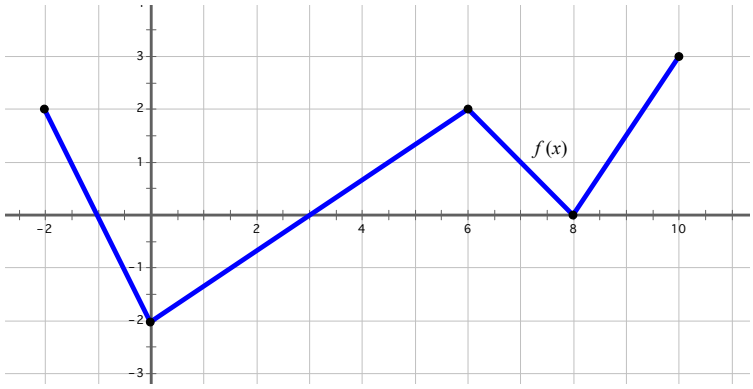
AP Calculus AB Name _____
Dr. Quattrin
2020 Spring Final #2A score _____



x	$g(x)$	$g'(x)$
-4	3	2
-2	5	1
0	7	0
2	5	-1
4	3	-2

1. Let h be the function defined by $h(x)=\sin(x)+e^{\cos 3x}$. Let $g(x)$ be a differentiable function with selected values for $g(x)$ and $g(x)$ given on the table above. And let $f(x)$ be the function whose graph is given above.

(a) Find the equation of the line tangent to $h(x)$ at $x=\frac{\pi}{2}$.

(b) Let K be the function defined by $K(x)=f(h(x))$. Find $K'\left(\frac{\pi}{2}\right)$.

(c) Let M be the function defined by $M(x) = g'(f(x)) \cdot f'(x)$. Find $\int_0^6 M(x)\,dx$.

(d) Let J be the function defined by $J(x) = g(2x) \cdot f(x)$. Find $J'(2)$.

2. Medieval alchemist Pol Maychrowitz believed that the Philosopher's Stone would help them to convert lead into gold. The Stone was never found, but, if it had been found and worked, Pol assumed he could convert 12 pounds of lead over a 72-hour period and that the conversion rate would follow a logistic growth curve $\frac{dG}{dt} = 1.2G\left(3 - \frac{G}{4}\right)$. (By the way, if he had succeeded, Pol would probably have been burned at the stake.)

a) If $G(0) = 4$, find the equation of the line tangent to $G(t)$. Use the tangent line to approximate $G(1)$.

b) How much gold would have been transmuted when the transformation way occurring the fastest?

c) Suppose Pol was incorrect and the actual growth rate followed the separable differential equation $\frac{dG}{dt}=1.2\left(3-\frac{G}{4}\right)$, instead of the logistic equation above. If $G(0)=1$, find the particular solution to the separable differential equation. Show the work that leads to your conclusion.