13-1 Free Response Homework

1. Answer: $2x^3 - x^2 + 3x + C$

$$\int \left(6x^2 - 2x + 3\right) dx$$
$$= 6\frac{x^3}{3} - 2\frac{x^2}{2} + 3x + c$$
$$= 2x^3 - x^2 + 3x + c$$

3. Answer: $3x^{2/3} + C$

$$\int \frac{2}{\sqrt[3]{x}}\, dx = \int 2x^{-1/3}\, dx$$
$$= 2\frac{x^{2/3}}{2/3} + c$$
$$= 3x^{2/3} + c$$

5. Answer: $\frac{2}{3}x^6 + \frac{5}{4}x^4 + C$

$$\int x^3\left(4x^2 + 5\right) dx$$
$$= \int \left(4x^5 + 5x^3\right) dx$$
$$= 4\frac{x^6}{6} + 5\frac{x^4}{4} + c$$
$$= \frac{2}{3}x^6 + \frac{5}{4}x^4 + c$$

7. Answer: $\frac{2}{3}x^{3/2} - 12x^{1/2} + C$

$$\int \left(\sqrt{x} - \frac{6}{\sqrt{x}}\right) dx$$
$$= \int \left(x^{1/2} - 6x^{-1/2}\right) dx$$
$$= \frac{x^{3/2}}{3/2} - 6\frac{x^{1/2}}{1/2} + c$$
$$= \frac{2}{3}x^{3/2} - 12x^{1/2} + c$$

9. Answer: $\frac{1}{4}x^4+x^3+\frac{3}{2}x^2+x+C$

$$\int (x+1)^3\,dx$$
$$=\int\left(x^3+3x^2+3x+1\right)\,dx$$
$$=\frac{x^4}{4}+3\frac{x^3}{3}+3\frac{x^2}{2}+x+c$$
$$=\frac{x^4}{4}+x^3+\frac{3}{2}x^2+x+c$$

11. Answer: $\frac{2}{3}x^{3/2}+\frac{6}{5}x^{5/2}-12x^{1/2}+C$

$$\int\left(\sqrt{x}+3\sqrt[2]{x^3}-\frac{6}{\sqrt{x}}\right)dx$$
$$=\int\left(x^{1/2}+3\,x^{3/2}-6x^{-1/2}\right)dx$$
$$=\frac{x^{3/2}}{3/2}+3\,\frac{x^{5/2}}{5/2}-6\frac{x^{1/2}}{1/2}+c$$
$$=\frac{2}{3}x^{3/2}+\frac{6}{5}\,x^{5/2}-12x^{1/2}+c$$

13. $f(x)=x^3-3x^2+3x+2$

$$f(x)=\int\left(3x^2-6x+3\right)dx$$
$$=x^3-3x^2+3x+c$$
$$f(0)=2=(0)^3-3(0)^2+3(0)+c$$
$$c=2$$

15. $f(x)=\frac{3}{2}x^2-\frac{10}{3}x^{3/2}-2x+\frac{32}{3}$

$f(x)=\int\left(\sqrt{x}-2\right)\left(3\sqrt{x}+1\right)\,dx$

$=\int\left(3x--5\sqrt{x}-2\right)\,dx$

$=\frac{3}{2}x^2-5\frac{x^{3/2}}{3/2}-2x+c$

$f(4)=1=\frac{3}{2}(1)^2-5\frac{(1)^{3/2}}{3/2}-2(1)+c$

$c=\frac{32}{3}$

17. $x(t)=\frac{1}{12}t^4-\frac{1}{3}t^3+2t^2+2t+4$

$v(t)=\int\left(t^2-2t+4\right)\,dt$

$=\frac{1}{3}t^3-t^2+4t+c_1$

$v(0)=2=\frac{1}{3}t^3-t^2+4t+c_1$

$c_1=2$

$x(t)=\int\left(\frac{1}{3}t^3-t^2+4t+2\right)\,dt$

$=\frac{1}{12}t^4-\frac{1}{3}t^3+2t^2+2t+c_2$

$x(0)=4=\frac{1}{12}t^4-\frac{1}{3}t^3+2t^2+2t+c_2$

$c_2=4$

<u>13-1 Multiple Choice Homework</u>

1. Answer: D

$\int\frac{1}{x^2}\,dx=\frac{x^{-1}}{-1}+c$

3. Answer: A

$$\int x\sqrt{3x}\ dx = \sqrt{3}\int x^{3/2}\ dx = \sqrt{3}\frac{x^{5/2}}{5/2} + c$$

