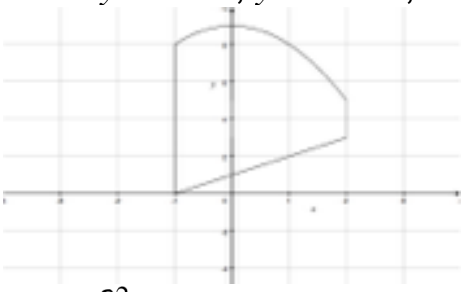
Chapter 6 Solution Manual

6.1 Free Response Solutions

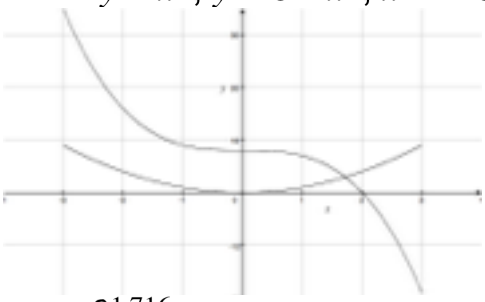
1. $y = x + 1,\ y = 9 - x^2,\ x = -1,\ x = 2$



$$A = \int_{-1}^{2} \left[(9 - x^2) - (x + 1) \right] dx = \int_{-1}^{2} (9 - x^2 - x - 1) dx = \int_{-1}^{2} (8 - x - x^2) dx$$

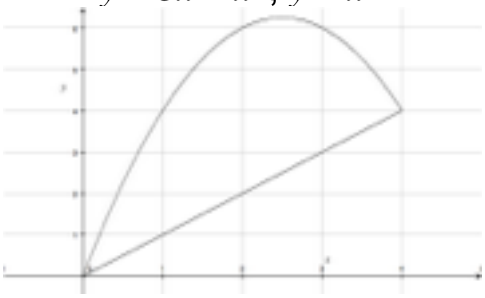
$$= \left[8x - \frac{x^2}{2} - \frac{x^3}{3} \right]_{-1}^{2} = \left[16 - \frac{4}{2} - \frac{8}{3} \right] - \left[-8 - \frac{1}{2} + \frac{1}{3} \right] = 19.5$$

3. $y = x^2,\ y = 8 - x^3,\ x = -3,\ x = 3$



$$A = \int_{-3}^{1.716} \left[(8 - x^3) - x^2 \right] dx + \int_{1.716}^{3} \left[x^2 - (8 - x^3) \right] dx = 60.252$$

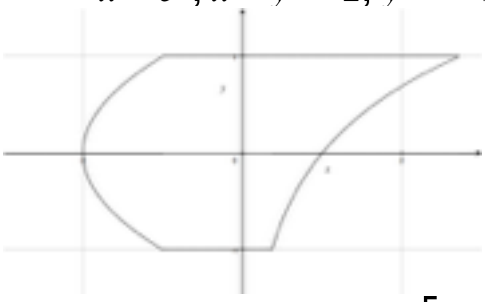
5. $y = 5x - x^2,\ y = x$



$$A = \int_0^4 \left[(5x - x^2) - x \right] dx = \int_0^4 \left[4x - x^2 \right] dx = \left[2x^2 - \frac{1}{3}x^3 \right]_0^4$$

$$= 32 - \frac{8}{3} = 32/3$$

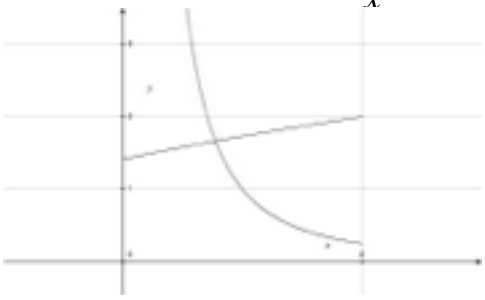
7. $x = e^y,\ x = y^2 - 2,\ y = -1,\ y = 1$



$$A = \int_{-1}^1 \left[e^y - (y^2 - 2) \right] dy = \left[e^y - \frac{1}{3}y^3 + 2y \right]_{-1}^1$$

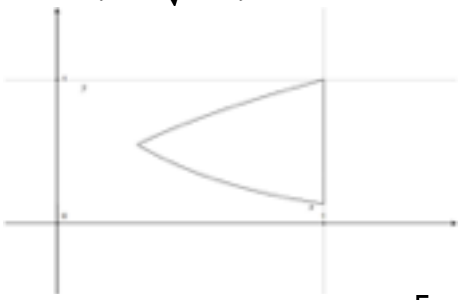
$$= \left[e^1 - \frac{1}{3} + 2 \right] - \left[\frac{1}{e} + \frac{1}{3} - 2 \right] = 5.684$$

9. $y = \sqrt{x+2},\ y = \frac{1}{x^2},\ x = 1,\ x = 2$



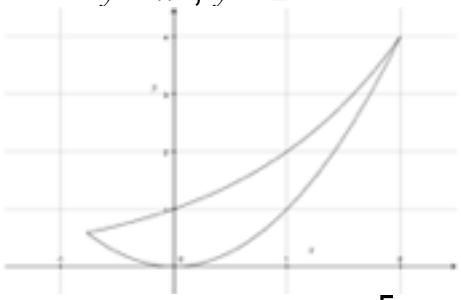
$$A = \int_1^2 \left(\sqrt{x+2} - \frac{1}{x^2} \right) dx = 1.369$$

11. $y=\sqrt{x}$, $y=e^{-2x}$, $x=1$



$$A=\int_{.301}^{1}\left(\sqrt{x}-e^{-2x}\right)dx=\left[\frac{2}{3}x^{3/2}+\frac{1}{2}e^{-2x}\right]_{.301}^{1}=.350$$

13. $y=x^2$, $y=2^x$



$$A=\int_{-.767}^{2}\left[2^x-x^2\right]dx=\left[\frac{2^x}{\ln 2}-\frac{1}{3}x^3\right]_{-.767}^{2}=2.106$$

6.1 Multiple Choice Solutions

1. $A=\int_{-9/8}^{9/8} e^{\left(\frac{-1}{1+x^2}\right)}dx=1.080$

The correct answer is B

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3. $Area=\int_{0}^{1.456}(2\sin x-x\ln(2x+1))dx=0.661$

The correct answer is B

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5. $$Area = \int_0^1 \left(e^{2x} - 1\right)dx = \left[\frac{1}{2}e^{2x} - x\right]_0^1 = \left(\frac{1}{2}e^2 - 2\right) - \left(\frac{1}{2}e^0 - 0\right)$$
$$= \frac{1}{2}\left(e^2 - 3\right)$$

The correct answer is B

6.2 Free Response Solutions

1. $y = 1 + 6x^{3/2}$ on $x \in [0, 1]$

$y' = 9x^{1/2} \rightarrow L = \int_0^1 \sqrt{1 + 81x}\, dx = 6.103$

3. $x = \frac{1}{3}\sqrt{y}(y - 3)$ on $y \in [1, 9]$

$x = \frac{1}{3}y^{3/2} - y^{1/2} \rightarrow x' = \frac{1}{2}y^{1/2} - \frac{1}{2}y^{-1/2}$

$L = \int_1^9 \sqrt{1 + \left(\frac{1}{2}y^{1/2} - \frac{1}{2}y^{-1/2}\right)^2}\, dy = \int_1^9 \sqrt{1 + \frac{1}{4}y - \frac{1}{2} + \frac{1}{4}y^{-1}}\, dy$

