AP Calculus BC '18-19
Volume Test
CALCULATOR ALLOWED

Name: Solution Key

Score: ______________

1. What is the area enclosed by $y = x\ln(2x+1)$ and $y = 2\sin x$?

(a) 0.334

(b) 0.661

(c) 3.526

(d) 0.825

(e) 2.983

$$\int_0^{1.9869313} \left(2\sin x - x\ln(2x+1)\right) dx$$

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2. A region is bounded by $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, where $m > 0$. The area of this region:

(a) is independent of *m*.
(b) increases as *m* increases.
(c) decreases as *m* increases.
(d) increases until $m = \frac{1}{2}$, then decreases.
(e) is none of the above

$$\int_m^{2m} \frac{1}{x}\, dx$$

$$= \ln x \Big]_m^{2m}$$

$$= \ln 2m - \ln m = \ln 2$$

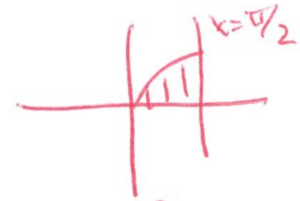
3. Let R be the region in the first quadrant bounded by $x = \sin^{-1} y$, the x-axis, and $x = \frac{\pi}{2}$. Which of the following integrals gives the volume of the solid generated when R is rotated about the x-axis?

$y = \sin x$

(a) $\pi \int_0^{\pi/2} y^2\, dy$ (b) $\pi \int_0^1 \left(\sin^{-1} y\right)^2 dy$

(c) $\pi \int_0^{\pi/2} \left(\sin^{-1} y\right)^2 dy$ (d) $\pi \int_0^1 (\sin x)^2\, dx$

(e) $\pi \int_0^{\pi/2} (\sin x)^2\, dx$



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4. Which of the following integrals gives the length of the graph $y = \sin^2 x$ from $x=a$ to $x=b$?

$\frac{dy}{dx} = 2\sin x \cos x$

(a) $\int_a^b \sqrt{1+4\sin^2 x \cos x}\, dx$ (b) $\int_a^b \sqrt{1+\sin^4 x}\, dx$

(c) $\int_a^b \sqrt{1+2\cos^2 x}\, dx$ (d) $\int_a^b \sqrt{1+\cos^4 x}\, dx$

(e) $\int_a^b \sqrt{1+4\sin^2 x \cos^2 x}\, dx$

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5. What is the slope of the line tangent to $\tan x + 2xy = 3y^2 - 3$ at the point $(0,-1)$?

(a) $\frac{1}{6}$ (b) 0 (c) $-\frac{1}{8}$ (d) $\frac{1}{3}$ (e) undefined

$$\sec^2 x + 2x\frac{dy}{dx} + 2y = 6y\frac{dy}{dx}$$

$$1 + 0 - 2 = -6\frac{dy}{dx}$$

6. Let $\boldsymbol{R}$ represent the region in the first quadrant bounded by $y = -3x + 6$. Which expression gives the volume of the solid with base $\boldsymbol{R}$ whose cross-section perpendicular to the x-axis are semi-circles?

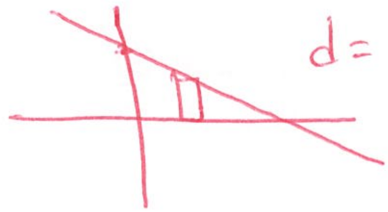
(a) $\pi \int_0^2 (-3x+6)^2 \, dx$

(b) $\pi \int_0^6 \left(\frac{6-y}{3}\right)^2 dy$

(c) $\frac{\pi}{4} \int_0^2 (-3x+6)^2 \, dx$

(d) $\frac{\pi}{8} \int_0^2 (-3x+6)^2 \, dx$

(d) $\frac{\pi}{2} \int_0^6 \left(\frac{6-y}{6}\right)^2 dy$



$d = -3x+6$

$r = \frac{-3x+6}{2}$

$V = \int_0^2 \frac{1}{2}\pi r^2 \, dx$

$= \int_0^2 \frac{\pi}{2}\left[\frac{-3x+6}{2}\right]^2 dx$

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7. What is the volume of the solid formed when the region bounded by $y = 2x + 5$, $x = -1$, and $y = 0$ is rotated around the y-axis?

