AP Calculus AB '20-21 Name Solution Key
Dr. Quattrin
Derivative Test v1

Part I: Multiple choice – Circle correct answer.

1. If $f(x)$ is a differentiable function where $f(2)=1$ and the tangent line approximation at $x=2$ for $f(2.1)$ is 0.7, what is $f'(2)$?

$y-1=m(x-2)$

$m=\frac{.7-1}{2.1-2}=\frac{-.3}{.1}$

(A) 0.7 (B) −3 (C) 0.3 (D) 7 (E) −2

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2. If $f(x)=\tan^{-1}(\cos x)$, then $f'(x)=$ $\frac{1}{1+\cos^2 x}(-\sin x)$

a) $\sec^{-2}(\cos x)$ b) $-\sin x\sec^{-2}(\cos x)$ c) $-\csc x$

d) $\frac{-\cos x}{1-\sin^2 x}$ e) $\frac{-\sin x}{\cos^2 x+1}$

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3. Which of the following statements must be **true**?

I. $\frac{d}{dx}\left(x^3+4x^2-\sqrt[3]{x^2}-\frac{1}{7x}\right)=3x^2+8x-\frac{3}{2}\cancel{x^{1/2}}+\frac{1}{7x^2}$

II. $\frac{d}{dx}e^{\csc x}=e^{\csc x}(-\csc^2 x)$

III. $\frac{d}{dx}\ln\left(1-x^3\right)=\frac{-3x^2}{1-x^3}$

a) I only b) III only c) II and III only

d) I and III only e) None of these

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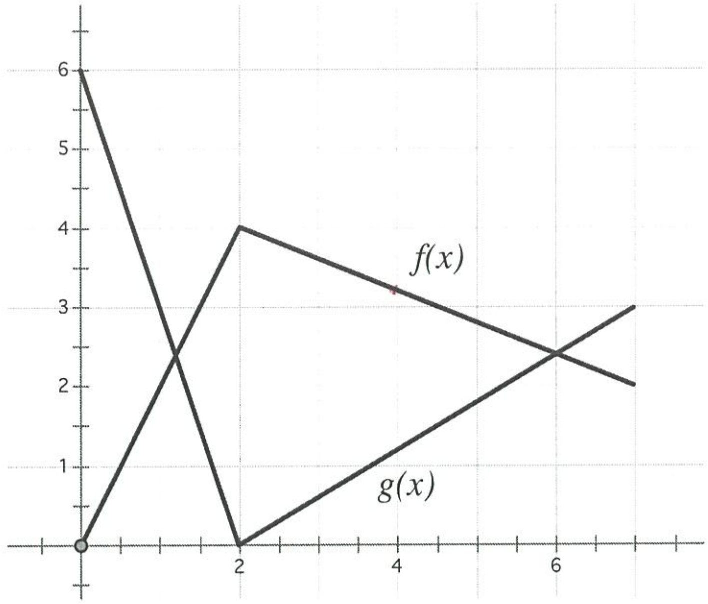
x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–3	7	–7	–6	7
–2	1	–5	0	5
–1	–3	–3	4	3
0	–5	–1	6	1
1	–5	1	6	–1
2	–3	3	4	–3
3	1	5	0	5

4. Given the table above, find $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$, when $x = 0$.

a) $\frac{7}{36}$ b) $\frac{5}{36}$ c) $\frac{5}{36}$ d) $\frac{1}{36}$ e) $-\frac{1}{36}$