$= \int_1^9 \sqrt{\frac{1}{2} + \frac{1}{4}y + \frac{1}{4}y^{-1}}\, dy = 10.667$

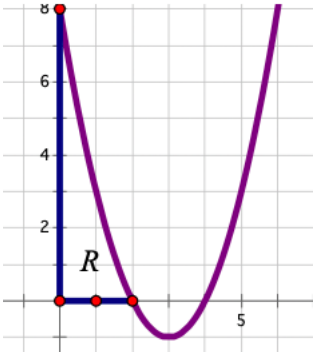
5. $y = \ln x^{3/2}$ on $x \in \left[1, \sqrt{3}\right]$

$y' = \frac{1}{x^{3/2}} \cdot \frac{3}{2}x^{1/2} = \frac{3}{2x} \rightarrow L = \int_1^{\sqrt{3}} \sqrt{1 + \frac{9}{4x^2}}\, dx = 1.106$

7. Find the length of the arc along $f(x) = \int_0^x \sqrt{\cos t}\, dt$ on $x \in \left[0, \frac{\pi}{2}\right]$.

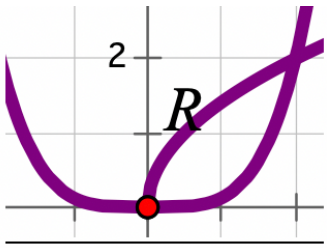
$f'(x) = \sqrt{\cos x} \rightarrow L = \int_0^{\pi/2} \sqrt{1 + \cos x}\, dx = 2$

9. Find the perimeter of the region R bounded by $y=(x-3)^2-1$, the x-axis, and the y-axis.



$$L=\int_0^2\sqrt{1+(2x-6)^2}\,dx+8+2=8.268+10=18.268$$

11. Find the perimeter of the region R bounded by $y=\sqrt{2x}$ and $y=\frac{1}{8}x^4$.



$$L=\int_0^2\sqrt{1+(0.5x^3)^2}\,dx+\int_0^2\sqrt{1+\left(\frac{1}{\sqrt{2x}}\right)^2}\,dx$$
$$=3.200+2.958=6.158$$

6.2 Multiple Choice Solutions

1. $\frac{dy}{dx}=\sqrt{x}\rightarrow L=\int_0^4\sqrt{1+(\sqrt{x})^2}\,dx=\int_0^4\sqrt{1+x}\,dx=$
$=\left[\frac{2}{3}(1+x)^{3/2}\right]_0^4=\frac{2}{3}(5\sqrt{5}-1)$

The correct answer is A

3. $\dfrac{dy}{dx} = \dfrac{1}{\cos x}(-\sin x) = -\tan x$

$$L = \int_0^{\pi/3} \sqrt{1 + (-\tan x)^2}\, dx = \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx =$$

$$= \ln\left|\sec x + \tan x\right|_0^{\pi/3} = \ln\left|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right| - \ln\left|\sec 0 + \tan 0\right|$$

$$= \ln\left|2 + \sqrt{3}\right| - \ln 1 = \ln\left|2 + \sqrt{3}\right|$$

The correct answer is D

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5. $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}} \to L = \displaystyle\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2}\, dx$

The correct answer is E

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7. $\displaystyle\int_2^8 \sqrt{1 + \frac{1}{4 - x^2}}\, dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$

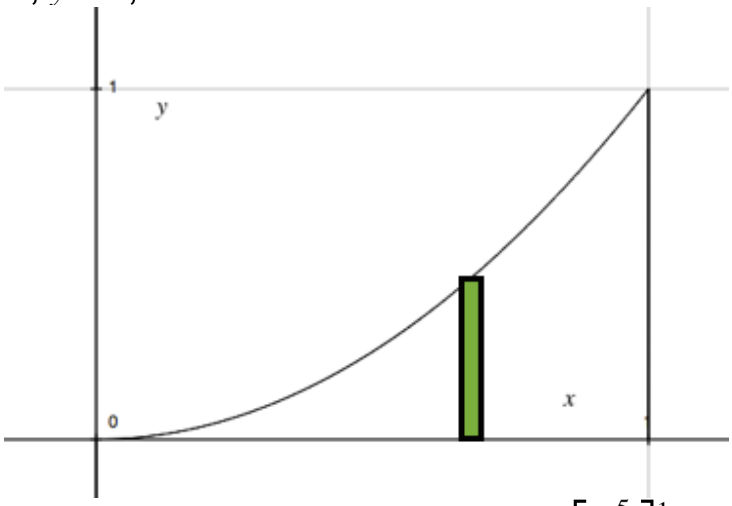
$$\frac{dy}{dx} = \sqrt{\frac{1}{4 - x^2}} = \frac{1}{\sqrt{4 - x^2}} \to y = \sin^{-1}\frac{x}{2} + c$$

$$(1, 0) \to 0 = \sin^{-1}\frac{1}{2} + c \to 0 = \frac{\pi}{6} + c \to c = -\frac{\pi}{6}$$

The correct answer is E

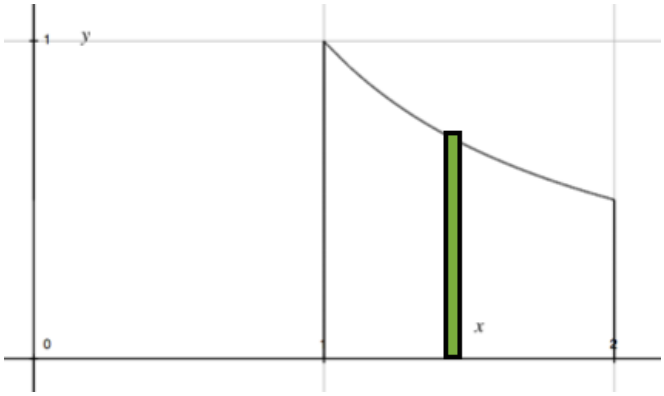
6.3 Free Response Solutions

1. $y = x^2,\ x = 1,\ y = 0$; about the $x-$axis.



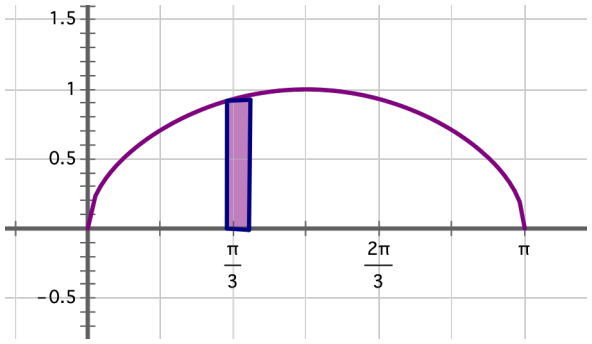
$$V = \pi \int_0^1 r^2 dx = \pi \int_0^1 (x^2)^2 dx = \pi \int_0^1 x^4 dx = \pi \left[\frac{x^5}{5} \right]_0^1 = \frac{\pi}{5}$$

3. $y = \frac{1}{x},\ x = 1,\ x = 2,\ y = 0$; about the $x-$axis.



$$V = \pi \int_1^2 r^2 dx = \pi \int_1^2 \left(\frac{1}{x} \right)^2 dx = \pi \int_1^2 x^{-2} dx = \pi \left[\frac{x^{-1}}{-1} \right]_1^2 = \pi/2$$

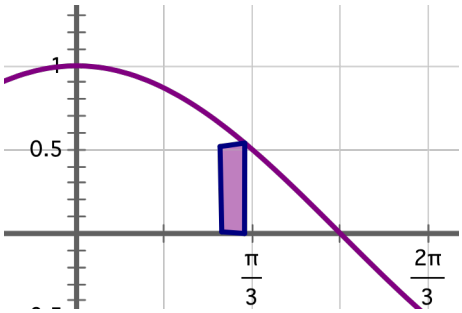
5. The region bounded by $y = \sqrt{\sin x}$ and $y = 0$.



