AP Calculus AB '21-22
Dr. Quattrin
Derivative Test

Name Solution Key

score ____________

Part I: Multiple choice – Circle correct answer.

1. Let $f(x)$ be the function given by $f(x)=\sqrt[3]{x+3}$. What is the slope of the line tangent to $f(x)$ at $(5,2)$?

$f' = \frac{1}{3}(x+3)^{-2/3}$

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{12}$ d) $\frac{1}{4\sqrt{2}}$ e) $\frac{7}{4\sqrt{2}}$

$m = \frac{1}{3}\left(\frac{1}{8^{2/3}}\right) = \frac{1}{12}$

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2. If $f(x)=\tan^2(3-x)$, then $f'(0)=$ $2\tan 3 \sec^2 3\,(-1)$

(a) $-2\sec^2 3$ (b) $-2\tan 3\sec 3$ (c) $-2\tan 3\sec^2 3$

(d) $2\tan 3\sec^2 3$ (e) $-6\tan 3\sec^2 3$

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3. Which of the following statements must be **False**?

F I. $\frac{d}{dx}\sqrt{e^{2x}+5}=\frac{e^{2x}(2)}{\sqrt{e^{2x}+5}}$

F II. $\frac{d}{dx}(\ln\sec x)=\cancel{\ne}\tan x$

III. $\frac{d}{dx}\left(6x^3-7^3+\sqrt[3]{x^8}-\frac{2}{x^3}\right)=18x^2-\cancel{147}+\frac{8}{3}\sqrt[3]{x^5}+\frac{6}{x^4}$

F a) I only b) II only c) III only

d) II and III only e) I, II, and III

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4. Let the function f be differentiable on the interval $[0,\ 2.5]$ and define g by $g(x) = f(f(x))$. Use the table to find $g'(1.5)$. $g' = f'(f(1.5)) \cdot f'(1.5)$

$= f'(2.5) \cdot f'(1.5) = (2.2)(1.1)$

x	0.0	0.5	1.0	1.5	2.0	2.5
$f(x)$	0.5	1.5	2.0	2.5	1.0	0.0
$f'(x)$	0.1	0.3	0.6	1.1	2.0	2.2

a) 0.0 b) 1.24 c) 1.65 d) 2.08 e) 2.42

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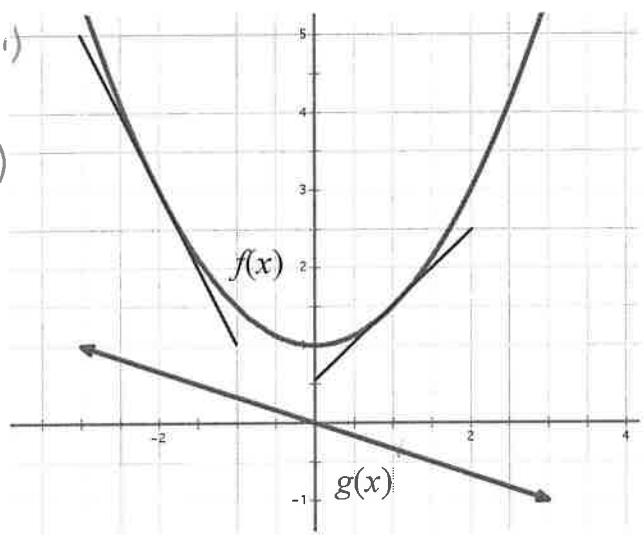
5. The figure below shows the graph of the functions f and g. The graphs of the lines tangent to the graph of f at $x = -2$ and $x = 1$ are also shown. If $B(x) = f(x) \cdot g(x)$, what is $B'(1)$?

$B' = f(1)g'(1) + g(1)f'(1)$

$= \left(\frac{3}{2}\right)\left(\frac{-1}{3}\right) + \left(\frac{-2}{3}\right)(1)$

$= -\frac{1}{2} - \frac{2}{3}$



a) $-\frac{5}{6}$ b) $-\frac{1}{2}$ c) $\frac{1}{6}$ d) $\frac{1}{3}$ e) $\frac{1}{2}$

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6. Let f and g be differentiable functions with the following properties:

i. $g(x) < 0$ for all x

ii. $f(2) = 3$

If $h(x) = f(x)g(x)$, and $h'(x) = f(x)g'(x)$, then $f'(x) =$

$h' = f \cdot g' + g \cdot f'$

a) $f'(x)$ b) $g(x)$ c) e^{-x}

$g \cdot f' = 0$

$f' = 0$

d) 0 e) 3

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7. If $y = \sin e^x$, then $\dfrac{d^2y}{dx^2} =$

