

AB Calculus '20-21

Name _____

Dx Apps II Test v1

Calculator allowed

Score _____

Directions: Show all work.

t in hours	0	12	24	36	48
$v(t)$ in km/sec	21	26.3	31.4	36.8	41.5

1. A Gravitational Slingshot Effect is sometimes used by space probes like Voyager 2 in order to increase its velocity without expending fuel. By flying close to the planet Saturn in a parabolic arc, the velocities on the table above were achieved by a probe. (In the original *Star Trek* episode "Tomorrow is Yesterday," the Enterprise used this effect around a black hole to time-travel to 1967.)

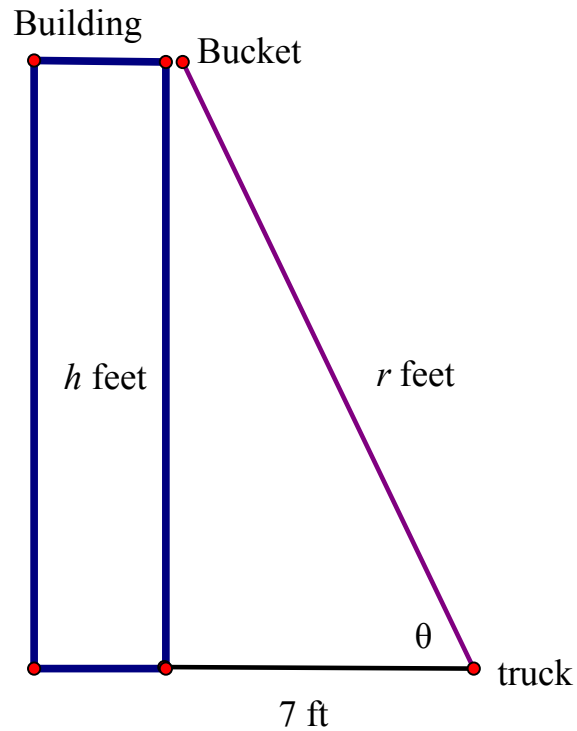
a. Approximate the probe's acceleration at $t = 30$.

b. Use a trapezoidal approximation for $\int_0^{48} v(t) dt$. Using the correct units, explain the meaning of this result.

c. Using your answer in b), approximate the average velocity of the probe between $t=0$ and $t=48$? Indicate the correct units.

d) The data on the table can be approximated by the equation $v(t) = 0.000027x^2 + 0.4396x + 21$. Based on this equation, find the total distance traveled by the probe between $t=0$ and $t=48$ hours. Indicate the units.

2. A fire truck is parked 7 feet away from the base of a building and its ladder is extended to the top of the building. The ladder retracts at a rate of 0.5 feet per second, while the angle of the ladder changes such that the bucket at the end of the ladder comes down vertically.



a) How far is the ladder extended when the bucket is 10 feet above the ground?

b) Find the rate at which the bucket is dropping vertically when the bucket is 10 feet above the ground.

c) What is the relationship between the angle θ and the height of the bucket? Find θ , in radians, when the bucket is 10 feet above the ground.

d) Find the rate, in radians per second, at which the angle the ladder forms with the ground is changing when the bucket is 10 feet above the ground.

3. A research team is studying a group of monkeys living on Monkey Island in Cambodia. When they begin observing the monkeys ($t = 0$), there are 20 monkeys on the island. The researchers determine that the population P grows logistically at a rate of $\frac{dP}{dt} = 3P - \frac{P^2}{40}$ monkeys per year.

a) According to this logistic model, what is the maximum population of monkeys on the island?

b) Further research shows the growth to be a bounded exponential function rather than a logistic growth function. Assuming a new model of $\frac{dM}{dt} = \frac{1}{40}(130 - M)$, find the particular solution to the differential equation, given that $M(0) = 20$.

c) Find $\lim_{t \rightarrow \infty} M$. Explain the meaning of this result, using the correct units.
