}}{2}$$
$$x = \begin{cases} \frac{\pi}{3} \pm 2\pi n \\ \frac{2\pi}{3} \pm 2\pi n \end{cases}$$
$$c.v.:\ x = -\frac{5\pi}{3},\ -\frac{4\pi}{3},\ \frac{\pi}{3},\ \frac{2\pi}{3},\ -2\pi, 2\pi$$

y'	END	+	0	−	0	+	0	−	0	+	END
x	-2π		$-\frac{5\pi}{3}$		$\frac{4\pi}{3}$		$\frac{\pi}{3}$		$\frac{2\pi}{3}$		2π

Extreme Points: $(-2\pi,\ -4.441),\ (2\pi, 6.441),\ \left(\frac{\pi}{3},\ 1.407\right),\ \left(\frac{2\pi}{3},\ 1.314\right),$
$\left(-\frac{5\pi}{3},\ -4.034\right),\ \left(-\frac{4\pi}{3},\ -4.128\right)$

2.

No fractions with a denominator of zero, no radicals, but there is a logarithm –

$$\begin{array}{cc} \tan x & 0 \quad + \quad \text{DNE} \quad - \quad 0 \quad + \quad \text{DNE} \quad - \quad 0 \\ x & \longleftarrow\!\!\!

Multiple Choice – No Calculator Allowed

5. E

$$f'(x) = \sec^2(2x) \cdot 2$$
$$f'\left(\frac{\pi}{6}\right) = \sec^2\left(\frac{\pi}{3}\right) \cdot 2 = 8$$

6. D

$$\frac{d}{dx}\left[\sin(xy) = x\right]$$
$$\cos(xy)\left[x \cdot \frac{dy}{dx} + y \cdot 1\right] = 1$$
$$x\cos(xy)\frac{dy}{dx} + y\cos(xy) = 1$$
$$x\cos(xy)\frac{dy}{dx} = 1 - y\cos(xy)$$
$$\frac{dy}{dx} = \frac{1 - y\cos(xy)}{x\cos(xy)}$$

7. A

$$y\big|_{x=0} = \arctan 0 = 0$$
$$y' = \frac{1}{1+\left(\frac{x}{3}\right)^2} \cdot \frac{1}{3}$$
$$y'_{x=0}\big| = \frac{1}{1+(0)^2} \cdot \frac{1}{3} = \frac{1}{3}$$

Point: $(0, 0)$

Slope: $\frac{1}{3}$

$$y - 0 = \frac{1}{3}(x - 0)$$
$$y = \frac{1}{3}x \Leftrightarrow 3y = x \Leftrightarrow x - 3y = 0$$

8. B

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h} \\
&\Rightarrow f(x) = \cos x \\
&\therefore f'(x) = -\sin x
\end{aligned}
$$

OR

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h} = \frac{0}{0} \\
&\overset{LH}{=} \lim_{h\to 0}\frac{-\sin(x+h)\cdot 1-0}{1} \\
&= \lim_{h\to 0}\frac{-\sin(x+h)}{1} = -\sin x
\end{aligned}
$$

Note: When using L' Hopital's Rule, you must take the derivative with respect to *h*.

Free Response – No Calculator Allowed

3. Domain: $x \in [-2\pi,\ 2\pi]$

Range: $y \in [-4.441,\ 6.441]$

y-int: $(0,\ 1)$

Axis Points:

$\cos x = 0$

$$x = \begin{cases} \frac{\pi}{2} \pm 2\pi n \\ -\frac{\pi}{2} \pm 2\pi n \end{cases} = \left\{ \pm\frac{\pi}{2} \pm 2\pi n \right.$$

$$\Rightarrow x = -\frac{3\pi}{2},\ -\frac{\pi}{2},\ \frac{\pi}{2},\ \frac{3\pi}{2}$$

$$\left(\frac{\pi}{2},\ \frac{\sqrt{3}\pi}{4}\right), \left(\frac{\pi}{2},\ -\frac{\sqrt{3}\pi}{4}\right), \left(\frac{3\pi}{2},\ \frac{3\sqrt{3}\pi}{4}\right), \left(-\frac{3\pi}{2},\ -\frac{3\sqrt{3}\pi}{4}\right)$$

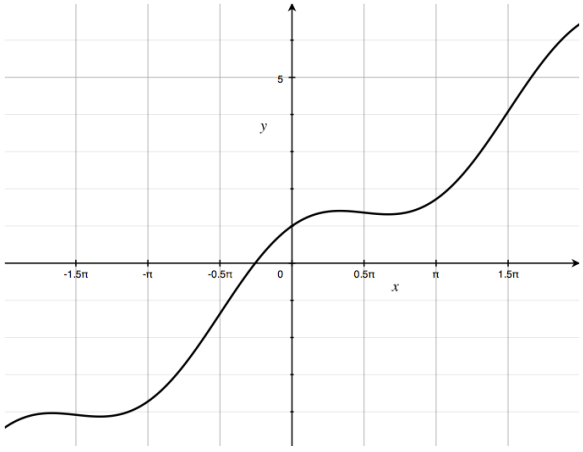
Extreme Points: $(-2\pi,\ -4.441), (2\pi,\ 6.441), \left(\frac{\pi}{3},\ 1.407\right), \left(\frac{2\pi}{3},\ 1.314\right),$

$$\left(-\frac{5\pi}{3},\ -4.034\right), \left(-\frac{4\pi}{3},\ -4.128\right)$$

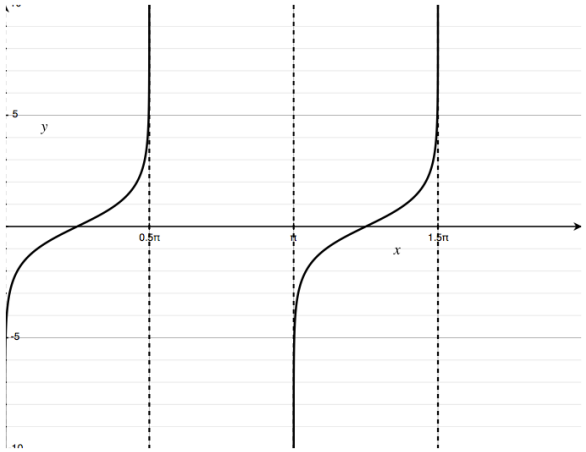
VAs: None

POEs: None

End Behavior: None



4. Domain: $x \in \left(0,\ \frac{\pi}{2}\right) \cup \left(\pi,\ \frac{3\pi}{2}\right)$



Range: $y \in$ All Reals

y-int: None

Zeros:

$$\ln(\tan x) = 0$$

$$e^{\ln(\tan x)} = e^0$$

$$\tan x = 1$$

$$x = \begin{cases} \frac{\pi}{4} \pm 2\pi n \\ \frac{5\pi}{4} \pm 2\pi n \end{cases} = \left\{ \frac{\pi}{4} \pm \pi n \right.$$

$$\Rightarrow x = \frac{\pi}{4},\ \frac{5\pi}{4}$$

$$\left(\frac{\pi}{4},\ 0\right),\ \left(\frac{5\pi}{4},\ 0\right)$$

Extreme Points: None

VAs: $x = 0,\ x = \frac{\pi}{2},\ x = \pi,\ x = \frac{3\pi}{2}$

POEs: None

End Behavior: None