$$V = \pi \int_0^{\pi} r^2 dx = \pi \int_0^{\pi} \left(\sqrt{\sin x}\right)^2 dx = \pi \int_0^{\pi} (\sin x) dx = \pi[-\cos x]_0^{\pi}$$

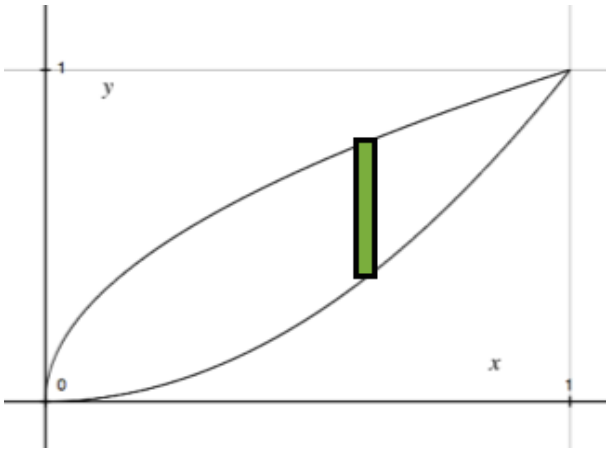
$$= 2\pi$$

7. The region bounded by $y = \cos x$, $x = 0$, and $y = 0$.



$$V = \pi \int_0^{\pi/2} r^2 dx = \pi \int_0^{\pi/2} (\cos^2 x) dx = \pi \left[\frac{1}{2}x + \frac{1}{4}\sin 2x\right]_0^{\pi} = \frac{\pi^2}{2}$$

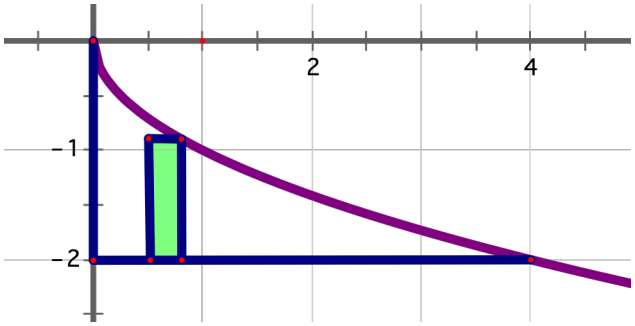
9. $y = x^2,\ x = y^2$; about the $x-$axis.



$$V = \pi \int_0^1 \left(R^2 - r^2\right)dx = \pi \int_0^1 \left[\left(\sqrt{x}\right)^2 - \left(x^2\right)^2\right]dx = \pi \int_0^1 \left(x - x^4\right)dx$$

$$= \pi\left[\frac{1}{2}x^2 - \frac{1}{5}x^5\Big|_0^1\right] = \frac{3\pi}{10}$$

11. The region bounded by $y = -\sqrt{x},\ y = -2$ and the y-axis.



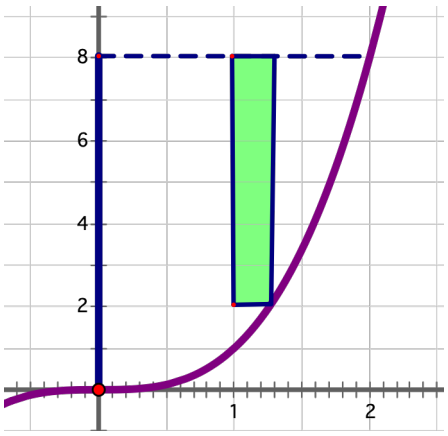
$$V = \pi \int_0^4 \left(R^2 - r^2\right) dx$$

$$= \pi \int_0^4 \left[(-2)^2 - \left(-\sqrt{x}\right)^2\right] dx$$

$$= \pi \int_0^4 (4 - x) dx$$

$$= \pi \left[4x - \frac{1}{2}x^2 \Big|_0^4 \right] = 8\pi$$

13. The region bounded by $y = x^3$, $y = 8$, and the y-axis.



$$V = \pi \int_0^2 \left(R^2 - r^2\right) dx = 2\pi \int_0^2 \left[(8)^2 - \left(x^3\right)^2\right] dx$$

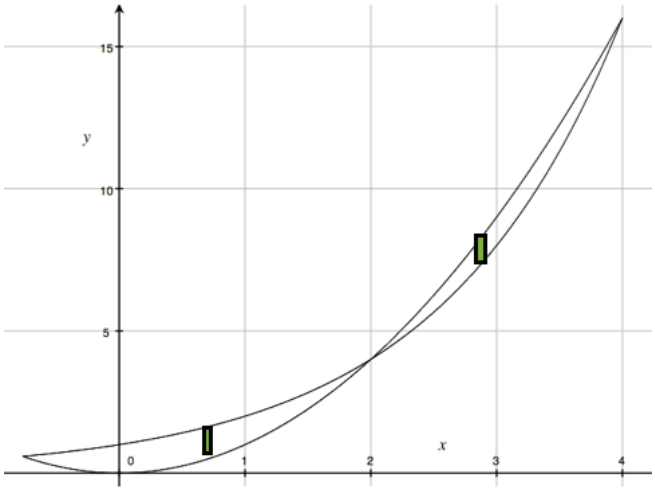
$$= \pi \int_0^2 \left[64 - x^6\right] dx$$

$$= \pi \left[64x - \frac{1}{7}x^7 \Big|_0^2 \right]$$

$$\pi \left[128 - \frac{128}{7} \right]$$

$$= \frac{768\pi}{7}$$

15. $y = x^2$, $y = 2^x$; about the $x-$axis.



$$V = \pi \int_{-.767}^{2} \left(R^2 - r^2\right) dx + \pi \int_{2}^{4} \left(R^2 - r^2\right) dx$$

$$V = \pi \int_{-.767}^{2} \left[\left(2^x\right)^2 - \left(x^2\right)^2\right] dx + \pi \int_{2}^{4} \left[\left(x^2\right)^2 - \left(2^x\right)^2\right] dx = 94.612$$

6.3 Multiple Choice Solutions

1. $V = \pi \int_0^4 \left(\left(\sqrt{x}\right)^2 - \left(\frac{1}{2}x\right)^2 \right) dx = \pi \int_0^4 \left(x - \frac{1}{4}x^2 \right) dx =$

$$\pi \left[\frac{1}{2}x^2 - \frac{1}{12}x^3 \right]_0^4 = \pi \left(8 - \frac{16}{3} \right) = \frac{8}{3}\pi$$

The correct answer is E

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3. $V = \pi \int_1^e \left(\sqrt{\ln x}\right)^2 dx = \pi \int_1^e (\ln x)\, dx = \pi$

The correct answer is C

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5. $V = \pi \int_0^1 \left(3^2 - \left(\tan^{-1} x\right)^2\right) dx = 27.505$

The correct answer is E

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7. $x^2 + 4y^2 = 1 \rightarrow y = \sqrt{1 - \frac{1}{4}x^2} \rightarrow V = \pi \int_{-2}^{2} \left(\sqrt{1 - \frac{1}{4}x^2}\right)^2 dx$

$V = 2\pi \int_0^2 \left(1 - \frac{1}{4}x^2\right) dx = 2\pi \left[x - \frac{1}{12}x^3\right]_0^2 = \frac{8\pi}{3}$

