AP Calculus AB '20-21
Integral Test v3

Name Solution Key

Score____________

NO CALCULATOR ALLOWED

1. Find $\int_2^4 x\,dx = \left.\frac{x^2}{2}\right]_2^4 = 8 - 2$

a) 2 b) 6 c) 12 d) 10 e) 16

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2. $-\int_0^{\ln 3} \frac{-e^x}{(1-e^x)^2+4}dx = -\int_1^3 \frac{1}{u^2+4}\,du = -\frac{1}{2}\tan^{-1}\left.\frac{u}{2}\right]_0^{-2}$

$= -\frac{1}{2}\tan^{-1}(-1) - 0$

$= \frac{1}{2}\left(\frac{\pi}{4}\right)$

a) $\frac{\pi}{8}$

b) $\frac{\pi}{4}$

c) $\ln 3$

d) $-\frac{1}{2}\tan^{-1}(\ln 3)$

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3. Given that $\int_2^3 P(t)\,dt = 7$ and $\int_2^7 P(t)\,dt = -2$, what is $\int_7^3 P(t)\,dt =$

a) -9 b) -5 c) 5 d) 9

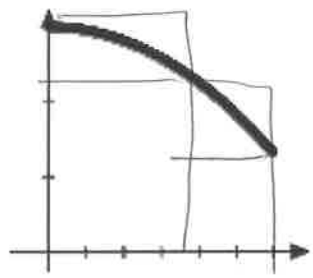
e) not enough information

$\int_2^7 = \int_2^3 + \int_3^7 = 7 - \boxed{\int_7^3 P(t)\,dt}$

$= -2$

$7 + 2 = \int_7^3$

4. The graph of the function f is shown below for $0 \leq x \leq 3$.



Of the following, which has the smallest value?

a) $\int_1^3 f(x)dx$

b) Left Riemann sum approximation of $\int_1^3 f(x)dx$ with 6 equal sub intervals.

c) Right Riemann sum approximation of $\int_1^3 f(x)dx$ with 6 equal sub intervals.

d) Midpoint Riemann sum approximation of $\int_1^3 f(x)dx$ with 6 equal sub intervals.

e) Trapezoidal sum approximation of $\int_1^3 f(x)dx$ with 6 equal sub intervals.

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5. $\int_2^3 \frac{1}{x\sqrt{x^2-9}}\,dx = \frac{1}{3}\sec^{-1}\frac{x}{3}\Big]_2^3 = \frac{1}{3}\sec^{-1}1 - \frac{1}{3}\sec^{-1}\frac{2}{3}$

a) $\frac{1}{3}\sec^{-1}\left(\frac{2}{3}\right)-1$

b) $1-\frac{1}{3}\sec^{-1}\left(\frac{2}{3}\right)$

c) $-\frac{1}{3}\sec^{-1}\left(\frac{2}{3}\right)$

d) $\frac{\pi}{2}-\frac{1}{3}\sec^{-1}\left(\frac{2}{3}\right)$

e) $\frac{1}{3}\sec^{-1}\left(\frac{2}{3}\right)-\frac{\pi}{2}$

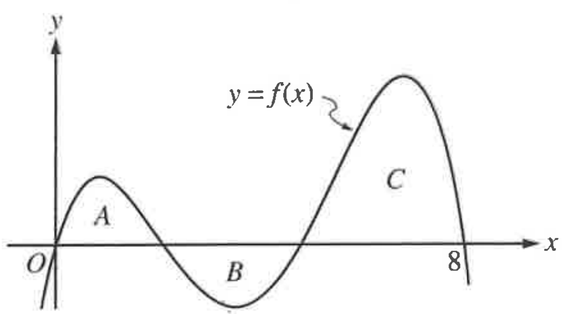
$A = \int_0^t (P - R)\, dt$ $A' = P - R = 0$

6. Starting at $t = 0$ hours, water is added to an empty tank at a rate of $P(t) = 4t\sin\left(\frac{t}{4}\right)$ gallons per hour. At the same time, water is pumped out of the tank at a rate of $R(t) = t^2 - 5t$ gallons per hour. On the interval $0 < t < 9$ hours, what is the maximum amount of water in the tank?

a) 8.435 gallons b) 19.006 gallons c) 29.115 gallons

8.435 d) 99.753 gallons e) 101.937 gallons

$\int_0^{8.435} P(x) - R(x)\, dx = 101.937$



$\int_0^8 = 14 - 16 + 50 = 48$

7. The regions *A*, *B*, and *C* in the figure above are bounded by the graph of the function *f* and the *x*-axis. The area of region *A* is 14, the area of region *B* is 16, and the area of region *C* is 50. What is the average value of *f* on the interval [0, 8] ?

