AP Calculus AB ’20-21
Integral Test v2

Name Solution Key

Score____________

NO CALCULATOR ALLOWED

1. Find $\int_3^6 x^2\,dx$ $= \left.\frac{x^3}{3}\right]_3^6 = \frac{216}{3} - 9 =$

a) 27 b) 63 c) 189 d) 81 e) 216

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2. $\int_e^{e^2} \frac{1}{x\ln x}\,dx = \int_1^2 \frac{1}{u}\,du = \ln|u|\Big]_1^2 = \ln 2$

$u = \ln x \quad du = \frac{1}{x}\,dx$

a) $\ln(\ln(2))$ b) $\frac{2}{e^2}$ c) $\ln 2$

d) $\frac{1-2e}{2e^2}$ e) DNE

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3. If $\int_{-3}^{-1} g(x)\,dx = -19$ and $\int_5^{-1} g(x)\,dx = -14$, then $\int_{-3}^{5} g(x)\,dx =$

a) -33 b) -5 c) 0 d) 5 e) 33

$\int_{-3}^{5} = \int_{-3}^{-1} + \int_{-1}^{5} = -19 + 14 = -5$

8. Find the average ~~rate of change~~ VALUE of $y = x^2 + 5x + 14$ on $x \in [-1, 2]$

a) 3 b) 6 c) 9 d) $\frac{65}{6}$ e) 20

17.5 52.5

$\frac{1}{2-(-1)} \int_{-1}^{2} (x^2+5x+14)\,dx$

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t (in minutes)	0	8	16	24	32	40	48
$V(t)$ (in m^3/min)	26	32	43	24	19	24	30

9. Waste flows through a sewage pipe. The table above shows the rate of flow at specific times. Using the right-hand rectangles, the approximate total volume of waste that flowed through the pipe over these 48 minutes is

a) $8[26+32+43+24+19+24+30]$

b) $8[26+32+43+24+19+24]$

c) $8[32+43+24+19+24+30]$

d) $4[26+32+43+24+19+24+30]$

e) $4[26+2(32)+2(43)+2(24)+2(19)+2(24)+30]$

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d

CALCULATOR ALLOWED
Directions: Show all work.

t months	0	1	2	3	4	5	6
$P(t)$	160.3	192.8	345.7	746.1	944.2	873.0	1128.6

t months	7	8	9	10	11	12
$P(t)$	928.3	851.3	751.3	535.5	216.4	150.7

1. A family leases solar panels on their house. At the end of the year, they receive a report, including the tables above, which shows the monthly production $P(t)$ of electricity, in kilowatts per month (kW/month), from the panels.

3

a) Use right-hand Riemann rectangles to approximate $\int_0^{12} P(t)\,dt$. Indicate the units.

$\approx (1)\,192.8 + (1)\,345.7 + (1)\,746.1 + (1)\,944.2 + (1)\,873 + (1)\,1128.6 + (1)\,928.3$

$+ (1)\,851.3 + (1)\,751.3 + (1)\,216.4 + (1)\,150.7 = $ KW

6278.4 KW

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2. A ski resort uses a snow machine to control the snow level on a ski slope. Over a 24-hour period the volume of snow added to the slope per hour is modeled by the equation $S(t) = 24 - t\sin^2\left(\frac{t}{14}\right)$. The rate that the snow melts is modeled by $M(t) = 10 + 8\cos\left(\frac{t}{3}\right)$. Both $M(t)$ and $S(t)$ are measured in $\frac{yd^3}{h}$ and t is measured in hours for $0 \le t \le 24$. At time t = 0 the slope holds $50yd^3$ of snow.

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a. How much snow has fallen on the mountain over the first 6 hours. Include the units.

$\int_0^6 S(t)\,dt = 142.413$

1

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b. Is the volume of the snow on the mountain increasing or decreasing at t = 4? Justify your answer.

$A = 50 + \int_0^t S(x) - M(x)\,dx$

2 $A' = S(4) - M(4) = 15.464$

$\therefore$ THE VOLUME IS INCREASING

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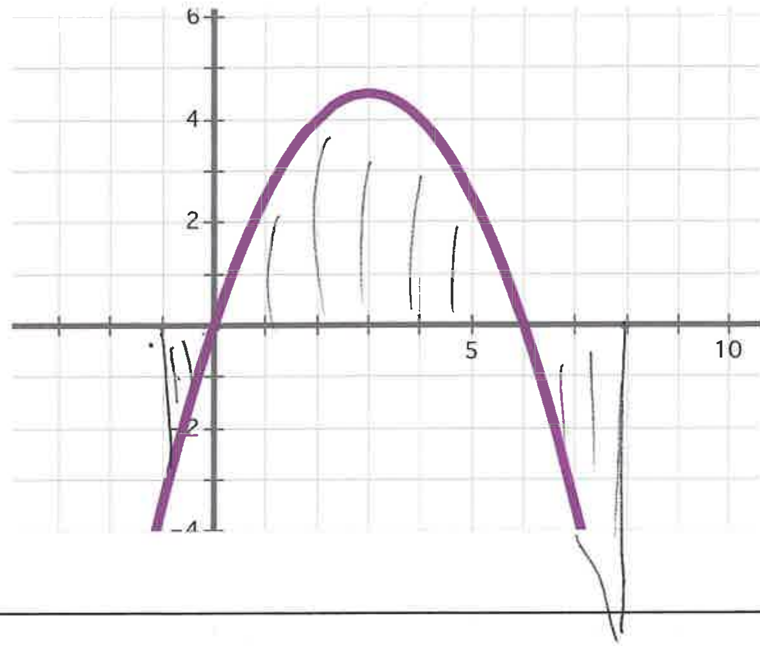
3. Consider the function $g(x)=3x-\frac{1}{2}x^2$ which has the graph below.



a) Find the zeros of $g(x)$. Show the work that leads to your answer.

(1)

$x = 0, 6$

$3x - \frac{1}{2}x^2 = 0$

$x\left(3 - \frac{1}{2}x\right) = 0$

b) Find the exact value of $\int_{-1}^{8.} g(x)\,dx$. Show the antiderivative and boundary

(3)

$\int_{-1}^{9}\left(3x - \frac{1}{2}x^2\right) dx \approx 9$

$= \left.\frac{3x^2}{2} - \frac{x^3}{6}\right]_{-1}^{8} = \left[48 + \frac{516}{6}\right] - \left[\left(\frac{3}{2} - \left(\frac{-1}{6}\right)\right)\right]$