The correct answer is B

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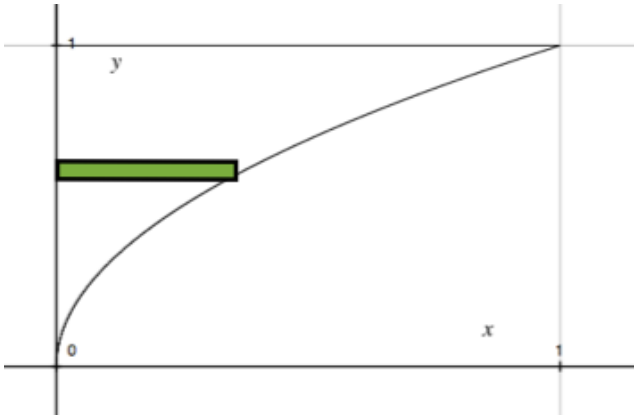
9. $Volume = \pi \int_0^2 \left(\pi^2 - \left(\frac{\pi}{2} - \sin^{-1}(x-1)\right)^2\right) dx + \pi \int_2^6 \left(\pi^2 - \left(\frac{\pi}{2}\sqrt{x-2}\right)^2\right) dx$

$= 105.585$

The correct answer is D

6.4 Free Response Solutions

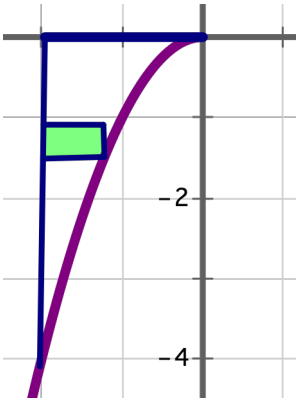
1. $y^2 = x,\ x = 0,\ y = 1$; about the $y-$axis.



$$V = \pi \int_0^1 r^2 dy = \pi \int_0^1 x^2 dy = \pi \int_0^1 (y^2)^2 dy = \pi \int_0^1 y^4 dy$$

$$= \pi \left[\frac{y^5}{5} \right]_0^1 = \frac{\pi}{5}$$

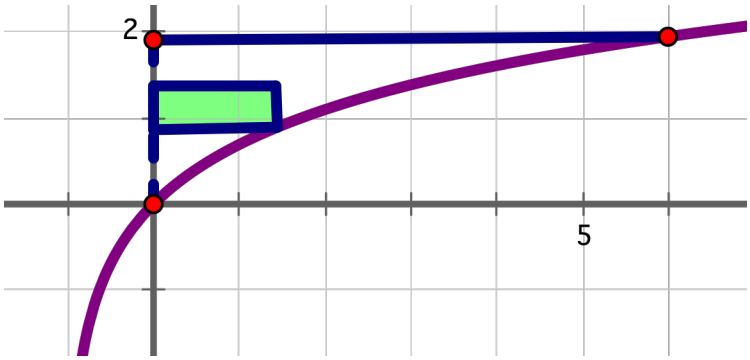
3. The region bounded by $x = -\sqrt{-y}$ on $y \in [-4, 0]$.



$$V = \pi \int_{-4}^0 r^2 dy = \pi \int_{-4}^0 x^2 dy = \pi \int_{-4}^0 \left(-\sqrt{-y}\right)^2 dy =$$

$$= \pi \int_{-4}^0 (y) dy = \pi \left[\frac{y^2}{2} \right]_{-4}^0 = 8\pi$$

5. The region bounded by $x = e^y - 1$, $x = 0$, and $y = \ln 3$.

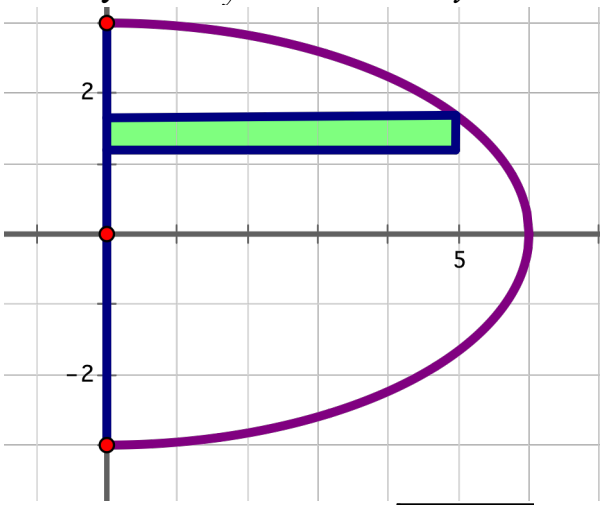


$$V = \pi \int_0^{\ln 3} r^2 dy = \pi \int_0^{\ln 3} \left(e^y - 1\right)^2 dy = \pi \int_0^{\ln 3} \left(e^{2y} - 2e^y + 1\right) dy$$

$$= \pi \left[\frac{1}{2} e^{2y} - 2e^y + y \right]_0^{\ln 3} =$$

$$= \pi \left[(4.5 - 6 + \ln 3) - (0.5 - 3 + 0) \right] = 2.986$$

7. The region bounded by $x^2 + 4y^2 = 36$ and the y-axis on $y \in [-3, 3]$.

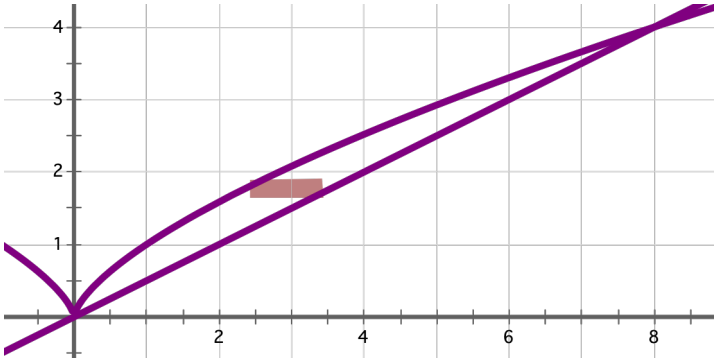


$$x^2 + 4y^2 = 36 \rightarrow x = \sqrt{36 - 4y^2}$$

$$V = \pi \int_{-3}^{3} r^2 dy = \pi \int_{-3}^{3} \left(\sqrt{36-4y^2}\right)^2 dy = 2\pi \int_{0}^{3} \left(36-4y^2\right) dy$$

$$= 2\pi \left[36y - \frac{4y^3}{3} \right]_0^3 = 144\pi$$

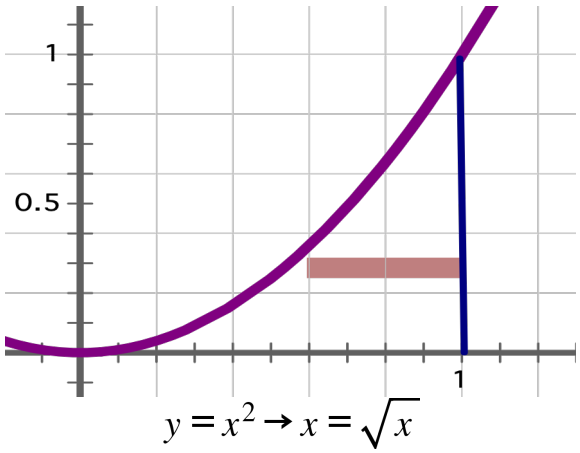
9. $y^2 = x$, $x = 2y$; about the $y-$axis.



$$V = \pi \int_{0}^{2} \left(R^2 - r^2\right) dy = \pi \int_{0}^{2} \left[(2y)^2 - (y)^2\right] dy \, \pi \int_{0}^{2} \left(4y^2 - y^4\right) dy$$

$$= \pi \left[\frac{4}{3} y^3 - \frac{1}{5} y^5 \Big|_0^2 \right] = \frac{64\pi}{15}$$

