Final Summary Sheet #2: Key Theorems, Key Ideas, Vocab, etc.

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Chapter 1: Review of Derivatives

Key Ideas:

1. Know the rules!
2. Difference between Power & Exponential rules = where the variable is:
   a. Power Rule: in the base
   b. Exponential Rule: in the exponent
3. The Chain Rule must be applied to any composite function
4. Work from the out-side-in
5. The derivatives will end up multiplying
6. Derivatives of all cos functions = neg. Non-cos = positive
7. Order of operations & Chain Rule
8. There is more than 1 equation for every curve
9. Local linearity & Rectilinear motion
10. You know you have a product if you have 2 variables or if x is in 2 places
11. Algebra before calculus
12. Factor problems at the end
13. 2nd derivative is just the derivative of the derivative
14. Simplify algebra throughout

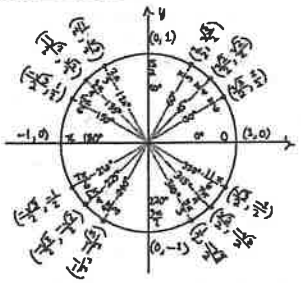
Vocab:

Derivative: the function that yields the slope of a tangent line
Numerical Derivative: the numerical value of the Δ of the tangent line @ x=a
Composite function: A function made up of 2 other functions — one within the other
Trigonometric: an equation w/ sin, cos, tan, sec, csc, or cot
Logarithmic: an equation w/ log or ln
Local linearity: function curve appears to be a line
Rectilinear Motion: movement that occurs in a straight line
Parametric Motion: movement that occurs in 2 dimensions
Velocity: directed speed
Ave. velocity: average rate of directed speed
Instantaneous Velocity: the rate @ an instant
Acceleration: the rate of change of velocity over time
Second Derivative: derivative of a derivative
Leibnitz: $\frac{d^2y}{dx^2}$ = d squared y, dx squared, $\frac{d^3y}{dx^3}$, ... $\frac{d^n y}{dx^n}$
Function: f''(x) = f double prime of x; f''(x)
Combination: y'' = y double prime

Missed Functions:

General Inverse: $\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))}$

Unit Circle:



Step-by-Step:

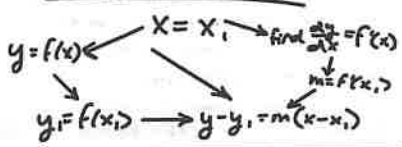
Simplifying Product or Quotient Rule:

1. Identify u & v
2. Use scratch work to find du & dv
3. Simplify scratch work
4. Plug u, v, du, & dv into formula
5. Simplify the formula
   → Factor
   → Common Denominator
   → Reduce

Key Phrases & Misc.:

When = solve for t
Where = solve for position
Which direction = is the velocity pos/neg?
Speeding up/ Slowing down = are v(t) & a(t) same/opposite direction

Tangent line Approx.:



Chapter 2: Anti-Derivative

Key Ideas:

1. Don't forget +C!
2. Derivatives distribute over addition/subtraction but not multiplication/division
3. Having an ordered pair lets you solve for +C
4. C is not always what they give for the y-value
5. Position, velocity, acceleration in that order
6. Put the +C in the formula as soon as you do the anti-derivative.
7. If you're going from acceleration to position, you're actually doing 2 problems one after the other & each one has +C that are different C's (& the first one will be integrated)
8. Products most often happen from doing the Chain Rule. U-Sub is how you reverse the Chain Rule.
9. U-Sub doesn't always work.
10. U is usually the inside of the composite function
11. You cannot mix variables.
12. You can put a number into the problem if you need it to make du, but you cannot put in a variable.
13. You should end with the variable you started with. Plug back in the original variable.
14. There are 4 ways to integrate fractions
    a) simplify Algebra through ÷
    b) U-Sub & eliminate ÷ → Integral
    c) U-Sub → $\int \frac{1}{u}\,du$ = ln|u|+C
    d) U-Sub → $\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a}$
15. When integrating different terms, you might have to use different u-subs.
16. Separate variables by ÷ × not + or −
17. dx & dt have to be in the numerator
18. Variable on top = y variable
    Variable on bottom = x variable
19. go through the pt. given, go to edge of screen, don't put arrows on ends.
20. MCQ's are about which are wrong; process of elimination

