

# AP Calculus AB '19-20

## Practice Fall Final Part IIa

Calculator Required

Name:

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1. At time  $t = 0$ , there are 120 gallons of oil in a tank. During the time interval  $0 \leq t \leq 10$  hours, oil flows into the tank at a rate of  $h(t) = 10 - \frac{t \cos(t)}{2}$  and out of the tank at a rate given by  $g(t) = 6 + \frac{e^{0.52t}}{t+1}$ . Both  $h$  and  $g$  are measured in gallons per hour.

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a) How much oil flows out of the tank during this 10-hour time period?

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b) Find the value of  $h(4.3) - g(4.3)$ . Using correct units, explain what this value represents in the context of this problem.

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c) Write an expression for  $A(t)$ , the total amount of oil in the tank at time  $t$ .

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d) Find the absolute maximum and minimum amount of oil in the tank during  $0 \leq t \leq 10$  hours.

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2. A test plane flies in a straight line with positive velocity  $v(t)$ , in miles per hour at  $t$  minutes, where  $v(t)$  is a differentiable function of  $t$ . Selected values of  $v(t)$  are shown below:

$t$	0	5	10	15	20	25	30	35	40
$v(t)$	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

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a) Estimate  $v'(23)$ . Show the work that leads to your answer. Indicate the units.

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b) Use a midpoint Riemann sum to estimate the total distance traveled by the plane over the time interval  $0 < t < 40$ . Indicate the units.

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c) The function  $f$ , defined by  $f(t) = 6 + \cos\frac{t}{10} + 3\sin\frac{7t}{40}$ , is used to model the velocity of the plane. According to this model, what is the acceleration at  $t = 23$ . Indicate the units of measure.

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d) According to the model  $f$  given in part c), what is the average velocity of the plane, in miles per minute, over the time interval  $0 < t < 40$ .

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Part IIa

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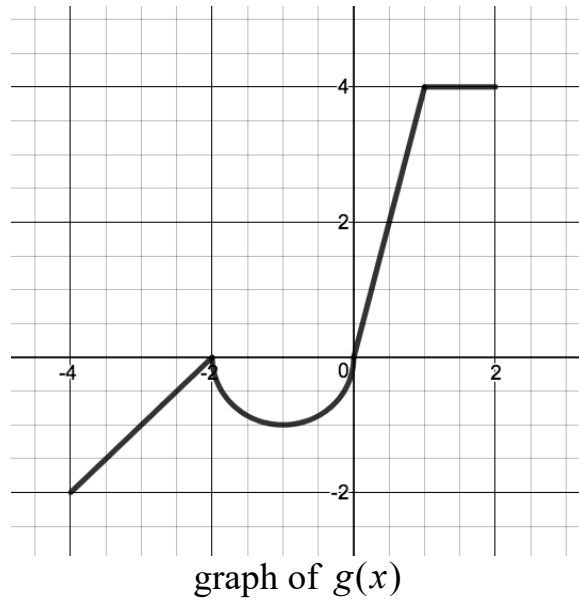
## Practice Fall Final Part IIb

No Calculator Allowed

Name:

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3. Below is the graph of  $g(x)$ , which is defined on  $-4 \leq x \leq 2$  and consists of three line segments and a semicircle. Let  $f(x) = \int_1^x g(t) dt$ .



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- a) Find  $f(-2)$ ,  $f'(-2)$ , and  $f''(-2)$

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- b) On what intervals, if any, is  $f(x)$  both decreasing and concave up? Justify your answer.
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c) Find the average rate of change of  $f(x)$  from  $x = -2$  to  $x = 2$ .

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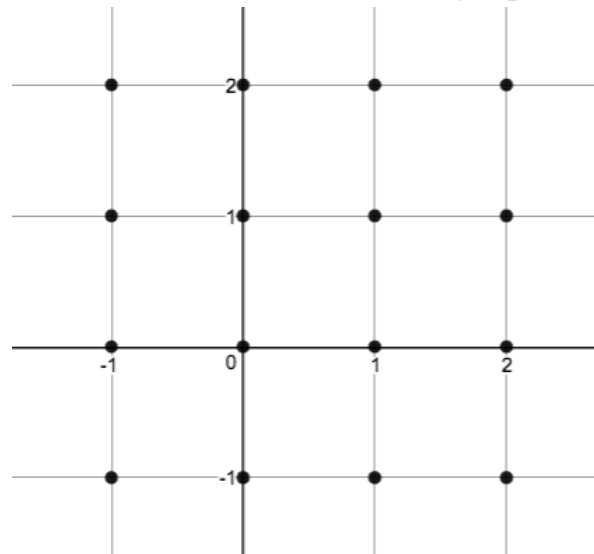
d) Identify all  $x$ -values in the open interval  $-4 < x < 2$  at which  $f(x)$  has a critical point, and classify each critical point as a local minimum, local maximum, or neither. Justify your answers.

4. A twice-differentiable function  $y = f(x)$  has derivative given by the differential equation  $\frac{dy}{dx} = \frac{x(y^2 - 1)}{y}$  and satisfies  $f(1) = -\sqrt{2}$ .

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a) Write the equation of the line **normal** to  $y = f(x)$  at the point  $(1, -\sqrt{2})$ .

b) Sketch the slope field for  $f(x)$  at the 16 integer points indicated.



c) Find the particular solution  $y = f(x)$  for which  $f(1) = -\sqrt{2}$ .

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