AP Calculus AB '19-20 Name___________________________

Integral Test

Score_____________

NO CALCULATOR ALLOWED

1. Find the average rate of change of $y = x^2 + 5x + 14$ on $x \in [-1, 2]$

a) 3 b) 6 c) 9 d) $\frac{65}{6}$ e) 18

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2. $\int_1^8 \frac{dx}{\left(1+\sqrt[3]{x}\right)^2 \sqrt[3]{x^2}}$

a) $-\frac{3}{2}$ b) $\frac{1}{2}$ c) $\frac{3}{2}$ d) $\ln 4$ e) $3\ln 4$

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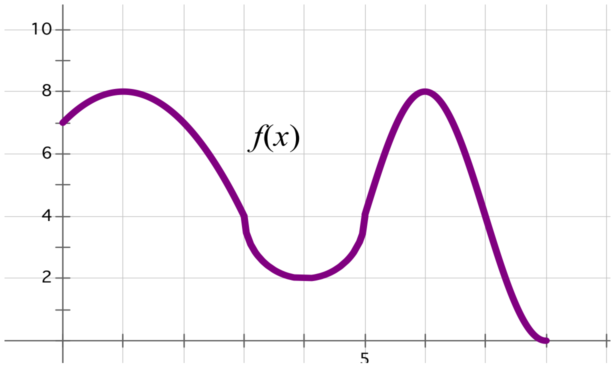
3. Consider the function f whose graph is shown below:



The approximate value of $\int_0^8 f(x)\,dx$, using eight right-hand rectangles with equal width, is

a) 18.5 b) 37 c) 40 d) 40.5 e) 44

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4. Let $R(t)$ represent the rate in gal/hr at which water is leaking out of a tank, where is measured in hours. Which of the following expressions represents the average rate of change of gallons of water per hour that leaks out in the first three hours?

a) $\int_0^3 R(t)\,dt$ b) $\frac{1}{3}\int_0^3 R(t)\,dt$ c) $\int_0^3 R'(t)\,dt$

d) $R(3)-R(0)$ e) $\frac{R(3)-R(0)}{3-0}$

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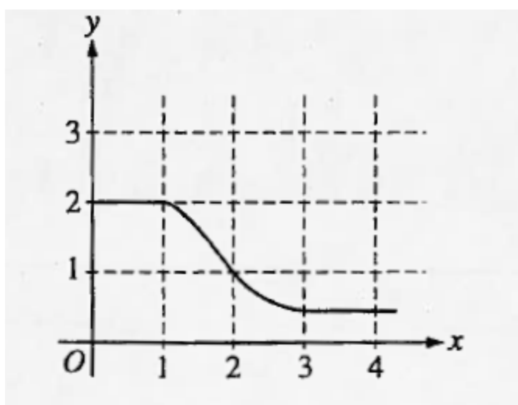
5. The cost, in dollars, to shred the confidential documents of a company is modeled by C, a differentiable function of the weight of documents in pounds. Of the following, which is the best interpretation of $C'(500) = 80$?

a) The cost to shred 500 pounds of documents is \$80.

b) The average cost to shred documents is $\frac{80}{500}$ dollar per pound.

c) Increasing the weight of documents by 500 pounds will increase the cost to shred the documents by approximately \$80.

d) The cost to shred documents is increasing at a rate of \$80 per pound when the weight of the documents is 500 pounds.



6. The graph of f is shown above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

a) 0.3 b) 1.3 c) 3.3 d) 4.3 e) 5.3

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7. A particle is moving along the x-axis and its velocity at time t is given by $v(t)=6-\frac{\ln t}{t}$. What is the total distance the particle travels between $t=1$ and $t=e$?

a) $6e-7$ b) $6e-1$ c) $6e+1$ d) $6e+6$ e) $e-7$

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t (in minutes)	0	8	16	24	32	40	48
$V(t)$ (in m^3/min)	26	32	43	24	19	24	30

8. Waste flows through a sewage pipe. The table above shows the rate of flow at specific times. Using the left Riemann rectangles, the approximate total volume of waste that flowed through the pipe over these 48 minutes is

a) $8\left[26+32+43+24+19+24+30\right]$

b) $8\left[26+32+43+24+19+24\right]$

c) $8\left[32+43+24+19+24+30\right]$

d) $4\left[26+32+43+24+19+24+30\right]$

e) $4\left[26+2(32)+2(43)+2(24)+2(19)+2(24)+30\right]$

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9. If $\int_{-5}^{2} f(x)\, dx = -17$ and $\int_{5}^{2} f(x)\, dx = -4$, then $\int_{-5}^{5} f(x)\, dx =$

a) -21 b) -13 c) 0 d) 13 e) 21

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CALCULATOR ALLOWED
Directions: Show all work.

Distance x (min)	0	1	5	6	8
Temperature T(t) (°C)	100	94	72	68	58

1. A metal wire of 8 cm is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius, of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

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a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate the units.

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b) Find $\int_0^8 T'(x)dx$ and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)dx$ in terms of the temperature of the wire.

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c) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature using a right-hand Riemann sum with four subintervals indicated by the data on the table. Indicate units of measure.

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2. For time $0 \le t \le 6$, a particle moves along the x-axis such that the velocity is determined by $v(t) = \frac{1}{2} e^{t/4} \cos\left(e^{t/4} \right)$. The particle is at position $x = 2$ at time $t = 0$.

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a) Find the average velocity of the particle for the time period $0 \le t \le 6$.

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b) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

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c) Is the velocity increasing or decreasing at time $t = 2$? Eplain your reasoning.

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d) For time $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

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3. A certain industrial chemical reaction produces synthetic oil at a rate of $S(t) = \frac{15t}{1+3t}$. At the same time, the oil is removed from the reaction vessel by a skimmer that has a rate of $R(t) = 2 + 5\sin\left(\frac{4\pi}{25}t\right)$. Both functions have units of gallons per hour, and the reaction runs from $t = 0$ to $t = 6$. At time of $t = 0$, the reaction vessel contains 2500 gallons of oil.

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a) How much oil will the skimmer remove from the reaction vessel in this six-hour period? Indicate units of measure.

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b) Write an expression for $P(t)$, the total number of gallons of oil in the reaction vessel at time t.

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c) Find the rate at which the total amount of oil is changing at $t = 4$.

d) For the interval indicated above, at what time t is the amount of oil in the reaction vessel at a minimum? What is the minimum value? Justify your answers.