11. The region bounded by $y = x^2$, $x = 1$, and $y = 0$.

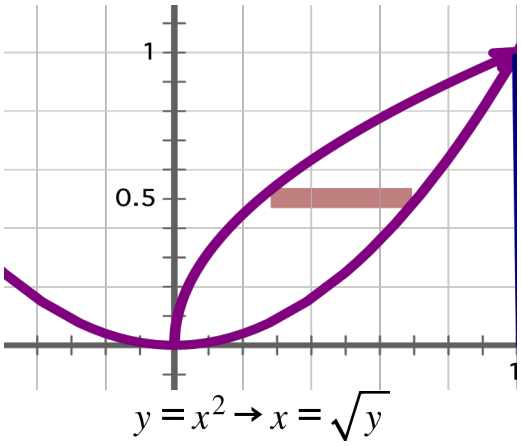


$y = x^2 \rightarrow x = \sqrt{x}$

$$V = \pi \int_0^1 \left(R^2 - r^2\right) dy = \pi \int_0^1 \left[(1)^2 - \left(y^{1/2}\right)^2\right] dy \, \pi \int_0^1 (1 - y) dy$$

$$= \pi \left[y - \frac{1}{2} y^2 \Big|_0^1 \right] = \frac{\pi}{2}$$

13. The region bounded by $y = x^2$ and $x = y^2$.

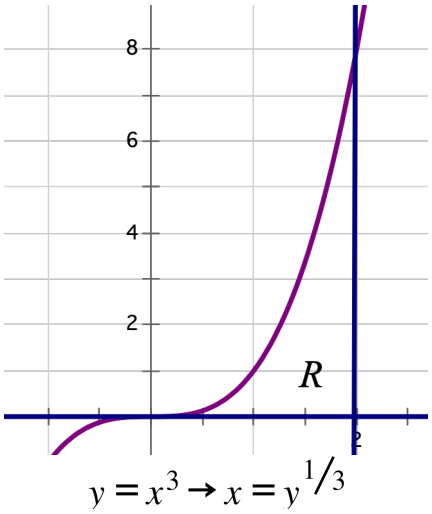


$y = x^2 \rightarrow x = \sqrt{y}$

$$V = \pi \int_0^1 \left(R^2 - r^2\right) dy = \pi \int_0^1 \left[\left(\sqrt{y}\right)^2 - \left(y^2\right)^2\right] dy = \pi \int_0^1 \left[y - y^4\right] dy$$

$$= \pi \int_0^1 \left[\frac{1}{2} y^2 - \frac{1}{5} y^5\right] dy = \frac{3\pi}{10}$$

15. Let R be the region bounded by $y = x^3$, $x = 2$, and $y = 0$.



$$V = \pi \int_0^8 \left(R^2 - r^2\right) dy = \pi \int_0^8 \left[(2)^2 - \left(y^{1/3}\right)^2\right] dy = \pi \int_0^8 \left[4 - y^{2/3}\right] dy$$

$$= \pi \int_0^8 \left[4y - \frac{3}{5} y^{5/3}\right] dy = \frac{128\pi}{5}$$

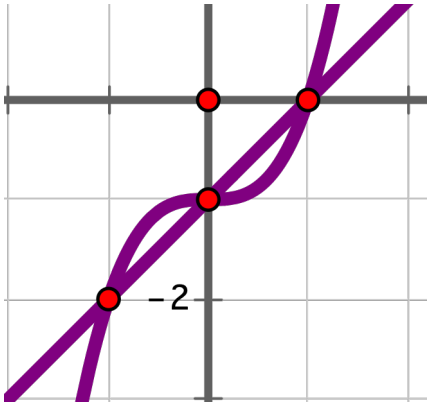
6.4 Multiple Choice Solutions

1. $$V = \pi \int_0^2 \left(\left(\sqrt{\frac{16x}{x^2+4}}\right)^2 - (x)^2\right) dx = \pi \int_0^2 \left(\frac{16x}{x^2+4} - x^2\right) dx$$

$$= \pi\left[8\ln\left(x^2+4\right) - \frac{1}{3}x^3\right] = \pi\left[\left(8\ln 8 - \frac{8}{3}\right) - (8\ln 4 - 0)\right] = \pi\left(24\ln 2 - \frac{8}{3} - 16\ln 2\right)$$

$$= \pi\left(8\ln 2 - \frac{8}{3}\right)$$

The correct answer is E



3. $y = x^3 - 1 \rightarrow x = (y+1)^{1/3}$; $y = x - 1 \rightarrow x = y + 1$

Boundaries: $y + 1 = (y+1)^{1/3} \rightarrow y = -2,\ -1,\ and\ 0$

$$Volume = \pi \int_{-2}^{0} \left((x_1)^2 - (x_2)^2 \right) dy = 2\pi \int_{-1}^{0} \left((y+1)^2 - (y+1)^{2/3} \right) dy = 6.462$$

The correct answer is E

---

5. The solid is generated when R is rotated about the y-axis, so B and C are incorrect. $y = \dfrac{\pi}{2}$ is a boundary, so E is incorrect. The formula for the Disk Method about the y-axis is $\pi \int_0^{\pi/2} x^2 dy$, so A is incorrect.

The correct answer is D

---

7. The solid is formed when the region is revolved about the x-axis, so only A or E can be correct. The graph shows the problem uses the Washer Method, but A uses the Disk Method.

The correct answer is E

6.5 Free Response Solutions

1. Washer Method

$$V=\pi\int_{-4}^{4}\left(R^2-r^2\right)dx=\pi\int_{-4}^{4}\left(\left(8-(-1)\right)^2-\left(\frac{1}{2}x^2-(-1)\right)^2\right)dx=$$

$$2\pi\int_{0}^{4}\left(81-\left(\frac{1}{4}x^4+x^2+1\right)\right)dx=2\pi\int_{0}^{4}\left(-\frac{1}{4}x^4-x^2+80\right)dx$$

$$=2\pi\left[-\frac{1}{20}x^5-\frac{1}{3}x^3+80x\right]_0^4=\frac{7424\pi}{15}$$

3. Washer Method

$$V=\pi\int_{1}^{3}\left(R^2-r^2\right)dx=\pi\int_{1}^{3}\left(\left(1+\frac{1}{x}\right)^2-1^2\right)dx=8.997$$

5. Washer Method

$$V=\pi\int_{0}^{1}\left(R^2-r^2\right)dx=\pi\int_{0}^{1}\left(\left(1-x^3\right)^2-\left(1-\sqrt{x}\right)^2\right)dx$$

$$\pi\int_{0}^{1}\left(\left(1-2x^3+x^6\right)-\left(1-2\sqrt{x}+x\right)\right)dx=\pi\int_{0}^{1}\left(x^6-2x^3-x+2x^{1/2}\right)dx$$

$$=\pi\left(\frac{1}{7}x^7-\frac{1}{2}x^4-\frac{1}{2}x^2+\frac{4}{3}x^{3/2}\right)_0^1=\frac{10\pi}{21}$$

7. $y=\sqrt{x+1}\rightarrow x=y^2-1$

Disk Method: $V=\pi\int_{0}^{2}\left(\left[3-\left(y^2-1\right)\right]^2\right)dy=\pi\int_{0}^{2}\left[9-\left(y^4-2y^2+1\right)\right]dy$

$$=\pi\left[-\frac{1}{5}y^5+\frac{2}{3}y^3+8\right]_0^2=\pi\left[-\frac{32}{5}+\frac{16}{3}+8\right]=\frac{104\pi}{15}$$