Vocab:

Rectilinear Motion: movement that occurs in a straight line.
Parametric Motion: movement that occurs in 2 dimensions.
Velocity: directed speed
Average Velocity: distance traveled divided by time or $\frac{x_2-x_1}{t_2-t_1}$
Instantaneous velocity: velocity at a particular time t
Acceleration: the rate of change of the velocity or $\frac{dv}{dt}$
Differential Equation: an equation that contains an unknown function & one or more derivatives
Seperable Differential Equation: a differential equation in which all terms with y's can be moved to one side of the equation & in which all terms with x's can be moved to the other side, by multiplication & division only
General Solution: All of the y-equations that would have the given equation as their derivative
Initial Condition: constraint placed on a differential equation (initial value)
Particular Solution: solution obtained from solving a differential equation when an initial condition
Slope field - given any function f, a slope field is drawn by taking evenly spaced points on the Cartesian coordinate system (usually points having integer coordinates) and, at each point, drawing a small line w/ the slope of the function.

Step-by-Step

U-subs Steps:

0.) Notice you are integrating a product or quotient
1.) Identify the inside function (u) of the composite function
2.) Find du from u
3.) If necessary, multiply a constant inside the integral to create du & balance it by multiplying the reciprocal of that constant outside the integral.
4.) Substitute u & du into the equation
5.) Perform the integration by the Anti-Power Rule
6.) Resubstitute the x-equivalent for u.

Steps to solving Differential Equation:

1. Separate the variables, using ÷ & x only (leave constants on the right side of equation)
2. Integrate both sides of the equation
3. Don't forget the +C (rt. side of equation)
4. Plug in the initial condition to solve for C & put it in the equation. If your integration resulted in a natural log (do 5 before 4)
5. Isolate y.

Key Phrases & Misc.:

When = solve for t
Where = solve for position
Which direction = is the velocity pos/neg?
Speeding up/ Slowing down = are v(t) & a(t) same/opposite direction

Products can result from:
- Chain Rule
- Differentiating a product
- Differentiating some trig. functions

Quotients can result from:
- Common denominators
- Differentiating a quotient
- Differentiating a log w/ composite function
- Differentiating some trig. inverse functions

Missed Formulas:

Anti-Exponential Rule: $\int (a^x)\,dx = \frac{a^x}{\ln a} + C$

Trig. Inverse Rules:
$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$
$\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$
$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$

Substitution:
$\int \tan u\,du = \ln|\sec u| + C$
$\int \cot u\,du = \ln|\sin u| + C$
$\int \sec u\,du = \ln|\sec u + \tan u| + C$
$\int \csc u\,du = \ln|\csc u - \cot u| + C$

Chapter 3: Fundamental Theorem of Calc

Key Ideas:

2. The definite integral – a number (not an equation) that represents the "area" under a curve
2. The integral could be negative
3. The most important idea in this chapter to solve a definite integral — do the anti-derivative, plug in the top number first, next the bottom one, then subtract
4. Don't need the +C bc it will drop anyway
5. The fundamental theorem does not work if there is a VA or POD between the boundaries
6. If you change the variable to u (u-sub) w/definite integrals, you have to change the boundaries
7. You do not change back to x bc u don't need to
8. Area is positive but integrals can be negative
9. When asked abt areas, you must graph the curve 1st!
10. If the curve crosses the x-axis, you need to split them into 2 separate integrals & account for the negative
11. Total Distance is like area. Total Distance is how far you traveled in total. Displacement is just the integral
12. The units are the most important thing about the set up
13. 4 ways to do an approximation
    1.) Left-hand Rectangles
    2.) Right-hand Rectangles
    3.) Midpoint Rectangles
    4.) Trapezoids
14. Trapezoids are figured out with the average of the heights · base.
15. Integrating is a result in total change
16. Answer the question asked

