AB Calculus '19-20
Volume Test
Calculator Allowed

Name SOLUTION KEY

Score: ______________

1. The area of the region in the first quadrant bounded by the graphs of $y = 7\cos x$, $y = 7\sin x$ and the y-axis is

(a) $7\sqrt{2}$ (b) 14 (c) $7\sqrt{2}+1$ (d) $7\left(\sqrt{2}-1\right)$ (e) $\dfrac{7\sqrt{2}}{2}$

$A = \int_0^{\pi/4} (7\cos x - 7\sin x)\, dx$

2. The base of a solid is the region bounded by $y = 2\sqrt{x}$, the x-axis, and $x = 2$. Each cross-section of the solid perpendicular to the x-axis is a square, with one side on the xy-plane. Which of the following expressions represents the volume of the solid?

(a) $\int_0^2 \left(2\sqrt{x}\right) dx$ (b) $\int_0^2 (4x)\, dx$ (c) $\int_0^2 (2x)\, dx$

(d) $\int_0^1 \left(2\sqrt{x}\right) dx$ (e) $\int_0^1 (4x)\, dx$

$A = (2\sqrt{x})^2$

$y = 2\sqrt{x}$

3. Which of the following definite integrals gives the length of $y = e^{4x}$ on $0 \le x \le 2$?

(a) $\int_0^2 \sqrt{1+e^{8x}}\, dx$ (b) $\int_0^2 \sqrt{1+16e^{8x}}\, dx$ (c) $\int_0^2 \sqrt{x+16e^{8x}}\, dx$

(d) $\int_0^2 \sqrt{x+e^{8x}}\, dx$ (e) $\int_0^2 \sqrt{e^{4x}+16e^{8x}}\, dx$

$\dfrac{dy}{dx} = e^{4x}(4)$

$L = \int_0^2 \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$

4. A region is bounded by $y = \ln x$, the x-axis, and the line $x = e$. Order from smallest to largest the volumes of the solids formed by rotating the region about the following lines:

I. $y = 0$ II. $y = 1$ III. $y = 2$

$\pi\int_0^e (\ln x)^2 = 2.257$ $\pi\int_0^e 1-(1-\ln x)^2 = 4.027$ $\pi\int_0^2 4-(2-(\ln x))^2 = 11.310$

(a) III < II < I (b) III < I < II (c) II < I < III

(d) I < II < III (e) I < III < II

5. The region enclosed by the graphs of $y = x^3 - 1$ and $y = x - 1$ is revolved about the y-axis. The volume of this solid is

(a) 0.360 (b) 0.972 (c) 1.944 (d) 3.032 (e) 6.462

$V = \pi\int_{-1}^{1} \left(\sqrt[3]{y+1}\right)^2 - (y+1)^2\,dy = 6.462$

6. The region bounded by the following graph $y = 3\sin x$ and the x-axis on $0 \le x \le \frac{\pi}{2}$, is rotated about the line $y = -2$. The volume of the solid is represented by

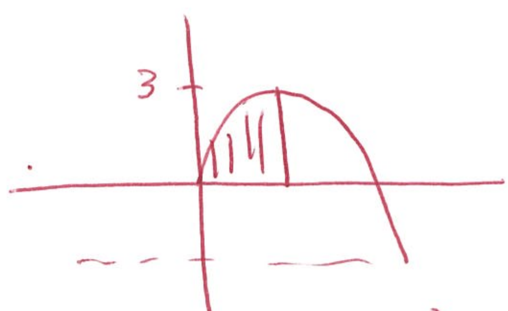
(a) $\pi\int_0^{\frac{\pi}{2}}\left((3\sin x + 2)^2 - 4\right)dx$

(b) $2\pi\int_0^{\frac{\pi}{2}}\left(9\sin^2 x + 2\right)dx$

(c) $\pi\int_0^{\frac{\pi}{2}}\left(9\sin^2 x - 4\right)dx$

(d) $2\pi\int_0^{\frac{\pi}{2}} 9\sin(x+2)^2\,dx$

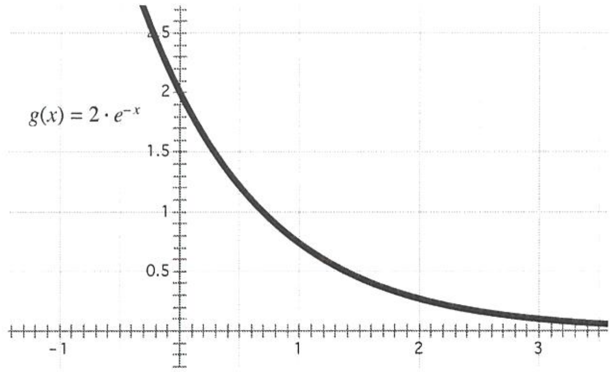
(e) $2\pi\int_0^{\frac{\pi}{2}}(3\sin x + 2)^2\,dx$