$y' = \cos e^x \cdot e^x$ $y'' = e^x (-\sin e^x) + \cos e^x \, (e^x)$

a) $\cos e^x$ b) $e^x \sin e^x + e^x \cos e^x$

c) $-e^x \sin e^x + e^x \cos e^x$ d) $e^x \sin e^x - e^x \cos e^x$

e) $e^x \cos e^x$

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8. If $h(t) = e^{2t}(t+1)$, then $h'(0) =$ $e^0(1) + (1)e^0(2) =$

a) 0 b) 1 c) 2 d) 3 e) 4

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9. Suppose f is a differentiable function such that $f(-1)=2$ and $f'(-1)=\frac{1}{2}$. Using the line tangent to the graph of $f(x)$ at $x=-1$, find the approximation of $f(-1.1)$

a) –3.05 b) –1.95 c) .95 d) 1.95 e) 3.05

$y-2=\frac{1}{2}(x+1)$

$f(-1.1)=2+\frac{1}{2}(-1.1+1) = 2-.05 = 1.95$

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Part II: Free Response – Show all work.

1a. $\frac{d}{dx}\left(\cos\left(e^{5x}\right)\right) = -\sin e^{5x}\, e^{5x}(5)$

$= -5e^{5x}\sin e^{5x}$

1b. $\frac{d}{dx}\left[-3x^7+13^x-\frac{7}{5}\sqrt[4]{x^7}-\frac{3}{\sqrt[5]{x^6}}-\frac{1}{2x^3}\right]$

$= \frac{d}{dx}\left[-3x^7+13^x-\frac{7}{5}x^{7/4}-3x^{-6/5}-\frac{1}{2}x^{-3}\right]$

$= -21x^6+13^x\ln 13-\frac{49}{20}x^{3/4}+\frac{18}{5}x^{-11/5}+\frac{3}{2}x^{-4}$

1c. $\frac{d}{dx}\left[\tan^{-1}\left(5x^2\right)\right] = \frac{1}{1+(5x^2)^2}(10x)$

$= \frac{10x}{1+25x^4}$

2a. $\frac{d}{dx}\left(15x^4 e^{-5x}\right)$

$u = 15x^4 \quad v = e^{-5x}$

$Du = 60x^3 \quad Dv = e^{-5x}(-5)$

$u\,Dv + v\,Du$

$15x^4 (-5e^{-5x}) + e^{-5x}(60x^3)$

$15x^3 e^{-5x}\left[-5x + 4\right]$

2b. $\frac{d}{dx}\left(\frac{3x}{5+x^2}\right)$ $= \frac{(5+x^2)(3) - 3x(2x)}{(5+x^2)^2}$

$= \frac{15 + 3x^2 - 6x^2}{(5+x^2)^2}$

$= \frac{15 - 3x^2}{(5+x^2)^2}$

3a. $\frac{d}{dx}\left(\ln^4\left(1-x^2\right)\right) = 4\left(\ln\left(1-x^2\right)\right)^3\left(\frac{1}{1-x^2}\right)(-2x)$

$$= \frac{-8x\ln^3\left(1-x^2\right)}{1-x^2}$$

3b. If $f(x)=4\sin^{-1}\frac{1}{2}x+\overset{u}{x}\overset{v}{\sqrt{4-x^2}}$, find $f'(x)$.

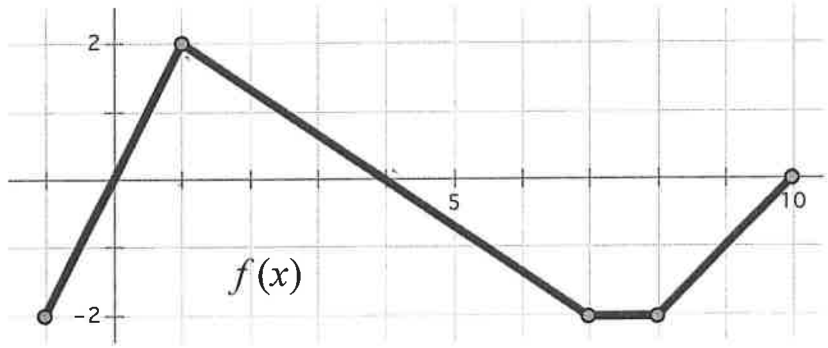
$$\qquad\qquad\qquad\qquad\qquad u \quad Dv \quad + v \quad Du$$

