AP Calculus BC '14-15
Integral Test

Name Solution Key

Score__________

NO CALCULATOR ALLOWED

1. Find $\int_0^1 (x+1)^2\,dx$ $= \left.\frac{(x+1)^3}{3}\right]_0^1 = \frac{2^3}{3} - \frac{1^3}{3}$

a. $\frac{1}{3}$ b. $\frac{2}{3}$ c. $\frac{4}{3}$ d. $\frac{5}{3}$ e. $\frac{7}{3}$

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2. $\int_0^{\frac{\pi}{4}} \cos 2x\,dx$ $= \left.\frac{1}{2}\sin 2x\right]_0^{\pi/4} = \frac{1}{2}\sin \pi/2 - \frac{1}{2}\sin 0$

a. $\frac{\sqrt{3}}{2}$ b. $\frac{\sqrt{2}}{4}$ c. $\frac{1}{\sqrt{2}}$ d. 2 e. $\frac{1}{2}$

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3. $\int_1^2 \frac{1}{\sqrt{4-t^2}}\,dt$ $= \left.\sin^{-1}\frac{t}{2}\right]_1^2 = \sin^{-1} 1 - \sin^{-1} 1/2 = \frac{\pi}{2} - \pi/6$

a. $\frac{\pi}{3}$ b. $\frac{2\pi}{3}$ c. $-\frac{\pi}{3}$ d. $\frac{\pi}{6}$ e. $-\frac{\pi}{6}$

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4. The average value of $y = \sin x$ on $x \in [0, \pi]$ is $\frac{1}{\pi - 0}\int_0^{\pi} \sin x$

$= \left.\frac{1}{\pi}(-\cos x)\right]_0^{\pi}$

$= \frac{1}{\pi}(-(-1) - (-1))$

a. $\frac{1}{\pi}$ b. $\frac{2}{\pi}$ c. $\frac{3}{\pi}$ d. $\frac{4}{\pi}$ e. 2π

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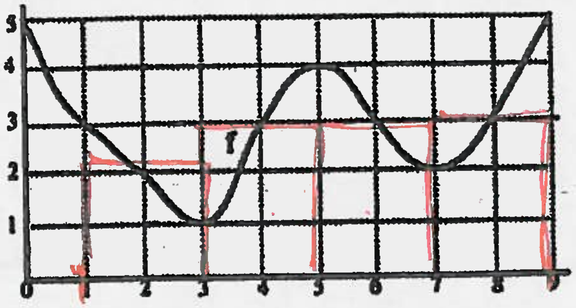
5. Consider the function f whose graph is shown below:



The approximate value of $\int_1^9 f(x)\,dx$, using four midpoint rectangles with equal width, is

a. 21 b. 22 c. 23 d. 24 e. 25

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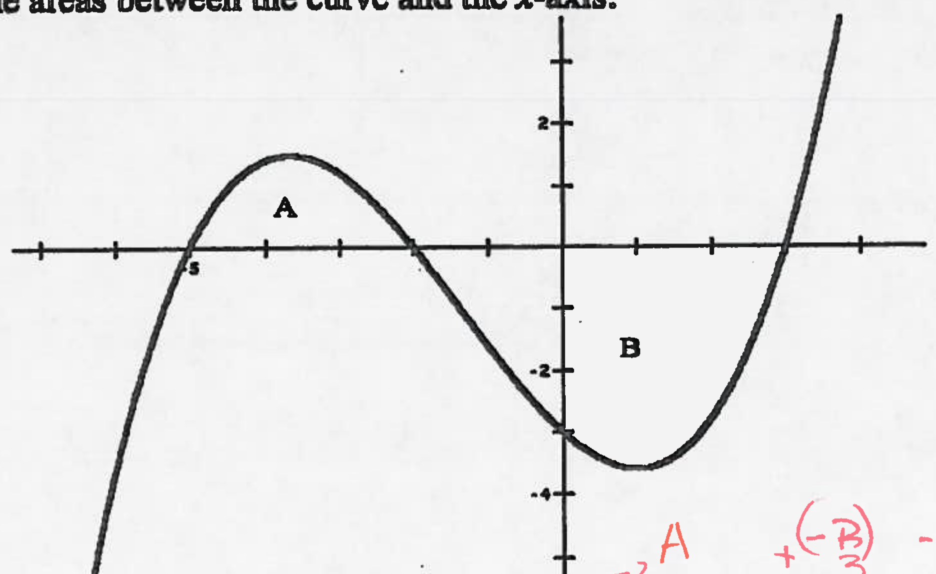
6. The graph of $y = f(x)$ is shown below. A and B are positive numbers that represent he areas between the curve and the x-axis.