$R = 3\sin x - (-2)$

$r = 2$

7. Let R be the region in the first quadrant enclosed by the graph of $y = 2e^{-x}$ and the line $x = k$.



(a) Find the area of *R*. Show the integration steps.

$$A = \int_0^k 2e^{-x}\,dx$$

$$= -2e^{-x}\Big]_0^k$$

$$= -2e^{-k} - (-2) = 2 - 2e^{-k}$$

(b) Find the volume, in terms of *k*, of the solid generated when R is revolved about the *x*-axis. Show the integration steps.

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$$V = \pi \int_0^k (2e^{-x})^2\,dx = 4\pi \int_0^k e^{-2x}\,dx$$

$$= 4\pi \left(\frac{-e^{-2x}}{2}\right]_0^k$$

$$= 2\pi\left[-e^{-2k} - (-1)\right]$$

$$= \cancel{4\pi}\ 2\pi\left(1 - e^{-2k}\right)$$

(c) If $k = 3$, find the volume of the solid where the cross-sections perpendicular to the x-axis are squares. Show the integration steps.

$$\int_0^3 (2e^{-x})^2 \, dx$$

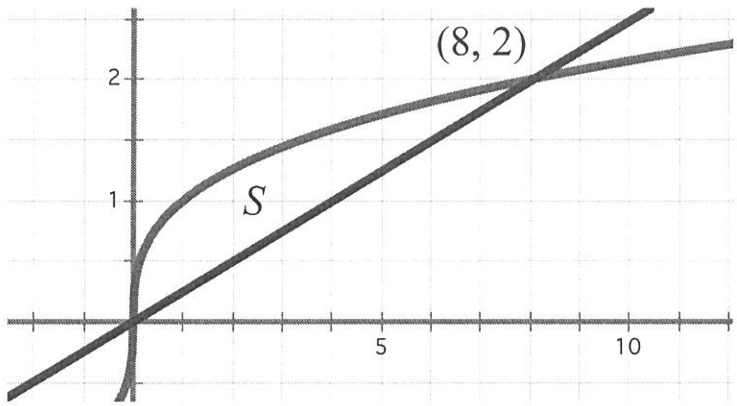
$$= 4\int_0^3 e^{-2x} \, dx$$

$$= 4\left[\frac{e^{-2x}}{-2}\right]_0^3$$

$$= -2e^{-6} - (-2)$$

$$= 1.995$$

8. Let S be the region in the first quadrant enclosed by the graph of $y=\sqrt[3]{x}$ and the line $y=\frac{1}{4}x$, as shown below.



a) Find the volume if the solid formed if S is rotated about the x-axis.

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$R = x^{1/3}$ $r = \frac{1}{4}x$

$$V = \pi \int_0^8 (x^{1/3})^2 - \left(\frac{1}{4}x\right)^2 dx =$$

$= 26.808$

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b) Find the volume if the solid formed if S is rotated about the line $y=3$.

$R = 3 - \frac{1}{4}x$ $r = 3 - (\sqrt[3]{x})$

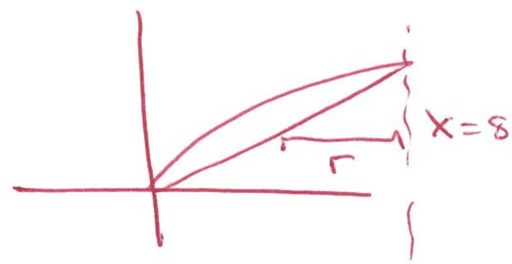
$$V = \pi \int_0^8 \left(3 - \frac{1}{4}x\right)^2 - (3 - x^{1/3})^2 dx$$

$= 48.590$

c) Set up, but do not solve, the equation for the volume of the solid formed if S is rotated about the line $x=8$.

$y=\sqrt[3]{x} \rightarrow x=y^3$

$y=\frac{1}{4}x \rightarrow x=4y$



$R=8-y^3$

$r=8-4y$

(4) $V=\pi\int_0^2 (8-y^3)^2-(8-4y)^2\,dy$