9. Washer Method: $V=\pi\int_{-1}^{1}\left(R^2-r^2\right)dy=\pi\int_{-1}^{1}\left(\left(3-x_1\right)^2-\left(3-x_2\right)^2\right)dy$

$$=\pi\int_{-1}^{1}\left(\left(3-y^2\right)^2-(3-1)^2\right)dy=\pi\int_{-1}^{1}\left(y^4-6y^2+9-4\right)dy$$

$$=2\pi\int_{0}^{1}\left(y^4-6y^2+5\right)dy=2\pi\left(\frac{1}{5}y^5-2y^3+7y\right)_{-1}^{1}=\frac{32\pi}{5}$$

11. Washer Method

$$V = \pi \int_{-.916}^{.916} \left(R^2 - r^2\right) dx = \pi \int_{-.916}^{.916} \left(\left(1 - y_1\right)^2 - \left(1 - y_2\right)^2\right) dx$$

$$= \pi \int_{-.916}^{.916} \left(\left(1 - \ln\left(x^2 + 1\right)\right)^2 - (1 - \cos x)^2\right) dx = 3.447$$

13. Washer Method $y = \sqrt{6x - x^2} \rightarrow y^2 = 6x - x^2 \rightarrow -y^2 = x^2 - 6x$

$$-y^2 + 9 = x^2 - 6x + 9 = (x - 3)^2 \rightarrow 9 - y^2 = (x - 3)^2 \rightarrow x = 3 + \sqrt{9 - y^2}$$

$$g(x) = y = -1 + e^{-0.5x} \rightarrow y + 1 = e^{-0.5x} \rightarrow \ln(y + 1) = -0.5x \rightarrow -2\ln(y + 1)$$

$$V = \pi \int_0^{2.9910233} \left[\left[(-2\ln(y + 1)) - (-1)\right]^2 - \left[\left(3 + \sqrt{9 - y^2}\right) - (-1)\right]^2\right] dy$$

$$= 374.525$$

6.5 Multiple Choice Solutions

1. This is a Washer Method problem. Both curves are below the line $y = 3$, so $V = \pi \int_a^b \left(\left[k - f(x)\right]^2 - \left[k - g(x)\right]^2\right) dx$. $y = 2x$ is closer to the line, so

The correct answer is C

---

3. This is a Disk Method problem. $V = \pi \int_0^1 \left[3 - \tan^{-1} x\right]^2 dx = 20.773$

The correct answer is E

---

5. $$Volume = \int_0^2 \left(\pi - \left(\frac{\pi}{2} - \sin^{-1}(x - 1)\right)\right)^2 dx + \int_2^6 \left(\pi - \frac{\pi}{2}\sqrt{x - 2}\right)^2 dx$$

$$= 39.111$$

The correct answer is C

6.6 Free Response Solutions

1. $A = \left(\sqrt{x}\right)^2 = x \qquad V = \int_0^4 x^2 dx = \left[\frac{1}{3}x^3\right]_0^4 = \frac{64}{3}$

3. $y = \sqrt{2x} \to x = \frac{y^2}{2}; \quad y = \frac{1}{8}x^4 \to x = \sqrt[4]{8y}$; Rectangle length = $\sqrt[4]{8y} - \frac{y^2}{2}$

$$A = 3s^2 = 3\left(\sqrt[4]{8y} - \frac{y^2}{2}\right)^2 \qquad V = \int_0^2 3\left(\sqrt[4]{8y} - \frac{y^2}{2}\right)^2 dy = 6.031$$

5. Rectangle length = $\sqrt{x} - \frac{1}{2}x$

$$A = s^2 = \left(\sqrt{x} - \frac{1}{2}x\right)^2 \qquad V = \int_0^4 \left(\sqrt{x} - \frac{1}{2}x\right)^2 dx = 0.533$$

7. Rectangle length = $e^{-.5x} - x^2 = w$

$$A = \frac{1}{2}w^2 = \frac{1}{2}\left(e^{-.5x} - x^2\right)^2 \qquad V = \int_{-1.429612}^{.81555342} \frac{1}{2}\left(e^{-.5x} - x^2\right)^2 dx = 0.687$$

9. $V = \int_{-2}^{2} \frac{1}{2} bh dx = \int_{-2}^{2} \frac{1}{2} \cdot \frac{2}{\sqrt{2}} y \cdot \frac{2}{\sqrt{2}} y dx = \int_{-2}^{2} y^2 dx = \int_{-2}^{2} \left(9 - \frac{9}{4}x^2\right) dx = 24$

11. $V = \int_{-1}^{1} (side)^2 dx$

$$= \int_{-1}^{1} \left(1 - x^2\right)^2 dx = 2\int_0^1 \left(1 - 2x^2 + x^4\right) dx = 2\left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right]_0^1 = \frac{16}{15}$$

13. $V = \int_{.301}^{1} \frac{1}{2}\pi r^2 dx = \frac{\pi}{2}\int_{.301}^{1} \left(\frac{\sqrt{x} - e^{-2x}}{2}\right)^2 dx = .085$

15. $V = \int_1^4 \frac{1}{x^2}dx = \left[\frac{x^{-1}}{-1}\right]_1^4 = -\frac{1}{4} - (-1) = \frac{3}{4}$

17(a) $V \approx (36\pi)\cdot 3 + (25\pi)\cdot 3 + (9\pi)\cdot 3 + (4\pi)\cdot 3 + (9\pi)\cdot 3 + (25\pi)\cdot 3 + (36\pi)\cdot 3 = 582\pi$

17(b) $V = \pi\int_0^{27}\left(4 + 2\cos\left(\frac{\pi}{27}h\right)\right)^2 dh = 13186.456$

6.6 Multiple Choice Solutions

1. $V = \int_0^3 \left(e^{-x^2/2}\right)^2 dx = \int_0^3 \left(e^{-x^2}\right) dx = 0.886$

The correct answer is A

---

3. Perpendicular to the *y*-axis means the *x* must be isolated:

$x + 3y = 5 \rightarrow x = 5 - 3y \rightarrow \quad V = \int_0^{5/3} (5-3y)^2\, dy = 13.889$

The correct answer is E

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5. $Volume = \int_0^{\pi/4} \frac{\pi}{2}\left(\frac{\cos x - \sin x}{2}\right)^2 dx = 0.112$

The correct answer is B

---

7. $x^2 + 4y^2 = 4 \rightarrow y = \sqrt{1 - \frac{1}{4}x^2} \rightarrow$

$$V = \int_{-2}^{2} \left(\sqrt{1 - \frac{1}{4}x^2} \right)^2 dx = 2\int_{0}^{2} \left(1 - \frac{1}{4}x^2 \right) dx = 2\left[x - \frac{1}{12}x^3 \right]_0^2 = 2\left[2 - \frac{2}{3} \right] = \frac{8}{3}$$

The correct answer is A

---

9. $V = \int_{0}^{2} \frac{1}{2}\left(\pi - \left(\frac{\pi}{2} - \sin^{-1}(x-1) \right) \right)^2 dx + \int_{2}^{6} \frac{1}{2}\left(\pi - \left(\frac{\pi}{2}\sqrt{x-2} \right) \right)^2 dx$

$= 6.225$

The correct answer is A.

6.7 Free Response Solutions

1(a) $Area = \int_0^2 \left(2x^2 - x^3\right) dx = \left[\frac{2}{3}x^3 - \frac{1}{4}x^4\right]_0^2 = \frac{16}{3} - 4 = \frac{4}{3}$

(b) $V = \frac{\pi}{8}\int_a^b r^2 dx = \frac{\pi}{8}\int_0^2 \left(\frac{2x^2 - x^3}{2}\right)^2 dx$

(c) The rectangles need to be horizontal, so $y = 2x^2 \rightarrow x = \sqrt{\frac{y}{2}}$ and $y = x^3 \rightarrow x = y^{1/3}$.

