BC Calculus '18-19 Name ~~Solution Key~~

Numerical Series Test

No Calculator Allowed Score__________.

1. Which of the following sequences ~~diverge~~ CON verge?

I. $\left\{\dfrac{4^n}{5^n}\right\}$ CON II. $\left\{\dfrac{4^n}{5n}\right\}$ DIV III. $\left\{\dfrac{4n}{5n}\right\}$ CON

(A) I only (B) I and II only (C) I and III only

(D) II and III only (E) III only

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2. Which of the following ~~sequences~~ SERIES converge?

I. $\sum_{n=1}^{\infty}\dfrac{(-1)^n}{n}$ C II. $\sum_{n=1}^{\infty}\dfrac{1}{n}$ D III. $\sum_{n=1}^{\infty}\dfrac{(-2)^n}{n}$ D

(A) I only (B) I and II only (C) I and III only

(D) II and III only (E) III only

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3. If $a_n = \dfrac{2n^3 - 1}{3n^3 + 8}$, which of the following converge?

I. $\{a_n\}$ CONV BY SEQ TEST

$\lim a_n = 2/3$

II. $\sum_{n=1}^{\infty} a_n$ DIV BY DIVERGENCE TEST

(A) I only (B) II only (C) Both I and II

(D) Neither I nor II (E) Not enough information

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4. Which of these series converge absolutely?

DIV

I. $\sum_{n=1}^{\infty} \dfrac{(-3)^n}{2^n} = (-1)^n \left(\frac{3}{2}\right)^n$

II. $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n^{1.5}}$ ABS CON

III. $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n^{0.3}}$ CONDITIONAL CON

(A) I only (B) II only (C) III only

(D) II and III only (D) I, II, and III

$a_2 = \frac{4}{49}$

$r = 4/7$

$S = \frac{4/49}{1 - 4/7}$

$= \frac{4}{21}$

5. The geometric series $\sum_{n=2}^{\infty} \frac{4^{n-1}}{7^n}$ is equal to:

(A) 0 (B) $\frac{4}{49}$ (C) $\frac{4}{21}$ (D) $\frac{4}{7}$ (E) $\frac{7}{3}$

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6. Which of these tests will prove the convergence or divergence of $\sum_{k=1}^{\infty} \frac{2k^3}{k^4 + k^2}$?

~~I.~~ Direct comparison with $\sum_{k=1}^{\infty} \frac{2}{k}$ THE GIVEN SERIES $< \frac{2}{k}$ BUT $\frac{2}{k}$ DIVERGES

II. Limit comparison with $\sum_{k=1}^{\infty} \frac{1}{k}$

~~III.~~ nth Term Test (Divergence Test)

(A) I only (B) II only (C) I and II only

(D) II and III only (D) I, II, and III

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7. If $\sum_{n=1}^{\infty} b_n$ is known to diverge, and $a_n \le b_n$ for all values of n, then we can conclude:

(A) $\sum_{n=1}^{\infty} a_n$ converges

(B) $\sum_{n=1}^{\infty} a_n$ diverges

(C) $\sum_{n=1}^{\infty} b_n$ converges

(D) $\lim_{n\to\infty} b_n = 0$

(E) No conclusion can be drawn from this information

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8. If $f(x) = x^n$, find the value of $\sum_{n=2}^{4} f(x)$ when $x = 2$.

$= \sum_{n=2}^{4} 2^n$

$= 2^2 + 2^3 + 2^4$

$= 4 + 8 + 16$

(A) 2^n
(B) $9 \cdot 2^n$
(C) $9n^2$
(D) 28
(E) 31

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9. Which of the following limits will lead to a conclusion for a series test?

~~I.~~ $\lim_{n\to\infty} a_n = 0$ II. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0$ III. $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ IF b_n CONV.

(A) I only (B) II only (C) I and III only

(D) II and III only (E) I, II, and III

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10. If $f(x) = \sum_{n=0}^{\infty} \left(\frac{3}{2} - \sec x\right)^n$, then $f\left(\frac{\pi}{3}\right) =$

(A) 2 (B) $\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) $\frac{3}{2}$ (E) divergent

$\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n = \frac{1}{1-(-1/2)} =$

BC Calculus '18-19
Numerical Series Test
Calculator Allowed

Name SOLUTION KEY

Score__________.

1. Use the Integral Test to determine if $\sum_{n=2}^{\infty} \frac{n^2}{\sqrt{n^3+1}}$ is convergent or divergent.

$\int_2^{\infty} \frac{x^2}{\sqrt{x^3+1}}\,dx \qquad u = x^3+1$
$\qquad du = 3x^2\,dx$

$= \lim_{b\to\infty} \frac{1}{3}\int_9^{b} u^{-1/2}\,du$

$= \lim_{b\to\infty} \frac{1}{3}\left.\frac{u^{1/2}}{1/2}\right]_9^{b}$

$\lim_{b\to\infty} \frac{2}{3}b^{1/2}$ DNE $\therefore$
THE $\int$ DIVERGES.

SINCE $\int$ DIVERGES, SO DOES THE Σ

2. Does the series $\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{\sqrt{n}}$ diverge, converge conditionally, or converge absolutely? Show the work that leads to your conclusion.

AST: $\lim_{n\to\infty} \left|\frac{\cos \pi n}{n^{1/2}}\right| = 0 \therefore$ CONVERGENT

p-SERIES $\sum \frac{1}{n^{1/2}}$ DIVERGES BECAUSE $p<1$

$\therefore \sum_{n=2}^{\infty} \frac{\cos \pi n}{\sqrt{n}}$ IS CONDITIONALLY CONVERGENT

3. Use the Ratio Test to determine if $\sum_{n=1}^{\infty} \frac{n2^n}{n!}$ is convergent or divergent.

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)2^{n+1}}{(n+1)!}}{\frac{n2^n}{n!}} \right| = \lim_{n\to\infty} \frac{2\cancel{(n+1)}\cancel{2^n}}{\cancel{(n+1)}\cancel{n!}} \cdot \frac{\cancel{n!}}{n\cancel{2^n}}$$

$$= \lim_{n\to\infty} \frac{2}{n} = 0 < 1$$

$\therefore$ THE SERIES IS CONVERGENT

4. Determine whether $\sum_{n=1}^{\infty} \frac{4^n - 1}{6^n}$ converges or diverges. Explain your reasoning.

COMPARE TO $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ WHICH CONVERGES BECAUSE IT IS A GEOMETRIC SERIES WITH $|r|<1$

$$\frac{4^n - 1}{6^n} \leq \left(\frac{2}{3}\right)^n$$

$\therefore \sum_{n=1}^{\infty} \frac{4^n - 1}{6^n}$ CONVERGES BECAUSE IT IS LESS THAN A CONVERGENT SERIES