13-2 Free Response Homework

1. Answer: $\frac{1}{20}(5x+3)^4 + C$

$u = 5x+3;\ du = 5dx$

$$\int (5x+3)^3\ dx$$

$$= \frac{1}{5}\int (5x+3)^3\ 5dx$$

$$= \frac{1}{5}\int u^3\ du$$

$$= \frac{1}{5}\frac{u^4}{4} + c$$

$$= \frac{1}{20}u^4 + c$$

3. Answer: $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$

$$\int \left(1 + 2x^3 + x^6\right) dx = x + 2\frac{x^4}{4} + \frac{x^7}{7} + c$$

5. Answer: $\frac{1}{6}\left(2x^2+3\right)^{3/2} + C$

$u = 2x^2+3;\ du = 4xdx$

$$\int \left(x\sqrt{2x^2+3}\right) dx = \frac{1}{2}\int \left(\sqrt{2x^2+3}\right) 2xdx$$

$$= \frac{1}{2}\int u^{1/2}\ du = \frac{1}{2}\frac{u^{3/2}}{3/2} + c = \frac{1}{3}\left(2x^2+3\right)^{3/2} + c$$

7. Answer: $\frac{1}{2}\left(1+x^4\right)^{1/2}+C$

$u=1+x^4;\ du=4x^3dx$

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx=\frac{1}{4}\int \frac{4x^3}{\sqrt{1+x^4}}\,dx$$

$$=\frac{1}{4}\int u^{-1/2}\,du=\frac{1}{4}\frac{u^{1/2}}{1/2}+c=\frac{1}{2}\sqrt{1+x^4}+c$$

9. Answer:

$\frac{1}{30}\left(x^2+4\right)^{15}-\frac{4}{7}\left(x^2+4\right)^{14}+\frac{16}{13}\left(x^2+4\right)^{13}+c$

$u=x^2+4;\ du=2xdx;\ x^2=u-4$

$$\int\left(x^5\left(x^2+4\right)^{12}\right)dx$$

$$=\frac{1}{2}\int\left(x^4\left(x^2+4\right)^{12}\right)2xdx$$

$$=\frac{1}{2}\int\left(u-4\right)^2u^{12}\,du$$

$$=\frac{1}{2}\int\left(u^2-8u-16\right)u^{12}\,du$$

$$=\frac{1}{2}\int\left(u^{14}-8u^{13}-16u^{12}\right)du$$

$$=\frac{1}{2}\frac{u^{15}}{15}-8\frac{u^{14}}{14}-16\frac{u^{13}}{13}+c$$

$$=\frac{1}{30}\left(x^2+4\right)^{15}-\frac{4}{7}\left(x^2+4\right)^{14}+\frac{16}{13}\left(x^2+4\right)^{13}+C$$

<u>13-2 Multiple Choice Homework</u>

1. Answer: C

$$\int \frac{x}{x^2-4}\,dx=\frac{1}{2}\int \frac{2x}{x^2-4}\,dx=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ln u+c$$

3. Answer: D

$$\int 60x\sqrt{1+x}\ dx = \int 60\left(u^2-1\right)u\ du = 60\int\left(u^3-u\right)\ du = 60\left(\frac{u^4}{4}-\frac{u^2}{2}\right)+c$$

5. Answer: E

$$\int x\left(x^2-1\right)^4 dx = \frac{1}{2}\int 2x\left(x^2-1\right)^4 dx = \frac{1}{2}\int u^4\,du = \frac{1}{2}\frac{u^5}{5}+c = \frac{1}{10}\left(x^2-1\right)^5+c$$

13-3 Free Response Homework

1. $\frac{1}{5}\sin x^5 + C$

$u = x^5;\ du = 5x^4dx$

$$\int\left(x^4\cos x^5\right)dx = \frac{1}{4}\int\left(\cos x^5\right)4x^4dx = \frac{1}{4}\int(\cos u)\,du = \frac{1}{4}\sin u + c$$

3. $\frac{1}{3}\tan(3x-1)+C$

$u = 3x-1;\ du = 3dx$

$$\int\left(\sec^2(3x-1)\right)dx = \frac{1}{3}\int\left(\sec^2(3x-1)\right)3dx = \frac{1}{3}\int\left(\sec^2 u\right)du = \frac{1}{3}\tan u + c$$