Washer Method: $V = \pi\int_0^8 \left(\left[y^{2/3}\right]^2 - \left[\sqrt{\frac{y}{2}}\right]^2\right)dy = \pi\int_0^8 \left(y^{4/3} - \frac{1}{2}y\right)dy =$

$= \pi\left[\frac{3}{7}y^{7/3} - \frac{1}{4}y^2\right]_0^8 = \pi\left(\frac{128}{7} - 16\right) = \frac{16\pi}{7}.$

3(a) $Area = \int_{-1}^3 (x+1)^{1/2}\, dx = \left[\frac{2}{3}(x+1)^{3/2}\right]_{-1}^3 = \frac{16}{3} - 0 = \frac{16}{3}$

(b) $Volume = \int_{-1}^3 (x+1)^{1/2}\left(2(x+1)^{1/2}\right) dx = \int_{-1}^3 (2x+2)\, dx =$

$\left[x^2 + 2x\right]_{-1}^3 = 15 - (-1) = 16$

(c) Washer Method with horizontal rectangles:

$y = \sqrt{x+1} \rightarrow x = y^2 - 1 \qquad V = \pi\int_0^2 \left(\left[4 - \left(y^2 - 1\right)\right]^2 - [4-3]^2\right)dy$

5(a) $A = \int_0^4 \left[-6\left(x^2-4x\right)e^{-x} - \left(x^2-4x\right)\right] dx = 23.326$

(b) $V = \pi \int_0^4 \left[\left[-6\left(x^2-4x\right)e^{-x}+4\right]^2 - \left(x^2-4x+4\right)^2\right] dx = 676.640$

(c) $V = \int_0^4 \left[-6\left(x^2-4x\right)e^{-x} - \left(x^2-4x\right)\right]^2 dx = 170.182$

7(a) $A = \int_0^3 \left[\frac{9}{2x+3}\right] dx = \frac{9}{2}\int_0^3 \left[\frac{1}{2x+3}\right] 2dx = \frac{9}{2}\int_3^9 \frac{1}{u}\, du = \frac{9}{2}\left[\ln u\right]_3^9$

$= \frac{9}{2}\left[\ln 9 - \ln 3\right] = \frac{9}{2}\ln 3$

(b) $y = \frac{9}{2x+3} \rightarrow \frac{dy}{dx} = -9(2x+3)^{-2}(2) = \frac{-18}{(2x+3)^2}$

$P = 3+3+1+\int_0^3 \sqrt{1+\left(\frac{-18}{(2x+3)^2}\right)^2}\, dx = 10.747$

(c) $V = \pi \int_0^3 \left(\frac{9}{2x+3}\right)^2 dx = \frac{81\pi}{2}\int_0^3 (2x+3)^{-2}\, 2dx = \frac{81\pi}{2}\left[\frac{(2x+3)^{-1}}{-1}\right]_0^3$

$= \frac{81\pi}{2}\left[-\frac{1}{9} - \left(-\frac{1}{3}\right)\right] = 9\pi = 28.274$

9(a) $A = \int_0^1 \left[4x(1-x) - \sqrt[4]{2x}(x-1)\right] dx = 1.089$

(b) $V = \int_0^1 \left[\left[3-4x(1-x)\right]^2 - \left[3-\sqrt[4]{2x}(x-1)\right]^2\right] dx = 6.630$

(c) $V = \int_0^1 \left[\left[4x(1-x)+2\right]^2 - \left[\sqrt[4]{2x}(x-1)+2\right]^2\right] dx = 17.665$

(d) $V = \int_0^1 \left[\left[k - x(1-x)\right]^2 - \left[k - \sqrt[4]{2x}(x-1)\right]^2\right] dx$

11(a) $Area_R = \int_0^{1.1418956} \left[\left(x^3 - 3x^2 + 2x + 4\right) - \left(2x\sqrt{4-x}\right)\right] dx = 2.463$

$Area_S = \int_{1.1418956}^{2.5444495} \left[\left(2x\sqrt{4-x}\right) - \left(x^3 - 3x^2 + 2x + 4\right)\right] dx = 1.551$

$Area_T = \int_0^4 \left[2x\sqrt{4-x}\right] dx - Area_S = 15.515$

(b) $V = \pi \int_1^3 \left(8 - 2x\sqrt{4-x}\right)^2 dx = 48.152$

(c) $V = \int_{1.1418956}^{2.5444495} \left[\left(2x\sqrt{4-x} - (-1)\right)^2 - \left(x^3 - 3x^2 + 2x + 4 - (-1)\right)^2\right] dx$
$= 55.727$

(d) $V = \int_0^{1.1418956} \left[\left(x^3 - 3x^2 + 2x + 4\right) - \left(2x\sqrt{4-x}\right)\right]^2 dx = 6.962$

13(a) $A = \int_0^{30} w(x)\ dx \approx \left(\frac{0 + 14.6}{2}\right)(5) + \left(\frac{14.6 + 13.5}{2}\right)(5) + \left(\frac{13.5 + 14.5}{2}\right)(5) +$
$+ \left(\frac{14.5 + 17.4}{2}\right)(5) + \left(\frac{17.4 + 11.6}{2}\right)(5) + \left(\frac{11.6 + 0}{2}\right)(5) = 359 ft^2$

(b) $V = \int_0^{30} w(x) \cdot h(x)\ dx \approx (3 \cdot 14.6)(10) + (6 \cdot 14.5)(10) + (8 \cdot 11.6)(10)$
$= 2236 ft^3$

(c) $w'(15) \approx \frac{17.4 - 13.5}{20 - 10} = .39$. Tangent Line:

(d) $x = 2t \Rightarrow$

$$f(2t) = -.0001565(2t+1)^4 + 0.007969(2t+1)^3 - 0.12954(2t+1)^2 + 1.06724(2t+1) - 0.931776$$

$$f'(2t) = -.000626(2t+1)^3(2) + 0.023907(2t+1)^2(2) - 0.25908(2t+1)(2) + 1.06724(2)$$

$$f'(2(8)) = 0.9929 \text{ ft/sec}$$

15(a) Disk Method with horizontal rectangles: $y = \frac{1}{8}x^5 \rightarrow x = (8y)^{1/5}$

$$V = \pi \int_0^4 \left[(8y)^{1/5}\right]^2 dy = \pi \int_0^4 \left[(8y)^{2/5}\right] dy = \frac{\pi}{8} \int_0^4 (8y)^{2/5} \, (8dy)$$

$$= \frac{\pi}{8}\left[\frac{5}{7}(8y)^{7/5}\right]_0^4 = \frac{20\pi}{7} \text{ ft}^3$$

(b) $$V = \pi \int_0^h (8y)^{2/5} dy = \frac{\pi}{8}\left[\frac{5}{7}(8y)^{7/5}\right]_0^h = \frac{5\pi}{56}(8h)^{7/5}$$

(c) $$\frac{d}{dt}\left[V = \frac{5\pi}{56}(8h)^{7/5}\right] \rightarrow \frac{dV}{dt} = \frac{5\pi}{56}\left[\frac{7}{5}(8h)^{2/5}(8)\right]\frac{dh}{dt} = \pi(8h)^{2/5}\frac{dh}{dt}$$

$$\left.\frac{dV}{dt}\right|_{h=3} = \pi(8(3))^{2/5}\left(-\frac{1}{5}\right) = -7.800 \text{ ft}^2/\text{hr}$$

