AP Calculus AB '19-20
Integral Practice Test

Name___________________________

Score____________

NO CALCULATOR ALLOWED

1. Find $\int_1^e \frac{1}{x}\,dx$

a. 1 b. e c. $\frac{1}{2}$ d. -1 e. -2

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2. $\int_0^{\frac{\pi}{3}} \sec x \tan x (1+\sec x)\,dx$

a. 4 b. $\frac{5}{2}$ c. 3 d. $\frac{9}{2}$ e. 5

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3. $\int_0^1 \frac{2}{1+x^2}\,dx$

a. $\frac{\pi}{2}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{6}$ d. 1 e. 2

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4. The average value of $y=\cos x(\sin(\sin x))$ on $x \in \left[0, \frac{\pi}{2}\right]$ is

a. 1 b. $\frac{2}{\pi}-\frac{1}{2}$ c. $\frac{2}{\pi}\cos 1$ d. $\frac{2}{\pi}-\frac{2}{\pi}\cos 1$ e. $\frac{1}{2}-\frac{2}{\pi}\cos 1$

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5. For $t \geq 0$ hours, H is a differentiable function of t that gives the change in temperature, in degrees Celsius per hour, at an Arctic weather station. Which of the following is the best interpretation of $\int_0^t H(x)dx$?

a) The change in temperature during the first t hours.
b) The change in temperature during the first day.
c) The average rate at which the temperature changed during the first t hours.
d) The rate at which the temperature is changing during the first day.
e) The rate at which the temperature is changing at the end of the 24th day.

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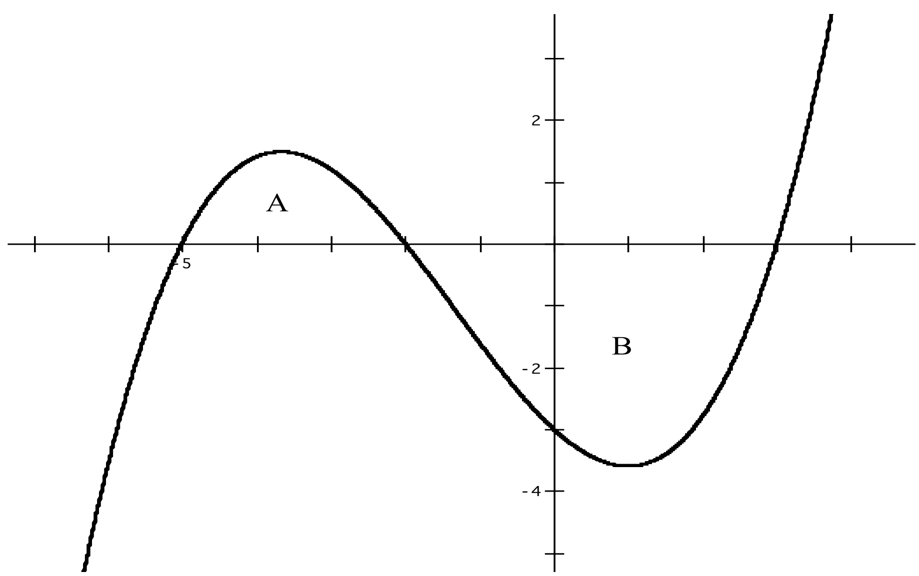
6. The graph of $y = f(x)$ is shown below. A and B are positive numbers that represent the areas between the curve and the x-axis.



In terms of A and B, $2\int_{-5}^{3} f(x)\, dx - \int_{-2}^{3} f(x)\, dx =$

a. A b. $A - B$ c. $2A - B$ d. $A + B$ e. $A + 2B$

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7. The following table lists the known values of a function $f(x)$.

x	1	2	3	4	5
$f(x)$	0	1.1	1.4	1.2	1.5

If the Trapezoidal Sum is used to approximate $\int_1^5 f(x)\,dx$, the result is

a) 3.7 b) 4.5 c) 4.6 d) 5.2

e) none of these

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CALCULATOR ALLOWED
Directions: Show all work.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is 55F°. The water is heated for 30 minutes, beginning at time $t=0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

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a. Use the data in the table to estimate $W'(t)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

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b. Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

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c. For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

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d. For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

2. A particle moves along the x-axis with velocity given by $v(t) = \frac{10\sin(0.4t^2)}{t^2 - t - 3}$ for time $0 \le t \le 3.5$. The particle is at position $x = -5$ at time $t = 0$.

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a) Find the acceleration of the particle at time $t = 3$.

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b) Find the position of the particle at time $t = 3$.

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c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

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d) A second particle moves along the *x*-axis with position given by $x_2 = t^2 - t$ for $0 \le t \le 3.5$. At what time *t* are the two particles moving with the same velocity?

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3. A tank at a sewage processing plant contains 125 gallons of raw sewage at

time $t = 0$. During the time interval $0 \le t \le 12$ hours, sewage is pumped into the tank at the rate $E(t) = 2 + \dfrac{10}{1 + \ln(t+1)}$. During the same time interval, sewage is pumped out at a rate of $L(t) = 12\sin\left(\dfrac{t^2}{47}\right)$.

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a) How many gallons of sewage are pumped into the tank during the time interval $0 \le t \le 12$ hours?

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b) Is the level of sewage rising or falling at $t = 6$? Explain your reasoning.

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c) How many gallons of sewage are in the tank at $t = 12$ hours?

d) At what time t, for $0 \le t \le 12$, is the volume of the sewage at an absolute maximum? Show the analysis that leads to your answer. If the sewage level ever exceeds 150 gallons, the tank overflows. Is there a time at which the tank overflows? Explain.