$\frac{1}{b-a}\int_a^b f(x)\, dx$ $\frac{48}{8}$

a) 6 b) 10 c) $\frac{40}{3}$ d) $\frac{80}{3}$ e) 48

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8. The average value of $y = e^x \cos x$ on $x \in \left[0, \frac{\pi}{2}\right]$ is $\frac{1}{\pi/2}\int_0^{\pi/2} e^x \cos x \, dx =$

a) 0 b) 1.213 c) 1.905 d) 2.425 e) 3.810

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t (in minutes)	0	8	16	24	32	40	48
$V(t)$ (in m^3/min)	26	32	43	24	19	24	30

9. Waste flows through a sewage pipe. The table above shows the rate of flow at specific times. Using the trapezoids, the approximate total volume of waste that flowed through the pipe over these 48 minutes is

a) $8\left[26+32+43+24+19+24+30\right]$

b) $8\left[26+32+43+24+19+24\right]$

c) $8\left[32+43+24+19+24+30\right]$

d) $4\left[26+32+43+24+19+24+30\right]$

e) $4\left[26+2(32)+2(43)+2(24)+2(19)+2(24)+30\right]$

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Score 27

CALCULATOR ALLOWED
Directions: Show all work.

1. Dr. Quattrin decides to lease solar panels from Sunrun Solar. After a year, he reanalyzes his PG&E bill to track both his consumption of electricity ($C_e(t)$) and his production of electricity ($P_e(t)$) over the course of a year. The tables below show the consumption of electricity, measured in kilowatts (kWs).

t months	0	1	2	3	4	5	6
$C_e(t)$	326.5	660.0	667.1	538.4	420.5	412.1	347.8

t months	7	8	9	10	11	12
$C_e(t)$	287.5	303.1	322.4	342.5	390.3	384.2

$P_e(t) = 407 - 374.2\cos\frac{\pi}{6}t$ models the production in kW per month that PG&E buys back.

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a) How much power does PG&E buy back from the Quattrins over the course of the year? Indicate the units.

1 $\int_0^{12} P_e(t)\,dt = 4854$ kW

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b) Using the trapezoidal sum, approximate the amount of power the Quattrins consume over the course of the year. Based on the estimates to (a) and (b), does Dr. Quattrin owe PG&E for electricity at the end of the year or does PG&E owe Dr. Quattrin a refund?

$\int \approx$ ~~5046.55~~ ~~5024.35~~ kW 5099.55

3

$\int_0^{12} C(t)\,dt > \int_0^{12} P(t)\,dt \therefore$ Dr Q owes PG&E

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c) Electricity costs \$0.28 per kW. Write an expression for amount due on the PG&E bill at time *t* months.

1

$E(t) = 0.28 \int_0^{t} [C_e(x) - P_e(x)]\,dt$

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d) Assume $C_e(t)$ can be modeled by $M_e(t) = 468 + 200\cos\frac{\pi}{6}(t-1)$ What is the ~~minimum~~ maximum due on the PG&E bill during the year?

4

$\underline{E' = 0.28\,M(t) - .28\,P(t)} = 0 \rightarrow$

CV: $t = 0,\ 3.555,\ 9.135,\ 12$

t	E(t)
0	0
3.555	-409.87
9.135	86.17
12	-204.96

MAXIMUM DUE is \$86.17

2. On Ocean Beach at the foot of Noriega is a shipwreck buried in the sand. In 1858, the clipper King Philip ran aground. It was stripped of salvageable material, but the 45% of its hull is still buried on the beach, and it appears every few years as the tide washes the sand in and out.

As the sand washes in or is added by the National Park Service, the height changes at a rate of

$$A(t)=\frac{7t}{1+2t},$$

and, as the tide washes sand out, the height changes at a rate of

$$B(t)=2+8\cos\left(\frac{\pi}{15}x\right)\sin\left(\frac{3\pi}{16}x\right)$$

Both $A(t)$ and $B(t)$ are both measured in in inches above the wreck per year for $0\le t\le 10$. At $t=0$, the height is 15 inches above the wreck.

a. Find the total inches of sand above the wreck which are washed out in the first five years. Indicate the correct units.