Vocab:

Displacement: How far apart the starting position & ending position are (pos. or neg.)
$= \int_a^b v\,dt$

Total Distance: how far you travel in total (only positive)
$= \int_a^b |v|\,dt$

Steps to Solving Integrals in Context:

1. Graph the function or find the sign pattern for the velocity
2. Identify the zeros (if any) between $x=a$ & $x=b$
3. Set up separate integrals to represent the sections above the x-axis & below the x-axis
4. Change the signs & the integrals representing the regions below the x-axis
5. Perform the integration & solve

Key Phrases:

Amount increasing (or decreasing): Total rate of change is pos. (or neg.)
Rate of change increasing (or decreasing): $\frac{d}{dt}$ (rate of change) is pos. (or neg.)
Amount increasing at an increasing rate: Total rate of change is pos. & $\frac{d}{dt}$ (rate of change) is pos.
Position at $x=a = x(a) + \int_a^b v\,dt$
Area $= \int_a^b f(x)\,dx - \int_c^b f(x)\,dx$

Step-by-Step:

U-Sub Steps:

1.) Notice you are trying to integrate a product
2.) Identify the inside function of composite & call it u
3.) Find du from u
4.) If necessary, introduce a constant inside the integral to create du. Balance w/a constant factor outside.
5.) Change the boundaries to what u would equal.
6.) Substitute u, du, & the new boundary #'s into the equation
7.) Perform the integration
8.) Plug the boundaries in & subtract
N.B.: Do not go back to x

Steps to Approximating an Integral w/Rectangles:

1. Draw the heights represented on the tables
2. Draw the top of the rectangles from the left endpoint, right endpoint, or midpoint values.
3. Calculate the areas of each rectangle
4. Add the areas together for your approximation
5. State answer using proper notation

Integral Approximations

→ Left Hand Rectangles are an overestimate if the curve is decreasing & an underestimate if that curve is decreasing.
→ Right Hand Rectangles are an underestimate if the curve is decreasing & an overestimate if the curve is decreasing
→ Midpoint Rectangles are an overestimate if the curve is concave down & underestimate if the curve is concave up
→ Midpoint Rectangles & Trapezoids are an underestimate if the curve is concave down & an overestimate if the curve is concave up

Units:

- $\int_a^b R(t)\,dt = \int_a^b \frac{\text{units}}{\text{time}}\,(\text{time}) = \text{sum of units} = \text{total units}$
- Amount increasing/decreasing $\Rightarrow \frac{d}{dt}(\text{units}) = \frac{\text{units}}{\text{time}}$
- Rate $\Rightarrow \frac{\text{units}}{\text{time}}$
- $\frac{f(b)-f(a)}{b-a} \Rightarrow \frac{\text{units}}{\text{time}}$
- $f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx \Rightarrow \frac{1}{\text{time}}\int_a^b \frac{\text{units}}{\text{time}}\,(\text{time}) = \frac{\text{units}}{\text{time}}$
- Amount increasing/decreasing $\Rightarrow \text{units}\left(\frac{1}{\text{time}}\right) = \frac{\text{units}}{\text{time}}$
- Amount increasing/decreasing at an increasing/decreasing rate $\Rightarrow (\text{units})\,\frac{1}{\text{time}}\cdot\frac{1}{\text{time}} = \frac{\text{units}}{\text{time}^2}$
- Rate increasing/decreasing $\Rightarrow \frac{d}{d(\text{time})}\left(\frac{d(\text{units})}{d(\text{time})}\right) = \frac{1}{\text{time}}\left(\frac{\text{units}}{\text{time}}\right) = \frac{\text{units}}{\text{time}^2}$

Rules of Logs:

① $\log_a x + \log_a y = \log_a(xy)$
② $\log_a x - \log_a y = \log_a\frac{x}{y}$
③ $\log_a x^n = n\log_a x$