$$f'(x) = 4\left(\frac{1}{\sqrt{1-\left(\frac{1}{2}x\right)^2}}\right)\left(\tfrac{1}{2}\right) + x\left(\tfrac{1}{2}\left(4-x^2\right)^{-1/2}(-2x)\right) + \left(4-x^2\right)^{1/2}(1)$$

$$= \frac{2}{\left(1-\frac{1}{4}x^2\right)^{1/2}} - \frac{x^2}{\left(4-x^2\right)^{1/2}} + \left(4-x^2\right)^{1/2}$$

$$= \frac{4}{\left(4-x^2\right)^{1/2}} - \frac{x^2}{\left(4-x^2\right)^{1/2}} + \frac{4-x^2}{\left(4-x^2\right)^{1/2}}$$

$$\frac{2\left(4-x^2\right)}{\left(4-x^2\right)^{1/2}} = 2\left(4-x^2\right)^{1/2}$$



x	$g(x)$	$g'(x)$
0	-1	1
2	1	3
4	3	6
6	6	12
8	4	8

4. Let $f(x)$ be the function whose graph is given above and let $g(x)$ be a differentiable function with selected values for $g(x)$ and $g'(x)$ given on the table above.

a) Find the equation of the line tangent to $f(x)$ at $x = 4$.

$f(4) = 0 \quad f'(4) = -2/3$

$y - 0 = \frac{-2}{3}(x - 4)$

b) Let K be the function defined by $K(x) = g(f(x))$. Find $K'(1)$.

$K' = g'(f(1)) \cdot f'(1)$

$= g'(2) \cdot f'(1)$

$= 3 \cdot \text{DNE} = \text{DNE}$

c) Let M be the function defined by $M(x) = g(x) \cdot f(x)$. Find $M'(4)$.

$$M' = g(4) \cdot f'(4) + f(4) \cdot g'(4)$$
$$= 3\left(-\frac{2}{3}\right) + (0)(6)$$
$$= -2$$

d) Let J be the function defined by $J(x) = \dfrac{f(2x)}{g(x)}$. Find $J'(2)$.

$$J' = \frac{g(2) f'(\cancel{2}\,4)(2) - f(\cancel{2}\,4) g'(2)}{(g(2))^2}$$

$$= \frac{1(-2/3)(2) - 0(3)}{(1)\cancel{3}^2}$$

$$= \frac{-4}{3}$$

5. If $g(x)=\sqrt{9-x^2}$, find $g''(x)$

$$g' = \frac{1}{\cancel{2}}(9-x^2)^{-1/2}(-\cancel{2}x) = \frac{-x}{(9-x^2)^{1/2}}$$

$$g''(x) = \frac{(9-x^2)^{1/2}(-1) - (-x)\left(\frac{-x}{(9-x^2)^{1/2}}\right)}{(9-x^2)^1}$$

$$= \frac{\frac{-(9-x^2) - x^2}{(9-x^2)^{1/2}}}{\frac{(9-x^2)^1}{1}} = \frac{-9}{(9-x^2)^{3/2}}$$

6. If $h(x)=\sin^3 2x$, find $h''(x)$

$$h' = 3\sin^2 2x \cos 2x\,(2) = 6\sin^2 2x \mid \cos 2x$$

$u = 6\sin^2 2x$

$Du = 12\sin 2x\cos 2x$

$DV = (-\sin 2x)(2) = -2\sin 2x$

$V = \cos 2x$

$$\begin{array}{ccccc} & u & Dv & + \; v & Du \end{array}$$

$$h'' = 6\sin^2 2x\,(-2\sin 2x) + \cos 2x\,(12\sin 2x\cos 2x)$$

$$= -12\sin^3 2x + 12\sin 2x\cos^2 2x$$

$$= 12\sin 2x\,(\cos^2 2x - \sin^2 2x)$$

7. Find the equations of the lines tangent and normal $y=(x+1)\ln x-3$ at $x=1$.

$y(1) = 2 \ln 1 - 3 = -3$

$\frac{dy}{dx} = (x+1)\left(\frac{1}{x}\right) + \ln x \,(1)$

$m = 2(1) + \ln(1) = 2$

TANGENT LINE $y+3 = 2(x-1)$

NORMAL LINE $y+3 = \frac{-1}{2}(x-1)$