5. $\frac{1}{5}\tan^5 x + C$

$u = \tan x;\ du = \sec^2 x\ dx$

$$\int\left(\tan^4 x\sec^2 x\right)dx = \int u^4\ du = \frac{1}{5}u^5 + c$$

7. $\frac{1}{6}e^{6x} + C$

$u = 6x;\ du = 6dx$

$$\int\left(e^{6x}\right)dx = \frac{1}{6}\int\left(e^{6x}\right)6dx = \frac{1}{6}\int\left(e^u\right)du = \frac{1}{6}e^u + c$$

9. $\frac{1}{4}\left(\ln\left(x^2+1\right)\right)^2+C$

$u=\ln\left(x^2+1\right);\ du=\frac{2x}{x^2+1}dx$

$\int\frac{x\ln\left(x^2+1\right)}{x^2+1}\,dx=\frac{1}{2}\int\ln\left(x^2+1\right)\frac{2x}{x^2+1}\,dx=\frac{1}{2}\int u\ du=\frac{1}{2}\frac{u^2}{2}+c=\frac{1}{4}u^2+c$

11. $-\frac{2}{3}\cot^{3/2}x+C$

$u=\cot x;\ du=-\csc^2 x dx$

$\int\left(\sqrt{\cot x}\,\csc^2 x\right)dx=-\int\sqrt{\cot x}\left(-\csc^2 x\right)dx=-\int u^{1/2}\ du=-\frac{2}{3}u^{3/2}+c$

13-3 Multiple Choice Homework

1. Answer: A

$\int\left(x^3+2+\frac{1}{x^2+1}\right)dx=\int\left(x^3\right)\ dx+\int(2)\ dx+\int\left(\frac{1}{x^2+1}\right)\ dx=\frac{x^4}{4}+2x+\tan^{-1}x+c$

3. Answer: C

$\int\frac{x-2}{x-1}\ dx=\int\left(1+\frac{-1}{x-1}\right)\ dx=x-\ln\left(x-1\right)+c$

5. Answer: D

$u=\cos x;\ du=-\sin x dx$

$\int 6\sin x\cos^2 x\ dx=-6\int u^2\ du=-2u^3+c$

13-4 Free Response Homework

1. Answer: 24

$\int_0^3\left(x^2+5\right)\ dx=\left[\frac{x^3}{3}+5x\right]_0^3=\left(\frac{3^3}{3}+5(3)\right)-\left(\frac{0^3}{3}+5(0)\right)=24$

3. Answer: 0

$u = 1 - x^2;\ du = 2x\ dx$

$u(-1) = 0,\ u(1) = 0$

$$\int_{-1}^{1} \left(x\sqrt{1-x^2} \right)\ dx = \frac{1}{2}\int_{-1}^{1} \left(2x\sqrt{1-x^2} \right)\ dx = \frac{1}{2}\int_{0}^{0} u^{1/2}\ dx = 0$$

5. Answer: $-33\frac{1}{3}$

$$\int_{-2}^{2} (x+5)\left(x^2-3\right)\ dx = \int_{-2}^{2} \left(x^3 + 5x^2 - 3x - 15 \right)\ dx = \left[\frac{x^4}{4} + 5\frac{x^3}{3} - 3\frac{x^2}{2} - 15x \right]_{-2}^{2} = -33\frac{1}{3}$$

7. Answer: 0.070

$u = \cos x;\ du = -\sin x\ dx$

$$u\left(\frac{\pi}{2}\right) = 0,\ u\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\int_{\pi/6}^{\pi/2} \cos^5 x \sin x\ dx = -\int_{\sqrt{3}/2}^{0} u^5\, du = \left[-\frac{u^6}{6} \right]_{\sqrt{3}/2}^{0} = .070$$

9. Answer: 0.549

$$\frac{1}{2}\int_{-1}^{2} \frac{2dx}{2x+5} = \frac{1}{2}\ln\left|2x+5\right|\Big]_{-1}^{2} = \frac{1}{2}\left[\ln 9 - \ln 3\right] = \frac{1}{2}\ln 3 = 0.549$$

11. Answer: 0.693

$u = \ln x;\ du = \frac{1}{x}\ dx$

$u(2) = \ln 2,\ u(4) = \ln 4$

$$\int_{2}^{4} \frac{dx}{x\ \ln x} = \int_{\ln 2}^{\ln 4} \frac{du}{u} = \left[\ln u\right]_{\ln 2}^{\ln 4} = \ln\ln 4 - \ln\ln 2 = 0.693$$

