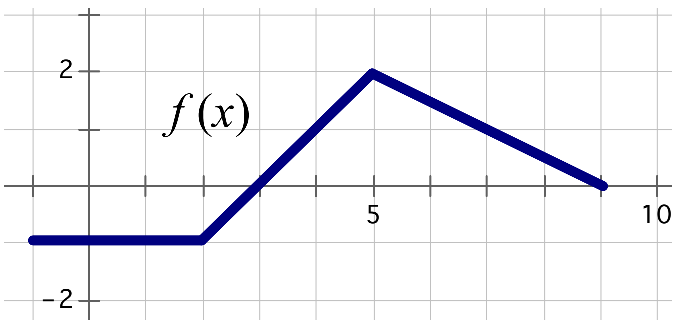
AP Calculus AB '20-21 Name________________________________
Dr. Quattrin
Fall Final Exam v2

Part II: Free Response – Show all work.



x	$g(x)$	$g'(x)$
0	-1	1
2	1	3
4	3	6
6	6	12
8	4	8

1. Let $f(x)$ be the function whose graph is given above and let $g(x)$ be a differentiable function with selected values for $g(x)$ and $g'(x)$ given on the table above. Furthermore, let h be the function defined by $h(x) = \ln(x^2 + 4)$.

(a) Find the equation of the line tangent to $g(x)$ at $x = 4$.

(b) Let K be the function defined by $K(x) = h(g(x))$. Find $K'(2)$.

(c) Let M be the function defined by $M(x) = h(x) \cdot g(x)$. Find $M'(6)$.

(d) Let J be the function defined by $J(x) = \frac{g(x)}{f(2x)}$. Find $J'(4)$.

---

2. Consider the differential equation $\frac{dy}{dx} = \frac{x^2 - 1}{y}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $y(0) = -2$. The function $y = f(x)$ is defined for all real numbers.

---

a) Find the equation of the line tangent to $y = f(x)$ at $y(0) = -2$

---

b) Use your answer to part a) to approximate $f(0.2)$.

---

c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $y(0) = -2$.

3. The search for treasure on Oak Island has been ongoing since 1795. In the last ten years, modern engineering techniques have been brought to bear by the Lagina brothers. They hired ROC Drilling to sink 8 foot-diameter metal tubes (called caissons) down to 200 feet and then use a hammer grab to excavate the "spoils" (what comes out of the caissons). The hammer grab brings up the spoils at a rate of

$$H(t) = 9 + 4\cos\left(\frac{t^2}{14}\right).$$

The spoils are then taken to a wash plant where they are rinsed and sorted into piles by size so they can be visually inspected to find artifacts. A shift supervisor measures their rate of output every few hours and records the findings in the chart below.

t = time in hours	0	1	3	5	8
$P(t)$ = Rate of spoils washed in yds^3/hour	0	8.1	14.5	12.3	3.2

At the beginning of the day, there are 2 yds^3 at the wash plant from the day before._

---

a. Find the total amount of spoils pulled from the caisson by the hammer grab during the eight hours described on the table. Indicate the correct units.

---

b. Use a Right Riemann sum with subintervals indicated by the table to approximate $\int_0^8 P(t)dt$. Using correct units, explain the meaning of this value in the context of the problem.

---

c. Assume the equation $W(t)=0.1x(10-x)^2$ models the $P(t)$ data on the table. Write an expression for $U(t)$, the amount of unwashed spoils at time *t*.

---

d. What is the absolute maximum amount of unwashed spoils waiting to be washed?