$$\frac{g(0)\cdot f'(0) - f(0)\, g'(0}{[g(0)]^2} = \frac{6(-1) - (-5)(1)}{6^2}$$

$$= \frac{-1}{36}$$



5. The graphs of $f(x)$ and $g(x)$ are shown above. If $h(x)=g(f(x))$, then $h'(4)=$

a) $-\frac{2}{5}$ b) $\frac{-6}{25}$ c) $\frac{6}{5}$ d) $\frac{14}{25}$

e) does not exist

$g'(f(4)) \cdot f'(4)$

$= g'(3.2) \cdot f'(4)$

$= \frac{3}{5}\left(\frac{-2}{5}\right) = \frac{-6}{25}$

6. If $\frac{d}{dx}[f(x)] = g(x)$ and if $h(x) = x^3$, then $\frac{d}{dx}[h(f(x))] =$

$h'(f(x)) \cdot f'(x)$

$3(f(x))^2 \cdot g(x)$

a) $f(x^3)$

b) $3x^2 g(x^3) + x^3 f(x^3)$

c) $x^3 g(x^3)$

d) $3[f(x)]^2 g(x)$

e) $3x^2 g(x^3)$

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7. If $y = e^{x^2}$, then $\frac{d^2y}{dx^2} =$

a) e^{x^2} b) $2e^{x^2}(2x^2+1)$ c) $2xe^{x^2}$

d) $4x^2e^{x^2}$ e) $2e^{x^2}(2x^2-1)$

$y' = 2x e^{x^2}$

$y'' = 2x e^{x^2}(2x) + e^{x^2}(2)$

$= 2e^{x^2}(2x^2+1)$

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Part II: Free Response – Show all work.

1a. $\frac{d}{dx}\left(\cot^{-1}\left(e^{2x}\right)\right) = \frac{-1}{1+e^{4x}} e^{2x} = \frac{-e^{2x}}{1+e^{4x}}$

b. $\frac{d}{dx}\left(3\cos\left(x^2+2x\right)\right) = -3\sin(x^2+2x)(2x+2)$

$= -6(x+1)\sin(x^2+2x)$

c. $\frac{d}{dx}\left[-4x^5+8^x-\frac{7}{5}\sqrt[4]{x^5}-\frac{3}{\sqrt[7]{x^6}}-\frac{1}{2x}\right]$

$= \frac{d}{dx}\left[-4x^5+8^x-\frac{7}{5}x^{5/4}-3x^{-6/7}-\frac{1}{2}x^{-1}\right]$

$= -20x^4+8^x\ln 8-\frac{7}{4}x^{1/4}-\frac{18}{7}x+\frac{1}{2}x^{-2}$

d. $\frac{d}{dx}\left(\frac{3x}{15+x^2}\right) = \frac{(15+x^2)(3)-(3x)(2x)}{(15+x^2)^2}$

$= \frac{-3x^2+45}{(15+x^2)^2}$

2. $\frac{d}{dx}\left(e^{-2x}\cos x\right)$

$= e^{-2x}(-\sin x) + \cos x \, e^{-2x}(-2)$

$= -e^{-2x}(\sin x + 2\cos x)$

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3. If $g(x) = \ln\left(9 - x^2\right)$, find $g''(x)$

$g' = \dfrac{-2x}{9-x^2}$

$g'' = \dfrac{(9-x^2)(-2) - (-2x)(-2x)}{(9-x^2)^2}$

$= \dfrac{-2x^2 - 18}{(9-x^2)^2}$

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4. Find $\frac{dy}{dx}$ in factored form if $y = 3\sin^{-1}\left(\frac{x}{3}\right) + \sqrt{9 - x^2}$.

$$= 3 \frac{1}{\sqrt{1 - x^2/9}} \left(\frac{1}{3}\right) + \frac{1\,(-2x)}{2(9-x^2)^{1/2}}$$

$$= \frac{3}{(9 - x^2)^{1/2}} + \frac{-\cancel{2}x}{\cancel{2}(9-x^2)^{1/2}}$$

$$= \frac{3 - x}{(9-x)^{1/2}}$$

5. Find the equations of the lines tangent and normal $y = \frac{-3x}{x^2+1}$ at $x = -1$.

$$y(-1) = \frac{-4}{2} = -2$$

$$y'(-1) = \frac{((-1)^2+1)(-3) - (3)(-2)}{((-1)^2+1)^2}$$

$$= \frac{3}{2}$$

TAN $y + 2 = \frac{3}{2}(x+1)$

Norm: $y + 2 = -\frac{2}{3}(x+1)$