13-4 Multiple Choice Homework

1. Answer: C

$$\int_2^6 \left(\frac{1}{x}+2x\right)\,dx = \left[\ln x + x^2\right]_2^6 = \left(\ln 6+36\right)-\left(\ln 2+4\right) = \ln 6-\ln 2+32 = \ln\frac{6}{2}+32$$

3. Answer: A

$u = \tan x;\ du = \sec^2 x\ dx$

$u\left(\frac{\pi}{4}\right)=1,\ u\left(\frac{\pi}{3}\right)=\sqrt{3}$

$$\int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{\tan x}\,dx = \int_1^{\sqrt{3}} \frac{du}{u} = \left[\ln u\right]_1^{\sqrt{3}} = \ln\sqrt{3}-\ln 1$$

5. Answer: D

$$\int_2^4 \left(f(x)+3\right)\,dx = \int_2^4 f(x)dx + 3\int_2^4 dx = 6+3x\Big]_2^4 = 6+\left(12-6\right) = 12$$

13-5 Free Response Homework

1. Answer: $\frac{8}{3}$

$$A = \int_{-1}^{1}\left(x^2+1\right)\,dx = \left[\frac{x^3}{3}+x\right]_{-1}^{1} = \left[\frac{1^3}{3}+1\right]-\left[\frac{(-1)^3}{3}+(-1)\right] = \frac{8}{3}$$

3. Answer: 4

$$A=\int_0^{2\pi}|\cos x|\,dx=\int_0^{\pi/2}\cos x\,dx-\int_{\pi/2}^{3\pi/2}\cos x\,dx+\int_{3\pi/2}^{2\pi}\cos x\,dx$$
$$=\left[\sin x\right]_0^{\pi/2}-\left[\sin x\right]_{\pi/2}^{3\pi/2}+\left[\sin x\right]_{3\pi/2}^{2\pi}$$
$$=\left[\sin\frac{\pi}{2}-\sin 0\right]_0^{\pi/2}-\left[\sin\frac{3\pi}{2}-\sin\frac{\pi}{2}\right]_{\pi/2}^{3\pi/2}+\left[\sin 2\pi-\sin\frac{3\pi}{2}\right]_{3\pi/2}^{2\pi}$$
$$=(1-0)-(-21-1)+\big(0-(-1)\big)$$
$$=4$$

5. Answer: 9

$$A=\int_{-3/2}^{3/2}\left|x\sqrt{9-4x^2}\right|dx=2\int_0^{3/2}x\sqrt{9-4x^2}\,dx=-\frac{1}{2}\int_0^{3/2}\sqrt{9-4x^2}\,(-4x\,dx)=-\frac{1}{2}\int_9^0\sqrt{u}\,du$$
$$=\left[-\frac{1}{2}\frac{u^{3/2}}{3/2}\right]_9^0=\left(0-\left(-\frac{1}{3}9^{3/2}\right)\right)=9$$

7. Answer: $4\sqrt{2}$

$$A=-\int_{-\pi}^0 y=3\sin x\sqrt{1-\cos x}\,dx=3\int_{-\pi}^0\sqrt{1-\cos x}\,(\sin x\,dx)=3\int_2^0\sqrt{u}\,du$$
$$=\left[3\frac{u^{3/2}}{3/2}\right]_2^0=2\left(0-\left(2^{3/2}\right)\right)=4\sqrt{2}$$

9a. Answer: $-\frac{3}{2}\text{m}$

$$\text{displacement}=\int_0^3(3t-5)dt=\left[\frac{3}{2}t^2-5t\right]_0^3=(13.5-15)-(0)=-1.5$$

9b. Answer: $\frac{41}{6}\text{m}$

$$\text{distance} = \int_0^3 |3t-5|\ dt = -\int_0^{5/3} (3t-5)\ dt + \int_{5/3}^3 (3t-5)\ dt$$

$$= -\left[\frac{3}{2}t^2 - 5t\right]_0^{5/3} + \left[\frac{3}{2}t^2 - 5t\right]_{5/3}^3 = \frac{41}{6}$$

11. Answer: 52 feet

$$\text{distance} = \int_1^5 (4t+1)dt = \left[2t^2 + t\right]_1^5 = (50+5) - (2+1) = 52$$