2

$\int_0^5 B(t)\,dt = 13.304$ INCHES ABOVE THE WRECK

b. Is the rate of change of the height increasing or decreasing at $t = 6$ years? Justify your answer.

$A'(6) - B'(6) = 0.777$ in/year²

SINCE $A'(6) - B'(6) > 0$, THE RATE OF CHANGE OF THE HEIGHT OF THE SAND ABOVE THE WRECK IS INCREASING AT $t = 6$ YEARS.

2

c. Write an expression for $H(t)$, the total number of inches above the wreck at time t.

1

$H(t) = 15 + \int_0^t A(x) - B(x)\,dx$

d. What is the absolute minimum number of inches that the sand is above (below) the wreck over the course of the ten years described?

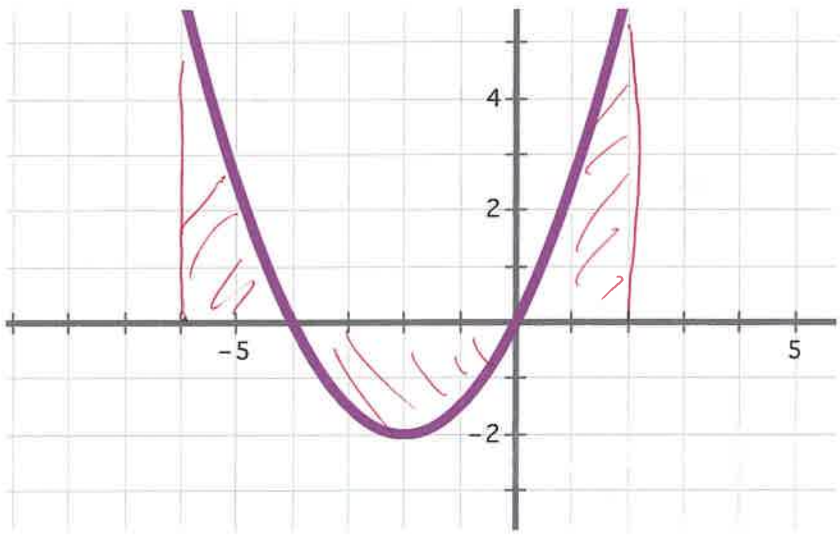
4

$H' = A(t) - B(t) = 0 \rightarrow t = 4.854,\ 8.292$

t	$H(t)$
0	15
4.854	-3.963
8.292	1.063
10	.112

THE MINIMUM LEVEL OF THE SAND ABOVE THE WRECK IS -3.963. IN OTHER WORDS, THE LOWEST LEVEL OF SAND IS 3.963 INCHES BELOW THE TOP OF THE WRECK.

3. Consider the function $g(x)=\frac{1}{2}x^2+2x$ which has the graph below.



a) Find the zeros of $g(x)$. Show the work that leads to your answer.

1

$\frac{1}{2}x^2+2x=0$

$\frac{1}{2}x(x+4)=0$ $x=0,-4$

b) Find the exact value of $\int_{-6}^{2} g(x)\,dx$. Show the antiderivative and boundary insertion steps.

$$\int_{-6}^{2} \left(\frac{1}{2}x^2 + 2x\right) dx = \left[\frac{x^3}{6} + x^2\right]_{-6}^{2}$$

3

$$\left(\frac{4}{3} + 4\right) - (-36 + 36)$$

$$= \frac{16}{3}$$

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c) Find the exact area between the x-axis and $g(x)$ on $x \in [-6, 2]$. Show the antiderivative and boundary insertion steps.

5

$$A = \int_{-6}^{-4} g(x)\,dx - \int_{-4}^{0} g(x)\,dx + \int_{0}^{2} g(x)\,dx$$

$$= \left[\frac{x^3}{6} + x^2\right]_{-6}^{-4} - \left[\frac{x^3}{6} + x^2\right]_{-4}^{0} + \left[\frac{x^3}{6} + x^2\right]_{0}^{2}$$

$$= \frac{16}{3} - \left(-\frac{16}{3}\right) + \frac{16}{3}$$

$$= 16$$