Properties of Definite Integrals:

① $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
② $\int_a^a f(x)\,dx = 0$
③ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where $a<c<b$

Missed Formulas:

- Trapezoidal Rule:
$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)\right]$
- Ave. Value Formula
$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

Chapter 4: Applications of the Derivative I: Graphing

1. A critical value of a function occurs when the derivative equals zero, the derivative doesn't exist, or at endpoints of a given domain.
2. An extreme's x-value must be a critical value, but a critical value doesn't have to be an extreme.
3. The derivative tests tell you if you have an extrema.
4. Max. & Min.'s = y-values
5. Whether an endpoint is at a max. or min. depends on the sign next to it.
6. On a table problem, if there are 2 y-values that are equal & the function is differentiable, then the mean value tells us there must be a place where the y-value is zero
7. Solving for critical values, when the derivative equals zero, use graphing to find P.O.I.
8. Don't forget the endpoints!
9. Set up a table to find the amount (y) values
10. Answer the question asked!
11. The 2nd derivative is the concavity, while the 1st derivative is increasing/decreasing
12. Increasing & decreasing ≠ indicate anything about concavity
13. Endpoints ≠ included on concavity integrals
14. POI's = where concavity changes → endpoints cannot be POI's
15. The 2nd Derivative Test tells you the same thing as the 1st Test, but 1st: sign pattern 2nd: concavity.
16. There are 3 equations & each one has a different sign pattern has a different interpretation to figure out what the y-graph looks like

Key Ideas ↑

Vocab:

Critical value: x-coordinate of the extreme
Max. value: y-coordinate of the high point
Min value: y-coordinate of the low point
Relative Extremes: highest/lowest point of any section of the curve
Absolute Extremes: highest/lowest point of the whole curve
Interval of Increasing: the interval of x-values for which the curve is rising left → right
Interval of Decreasing: the interval of x-values for which the curve is dropping left → right
Concavity: the way the curve bends
Concave up: curve bends counterclockwise
Concave down: curve bends clockwise
Inflection Value: the x value @ which the concavity of the curve switches (can be a VA)
Point of Inflection: the point @ which the concavity of the curve switches from up → down

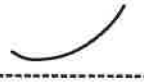
Derivative Tests:

1st Derivative Test: critical values
i. rel. max. point if the sign changes from + to −
ii. rel. min. point if the sign changes from − to +
iii. non-extreme if the sign is the same

2nd Derivative Test → Won't work for DNE's or endpoints
1. If $f'(c)=0$ & $f''(c)>0$, then $f(x)$ has a rel. min. value at $x=c$.
2. If $f'(c)=0$ & $f''(c)<0$, then $f(x)$ has a rel. max. value at $x=c$.

MISC.

Derivative	positive	negative
$f'(x)$	increasing	decreasing
$f''(x)$	concave ↑	concave ↓

Concave up	Concave down
	

Theorems/Step-by-Step:

Mean Value Theorem:
i. $f(x)$ is continuous on the closed interval $[a,b]$
ii. differentiable on the open interval (a,b)
→ There is at least 1 number c in the interval (a,b) $f'(c) = \frac{f(b)-f(a)}{b-a}$

Rolle's Theorem:
i. $f(x)$ is continuous on the closed interval $[a,b]$
ii. differentiable on the open interval (a,b)
iii. $f(a)=f(b)$
→ There is at least 1 number c in the interval (a,b) $f'(c)=0$

Extreme Value Theorem:
If f is continuous on the closed interval $[a,b]$, then $f(x)$ attains a maximum & a minimum at least once each.

ch. 3
Fundamental Theorem: If $f(x)$ is a continuous function on $[a,b]$, then
1.) $\frac{d}{dx}\int_c^x f(t)\,dt = f(x)$ or $\frac{d}{dx}\int_c^u f(t)\,dt = f(u)\cdot D_x$
2.) $F' = f(x)$ then $\int_a^b f(x)\,dx = F(b) - F(a)$