<u>13-5 Multiple Choice Homework</u>

1. Answer: C

$$\int_1^2 \left(4x^3 + 6x - \frac{1}{x}\right)dx = \left[x^4 + 3x^2 - \ln x\right]_1^2$$

$$= \left[2^4 + 3(2)^2 - \ln 2\right] - \left[1^4 + 3(1)^2 - \ln 1\right] = 24 - \ln 2 \text{ units}^2$$

3. Answer: D

$$a(t) = 12t^2$$

$$v(t) = \int 12t^2\ dt$$

$$= 4t^3 + c$$

$$v(0) = 6 = 4(0)^3 + c$$

$$c = 6$$

$$x(t) = \int_0^2 \left(4t^3 + 6\right)\ dt = \left[t^4 + 6t\right]_0^2 = 16 + 12 = 28$$

5. Answer: B

$$\int_0^{\sqrt{3}} \frac{x}{\sqrt{1+x^2}}\ dx = = \frac{1}{2}\int_1^4 \frac{2x}{\sqrt{1+x^2}}\ dx = \left[\frac{1}{2}\frac{\sqrt{u}}{1/2}\right]_1^4 = 2 - 1 = 1$$

Integrals Practice Test
Part 1: CALCULATOR REQUIRED

Multiple Choice (3 pts. each)

1. Answer: B

$$x(t) = 2 + \int_0^1 \left(3t^2 + 6t\right) dt = 2 + \left[t^3 + 3t^2\right]_0^1 = 6$$

2. Answer: B

$$\int_{-5}^{5} f(x)\, dx = \int_{-5}^{2} f(x)\, dx - \int_{5}^{2} f(x)\, dx = -17 - (-4) = -13$$

3. Answer: C

$$\int \frac{x^2}{e^{x^3}}\, dx = -\frac{1}{3}\int \frac{-3x^2}{e^{x^3}}\, dx = -\frac{1}{3}\int e^u\, du = -\frac{1}{3}e^u + c = -\frac{1}{3}e^{-x^3} + c$$

4. Answer: E

$$\int_0^x \sin t\, dt = \left[-\cos t\right]_0^x = -\cos x - (-\cos 0) = 1 - \cos x$$

5. Answer: B

$$\int_1^e \frac{x^2+1}{x}\, dx = \int_1^e \left(x + \frac{1}{x}\right) dx = \left[\frac{x^2}{2} + \ln x\right]_1^e = \left(\frac{e^2}{2} + 1\right) - \left(\frac{1}{2} - 0\right) = \frac{e^2}{2} + \frac{1}{2}$$

6. Answer: E

$$\int x\sqrt{4-x^2}\, dx = -\frac{1}{2}\int \sqrt{4-x^2}\,(-2x dx)$$

$$= -\frac{1}{2}\frac{\left(4-x^2\right)^{3/2}}{3/2} + c$$

$$= -\frac{1}{3}\left(4-x^2\right)^{3/2} + c$$

7. Answer: D

$$\int_{-2}^{2}\left(x^7+k\right)\,dx=16$$

$$\left[\frac{x^8}{8}+kx\right]_{-2}^{2}=16$$

$$\left(\frac{2^8}{8}+2k\right)-\left(\frac{(-2)^8}{8}-2k\right)=16$$

$$4k=16$$

$$k=4$$

8. Answer: D

$$\int\sin(2x+3)\,dx=\frac{1}{2}\int\sin(2x+3)\,(2dx)$$

$$=\frac{1}{2}\int\sin u\,du$$

$$=\frac{1}{2}(-\cos u)+c$$

$$=-\frac{1}{2}\cos(2x+3)+C$$

Integrals Practice Test
Part 2: NO CALCULATOR ALLOWED
Free Response

1. Answer: Area $= \frac{1}{2}$

$$A = \int_1^2 \frac{1}{x^2}\,dx = \left[\frac{-1}{x}\right]_1^2 = \frac{-1}{2} - (-1) = \frac{1}{2}$$

2. Answer: Area $= 2\ln 5$

$$A = \int_{-2}^{2} \left|\frac{x}{1+x^2}\right|\,dx = \frac{1}{2}\int_0^2 \left|\frac{2x}{1+x^2}\right|\,dx = \left[\ln\left(1+x^2\right)\right]_0^2 = 2\left(\ln 5 - 0\right) = 2\ln 5$$

3. Answer:

$$\text{Displacement} = \int_0^5 \left(t^2 - t\right)\,dt$$

$$\text{Total Distance Traveled} = -\int_0^1 \left(t^2 - t\right)\,dt + \int_1^5 \left(t^2 - t\right)\,dt$$