(d) $A = \pi r^2$ and $r = x = (8y)^{1/5}$, so

$$\frac{dV}{dt} = \pi(8h)^{2/5}\frac{dh}{dt} = \pi\left[(8h)^{1/5}\right]^2\frac{dh}{dt} = A\frac{dh}{dt}$$

17(a) $$V = \pi \int_0^4 \left(5x^{1/2}\right)^2 dx = 25\pi \int_0^4 x \, dx = 25\pi\left[\frac{x^2}{2}\right]_0^4 = 200\pi = 628.319 in^3$$

(b) $$y = 5\sqrt{x} \rightarrow \frac{dy}{dx} = \frac{5}{2}x^{-1/2}. \quad L = \int_{-4}^4 \sqrt{1 + \left(\frac{5}{2}x^{-1/2}\right)^2} \, dx = 21.965 \, in$$

(c) $\tan\theta = \dfrac{y}{4-x}$

(d) $\dfrac{d}{dt}\left[\tan\theta = \dfrac{y}{4-x}\right] \rightarrow \sec^2\theta \dfrac{d\theta}{dt} = \dfrac{(4-x)\frac{dy}{dt} - y\left(-\frac{dx}{dt}\right)}{(4-x)^2}$

$\theta = \tan^{-1}\dfrac{5}{3} = 1.030$

$\left(\sec^2 1.030\right)\dfrac{d\theta}{dt} = \dfrac{3(-.1)-5(-.03)}{3^2} \rightarrow \dfrac{d\theta}{dt} = -0.013 \ {}^{rad}/_{\min}$

19(a) $V = \pi\int_{-18}^{18}\left(\frac{1}{108}x^2 - 14\right)^2 dx = \pi\int_{-18}^{18}\left(\frac{1}{11664}x^4 - \frac{7}{27}x^2 - 14\right) dy$

$V = \pi\left[\frac{1}{11664}\frac{x^5}{5} - \frac{7}{27}\frac{x^3}{3} - 14x\right]_{-18}^{18} = 19{,}203.928 in^3$

(b) $A = 2\pi(11)^2 + 2\pi\int_{-18}^{18}\left(\frac{1}{108}x^2 - 14\right)\sqrt{1+\left[\frac{1}{54}x\right]^2} dx = 760.265 + 2990.835 = 3751.100 \ in^2$

(c) $V_{cyl} = \pi(12.5)^2 h \rightarrow \frac{d}{dt}\left[V_{cyl} = \pi(12.5)^2 h\right] \rightarrow \frac{dV}{dt} = (12.5)^2\pi\frac{dh}{dt}$

$-4800 = (12.5)^2\pi\frac{dh}{dt} \rightarrow \frac{dh}{dt} = -9.778 \ {}^{in}/_{hr}$

21(a) $Area = \int_{-4}^{4}\left(8 - \frac{1}{512}r^6\right)dr \rightarrow 2\left[8r - \frac{1}{512}\left(\frac{r^7}{7}\right)\right]_0^4 =$

$Area = 2\left(32 - \frac{1}{512}\left(\frac{4^7}{7}\right)\right) = 64 - \left(\frac{32}{7}\right) = \frac{384}{7}$

(b) $V = \int_0^8 (512h)^{1/3} dh$

(c) $\dfrac{d}{dt}\left[V = \int_0^h (512x)^{1/3} dx\right] \rightarrow \dfrac{dV}{dt} = (512h)^{1/3}\dfrac{dh}{dt}$

$$\left.\frac{dV}{dt}\right|_{t=4} = (512(4))^{1/3}(0.16) = 16(0.16) = 2.56$$

23(a) $$A = \int_0^{12}\left[\frac{2}{5}x + \frac{1}{2}\sin\left(\frac{\pi}{4}x\right)\right]dx$$

$$= \left[\frac{1}{5}x^2 + \frac{1}{2}\frac{\left(-\cos\left(\frac{\pi}{4}x\right)\right)}{\pi/4}\right]_0^{12} = \frac{144}{5} + \frac{2}{\pi}$$

(b) $$V = \frac{\pi}{2}\int_0^x\left[\frac{2}{5}t + \frac{1}{2}\sin\left(\frac{\pi}{4}t\right)\right]^2 dt$$

(c) $$\frac{d}{dt}\left[V_{air} = 150\pi - \frac{25}{2}\pi x - V_K(x)\right] =$$

$$\frac{dV_{air}}{dt} = -\frac{25}{2}\pi - \frac{\pi}{2}\left(\frac{2}{5}x + \frac{1}{2}\sin\left(\frac{\pi}{4}x\right)\right)\frac{dx}{dt}$$

$$\left[\frac{dV_{air}}{dt}\right]_{x=6} = -\frac{25}{2}\pi - \frac{\pi}{2}\left(\frac{2}{5}(6) + \frac{1}{2}\sin\left(\frac{\pi}{4}(6)\right)\right)(300)$$

$$= -\frac{25}{2}\pi - \frac{\pi}{2}\left(\frac{12}{5} + \frac{1}{2}(-1)\right)(300) = -\frac{25}{2}\pi - \frac{570\pi}{2}$$

$$= -\frac{595\pi}{2}\, ft^3/sec$$

6.8 Free Response Solutions

1. $V = \int_0^{\pi/6} 2\pi rl\,dx = \int_0^{\pi/6} 2\pi \cdot x \cdot (\sec x - 1)\,dx = .064$

3. $V = \int_1^8 2\pi rl\,dx = \int_1^8 2\pi \cdot x \cdot \left(x^{-2/3}\right)dx = 70.689$

5. $V = \int_0^1 2\pi rl\,dx = \int_0^1 2\pi \cdot x \cdot y\,dx = \int_0^1 2\pi x(x-1)^2\,dx = .209$

7. $V = \int_{-.916}^{.916} 2\pi rl\,dx = \int_{-.916}^{.916} 2\pi(2-x)\left(\cos x - \ln\left(x^2+1\right)\right)dx - 14.676$

9. $V = \int_{-.767}^{2} 2\pi rl\,dx + \int_2^4 2\pi rl\,dx$

$= \int_{-.767}^{2} 2\pi(1+x)\left(2^x - x^2\right)dx + \int_2^4 2\pi(1+x)\left(x^2 - 2^x\right)dx$

$= 55.428$

Volume Practice Test—Part 1

1. Points of intersection are $x = \pm 1.137292$.

$$A = \int_{-1.137292}^{1.137292} \left[\sqrt{4 - x^2} - \left(s^{x^2} - 2\right)\right] = 5.050$$

The correct answer is D

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3. The rotation is about the *x*-axis, so C and D are incorrect. Washer Method formula is $V = \pi \int_a^b \left(R^2 - r^2\right) dx$, so

The correct answer is B

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5. About the *y*-axis means the *x* is isolated: $x = \sin y$ and $y \in \left[0, \frac{\pi}{2}\right]$.

The correct answer is D

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7. $V = \pi \int_0^{0.833} \left[2^2 - \left[2 - \left(2 - e^{x^2}\right)\right]^2\right]^2 dx = 5.912$

The correct answer is C

Volume Practice Test—Part 2

1(a) $Area = \int_0^8 \frac{8}{x^2+4}dx = 8\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right)_0^8 = 4\tan^{-1}4 = 5.303$

(b) $Volume = \pi\int_0^8 \left(\frac{8}{x^2+4}\right)^2 dx = 0.771$

(c) semicircle area = $\frac{\pi}{2}r^2 = \frac{\pi}{2}\left(\frac{4}{x^2+4}\right)^2$

$Volume = \frac{\pi}{2}\int_0^8 \left(\frac{4}{x^2+4}\right)^2 dx = 2.452$