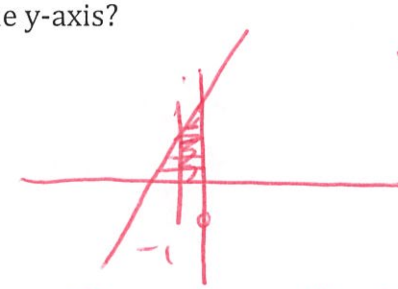
(a) 9.75

(b) 6.75

(c) 21.205

(d) 30.630

(e) 179.594



$x = \frac{y-5}{2}$

$V = \pi \int_0^3 \left(\frac{y-3}{2}\right)^2 dy$

8. Let ***S*** be the region bounded by $y = x^2$ and $y = 4$.

a. Find the area of ***S.*** Show the anti-differentiation steps.

$$\cancel{y}\ A = \int_{-2}^{2} (4 - x^2)\,dx = \cancel{2}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 - \frac{-8}{3}\right)$$

$$= \frac{32}{3} = 10.667$$

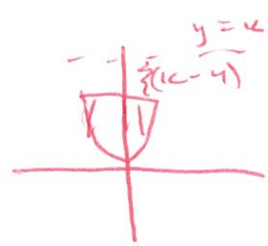
b. Find the volume of the solid generated by revolving ***S*** around the x-axis.

$$\cancel{V = \pi\int_{-2}^{2} (4 - x^2)^2\,dx = 107.233}$$

$$V = \pi\int_{-2}^{2} 4^2 - (x^2)^2\,dx = 160.850$$

c. There exists a number k, $k > 4$, such that when ***S*** is revolved around the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of *k*.

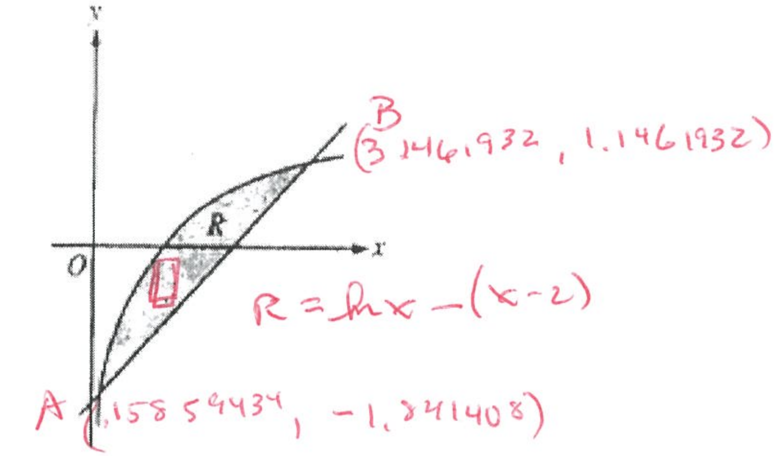
$$V = \pi\int_{-2}^{2} \left[(K - x^2)^2 - (K - 4)^2\right] dx = \cancel{107.233}\ 160.850$$



$r = K - 4$

$R = K - x^2$

9. Let $\boldsymbol{R}$ be the region bounded by $y = \ln x$ and $y = x - 2$.



a. Find the volume of the solid whose base is $\boldsymbol{R}$ and whose cross-sections perpendicular to the x-axis are equilateral triangles with one side in the base.

$$V = \int A\,dx = \int_A^B \frac{\sqrt{3}}{4}\left(\ln x - x + 2\right)^2 dx = .669$$

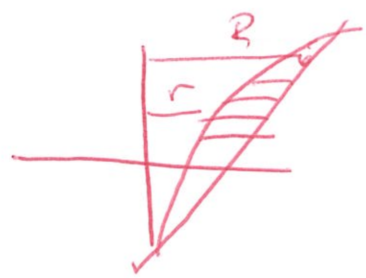
b. Find the volume of the solid formed by revolving $\boldsymbol{R}$ around the y-axis.

$x = e^y \quad x = y + 2$

$$V = \pi \int_{-1.841}^{1.146} \left((y+2)^2 - (e^y)^2\right) dy$$

$= \cancel{4.853\pi = 17.099}$

$= 17.099$



$R = y + 2$

$r = e^y$

10. Let T be the region bounded by $y = \csc\left(\frac{x}{2}\right)$, $x = \frac{\pi}{2}$, and $x = \frac{3\pi}{2}$

a. Find the perimeter of T.

$\frac{dy}{dx} = -\csc\frac{x}{2}\cot\frac{x}{2}\left(\frac{1}{2}\right.$

$P = \csc\pi/4 + \pi + \csc\frac{3\pi}{4} + \int_{\pi/2}^{3\pi/2}\sqrt{1+\left(-\frac{1}{2}\csc\frac{x}{2}\cot\frac{x}{2}\right)^2}$

$= \sqrt{2} + \pi + \sqrt{2} + 3.249$

$= 9.269$

b. Find the volume of the solid formed by revolving T around the x-axis. Show the anti-differentiation steps.

$V = \pi\int_{\pi/2}^{3\pi/2} \csc^2\frac{x}{2}\,dx$ $\quad u = \frac{x}{2} \quad du = \frac{1}{2}x \quad u\left(\frac{\pi}{2}\right) = \pi/4 \quad u\left(\frac{3\pi}{2}\right) = 3\pi/4$

$= 2\pi\int_{\pi/4}^{3\pi/4} \csc^2 u\,du = 2\pi\left(-\cot u\right]_{\pi/4}^{3\pi/4}$

$= 2\pi\left(1 - (-1)\right)$

$= 4\pi$