In terms of A and B, $\int_{-5}^{3} f(x)\,dx - \int_{-2}^{3} f(x)\,dx =$ $\underbrace{\int_{-5}^{-2} f(x)dx}_{A} + \underbrace{\int_{-2}^{3}}_{+(-B)} - \underbrace{\int_{-2}^{3}}_{-(-B)} = A$

a. A b. $A-B$ c. $2A-B$ d. $A+B$ e. $A+2B$

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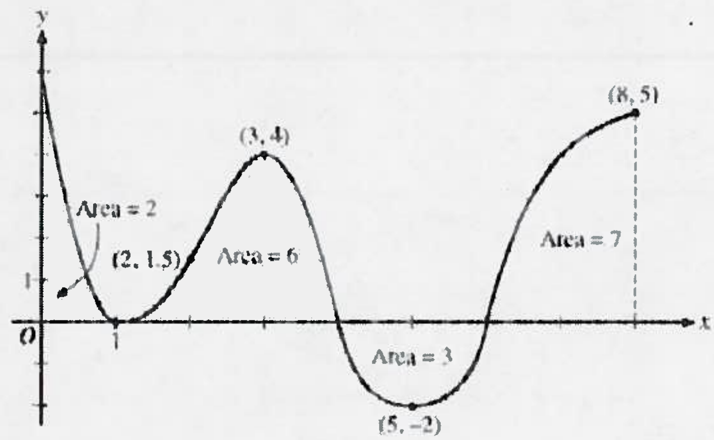
1. The figure below shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of has horizontal tangent lines at $x=1$, $x=3$, and $x=5$. The areas of the regions between the graph of and the -axis are labeled in the figure. The function is defined for all real numbers and satisfies $f(8)=4$.



Graph of f'

1

a. Find all values of x on the open interval $0 < x < 8$ for which the function has a local minimum. Justify your answer.

X = 6 IS A MIN BECAUSE IT IS A ZERO FOR $f'(x)$ AND f' SWITCHES FROM − TO +

3

b. Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

POSSIBLE MINS AT $X=0$ & $X=6$

$f(0) = f(8) + \int_8^0 f'(x)\,dx = \cancel{8}\ 4-12=-8$

$f(6) = f(8) + \int_8^6 f'(x)\,dx = 4-7=-3$

ABS MIN = −8

2

c. On what open intervals contained in $0<x<8$ is the graph of f both concave down and increasing? Explain your reasoning.

f INCREASING MEANS $f'>0$ $\therefore$ f' GRAPH IS ABOVE THE X-AXIS

f CONCAVE DOWN MEANS $f''<0$, WHICH MEANS f' GRAPH IS DECREASING.

f' ABOVE X-AXIS AND DECREASING IS

$x \in (0,1)$ AND $(3,4)$

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d. The function g is defined by $g(x)=(f(x))^3$. If $g(3)=-\frac{5}{2}$ find the slope of the line tangent to the graph of at $x=3$.

$g(3)=(f(3))^3=\frac{-5}{2} \rightarrow f(3)=\left(\frac{-5}{2}\right)^{1/3}$

$g'(3)=3(f(3))^2 f'(3)$

$=3\left(\left(\frac{-5}{2}\right)^{1/3}\right)^2(4)=12(-2.5)^{2/3}=22.144$

t hours	0	2	5	7	8
E(t) (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

1

a. Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

$$\frac{21-13}{7-5} = 4$$

3

b. Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

$$\approx \frac{1}{8}\left[2\left(\frac{4+0}{2}\right) + 3\left(\frac{13+4}{2}\right) + 2\left(\frac{21+13}{2}\right) + 1\left(\frac{23+21}{2}\right)\right]$$

$$= \frac{85.5}{8} = 10.687$$

MEANS THE AVERAGE # OF HUNDREDS OF ENTRIES OVER THE 8 HOUR PERIOD

2

c. At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$) ?

$= 23 - \int_{8}^{12} P(t)\,dt = 23 - 16 = 7$ HUNDRED ENTRIES

3

d. According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

$P'(t) = 0 = 3t^2 - 60t + 298$

$t = 9.184,\ 10.816$

+ 0 − 0 +

8 9.184 10.816 12

CHECK END POINTS

t	$P(t)$
8	0
9.184	5.089
10.816	2.911
12	8

$P(t)$ IS MAX AT $t = 12$

3. The number of parts per million (ppm), $C(t)$, of chlorine in a pool changes at the rate of $P'(t)=1-3e^{-0.3\sqrt{t}}$ ounces per day, where t is measured in days. There are 50 ppm of chlorine in the pool at time $t=0$. Chlorine should be added to the pool if the level drops below 40 ppm.

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a. Is the amount of chorine increasing or decreasing at $t=9$? Why or why not?

2

$P'(9) = -.220 < 0 \therefore$ THE AMOUNT IS DECREASING

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b. For what value of t is the amount of chlorine at a minimum? Justify your answer.

3

$$P'(t) = 1-3e^{-.3\sqrt{t}} = 0$$

$$t = 13.411$$

P' SWITCHES FROM $-$ TO $+$ $\therefore$ MINIMUM

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c. **When the value of chlorine is at a minimum, does chlorine need to be added? Justify your answer.**

4

$$P(13.411) = 50 + \int_0^{13.411} p'(t)\,dt$$

$$= 43.380 > 40$$

$\therefore$ NO CHLORINE NEEDS TO BE ADDED.