

**AP CALCULUS BC**  
**Stuff you MUST Know Cold**

**l'Hopital's Rule**

If  $\frac{f(a)}{g(a)} = \frac{0}{0}$  or  $\frac{\infty}{\infty}$ ,

then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

**Average Rate of Change  
(slope of the secant line)**

If the points  $(a, f(a))$  and  $(b, f(b))$  are on the graph of  $f(x)$  the average rate of change of  $f(x)$  on the interval  $[a, b]$  is

$$\frac{f(b) - f(a)}{b - a}$$

**Definition of Derivative  
(slope of the tangent line)**

$$f'(x) = \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} \right)$$

**Derivatives**

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \tan x \sec x$$

$$\frac{d}{dx}(\csc x) = -\cot x \csc x$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u} du$$

$$\frac{d}{dx}(e^u) = e^u du$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(a^u) = a^u (\ln a) du$$

**Properties of Log and Ln**

1.  $\ln 1 = 0$
2.  $\ln e^a = a$
3.  $e^{\ln x} = x$
4.  $\ln x^n = n \ln x$
5.  $\ln(ab) = \ln a + \ln b$
6.  $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

**Differentiation Rules**

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u) \frac{du}{dx}$$

Product Rule

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**Mean Value & Rolle's Theorem**

If the function  $f(x)$  is continuous on  $[a, b]$  and the first derivative exists on the interval  $(a, b)$ , then there exists a number  $x = c$  on  $(a, b)$  such

that  $f'(c) = \frac{f(b) - f(a)}{b - a}$

if  $f(a) = f(b)$ , then  $f'(c) = 0$ .

**Curve sketching and analysis**

$y = f(x)$  must be continuous at each:

critical point:  $\frac{dy}{dx} = 0$  or undefined.

local minimum:  $\frac{dy}{dx}$  goes  $(-, 0, +)$  or

$$(-, \text{und}, +) \text{ or } \frac{d^2 y}{dx^2} > 0$$

local maximum:  $\frac{dy}{dx}$  goes  $(+, 0, -)$  or

$$(+, \text{und}, -) \text{ or } \frac{d^2 y}{dx^2} < 0$$

Absolute Max/Min.: Compare local extreme values to values at endpoints.

pt of inflection : concavity changes.

$$\frac{d^2 y}{dx^2} \text{ goes } (+, 0, -), (-, 0, +),$$

$$(+, \text{und}, -), \text{ or } (-, \text{und}, +)$$

**"PLUS A CONSTANT"**

**The Fundamental Theorem of Calculus**

$$\int_a^b f(x) dx = F(b) - F(a)$$

where  $F'(x) = f(x)$

**2<sup>nd</sup> Fundamental Theorem of Calculus**

$$\frac{d}{dx} \int_{\#}^{g(x)} f(x) dx = f(g(x)) \cdot g'(x)$$

**Average Value**

If the function  $f(x)$  is continuous on  $[a, b]$  and the first derivative exist on the interval  $(a, b)$ , then there exists a number  $x = c$  on  $(a, b)$  such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx$$

$f(c)$  is the average value

**Euler's Method**

If given that  $\frac{dy}{dx} = f(x, y)$  and

that the solution passes through

$(x_0, y_0)$ , then

$$x_{\text{new}} = x_{\text{old}} + \Delta x$$

$$y_{\text{new}} = y_{\text{old}} + \frac{dy}{dx}(x_{\text{old}}, y_{\text{old}}) \cdot \Delta x$$

**Logistics Curves**

$$P(t) = \frac{L}{1 + Ce^{-(Lk)t}}$$

where  $L$  is carrying capacity  
Maximum growth rate occurs when  $P = \frac{1}{2} L$

$$\frac{dP}{dt} = kP(L - P) \text{ or}$$

$$\frac{dP}{dt} = (Lk)P\left(1 - \frac{P}{L}\right)$$

## Integrals

$$\int kf(u)du = k \int f(u)du$$

$$\int du = u + C$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{1}{u} du = \ln |u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \left( \frac{1}{\ln a} \right) a^u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \tan u du = -\ln |\cos u| + C$$

$$\int \cot u du = \ln |\sin u| + C$$

$$\int \sec u du = \ln |\sec u + \tan u| + C$$

$$\int \csc u du = -\ln |\csc u + \cot u| + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \left( \frac{|u|}{a} \right) + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \left( \frac{u}{a} \right) + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \left( \frac{u}{a} \right) + C$$

## Integration by Parts

$$\int u dv = uv - \int v du$$

## Arc Length

For a function,  $f(x)$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

For a polar graph,  $r(\theta)$

$$L = \int_{\theta_1}^{\theta_2} \sqrt{[r(\theta)]^2 + [r'(\theta)]^2} d\theta$$

## Lagrange Error Bound

If  $P_n(x)$  is the  $n$ th degree Taylor polynomial of  $f(x)$  about  $a$ , then

$$|f(b) - P_n(b)| \leq \frac{\max |f^{(n+1)}(c)|}{(n+1)!} |b-a|^{n+1}$$

for all  $c$  between  $a$  and  $b$ .

## Distance, velocity and Acceleration

$$\text{Velocity} = \frac{d}{dt} (\text{position})$$

$$\text{Acceleration} = \frac{d}{dt} (\text{velocity})$$

$$\text{Velocity Vector} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$$

$$\text{Speed} = |v(t)| = \sqrt{(x')^2 + (y')^2}$$

Distance Traveled =

$$\int_{\text{initial time}}^{\text{final time}} |v(t)| dt = \int_{\text{initial time}}^{\text{final time}} \sqrt{(x')^2 + (y')^2} dt$$

$$x(b) = x(a) + \int_a^b x'(t) dt$$

$$y(b) = y(a) + \int_a^b y'(t) dt$$

## Polar Curves

For a polar curve  $r(\theta)$ , the

$$\text{Area inside a "leaf" is } \frac{1}{2} \int_{\theta_1}^{\theta_2} [r(\theta)]^2 d\theta$$

where  $\theta_1$  and  $\theta_2$  are the "first" two times that  $r = 0$ .

The slope of  $r(\theta)$  at a given  $\theta$  is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta} [r(\theta) \sin \theta]}{\frac{d}{d\theta} [r(\theta) \cos \theta]}$$

## Ratio Test

(use for interval of convergence)

The series  $\sum_{n=0}^{\infty} a_n$  converges if

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1 \quad \text{CHECK ENDPOINTS}$$

## Alternating Series Error Bound

If  $S_N = \sum_{n=1}^N (-1)^n a_n$  is the  $N^{\text{th}}$  partial sum of a convergent alternating series, then

$$|S_{\infty} - S_N| \leq |a_{N+1}|$$

## Volume

### Solids of Revolution

$$\text{Disk Method: } V = \pi \int_a^b [R(x)]^2 dx$$

Washer Method:

$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$$

$$\text{Shell Method: } V = 2\pi \int_a^b r(x)h(x)dx$$

### Volume of Known Cross Sections

Perpendicular to

x-axis:

$$V = \int_a^b A(x)dx$$

y-axis:

$$V = \int_c^d A(y)dy$$

## Taylor Series

If the function  $f$  is "smooth" at  $x = c$ , then it can be approximated by the  $n$ th degree polynomial

$$f(x) \approx f(c) + f'(c)(x-c)$$

$$+ \frac{f''(c)}{2!} (x-c)^2 + \dots$$

$$+ \frac{f'''(c)}{3!} (x-c)^3 + \dots$$

$$+ \frac{f^{(n)}(c)}{n!} (x-c)^n$$

## Elementary Functions

Centered at  $x = 0$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

## Most Common Series

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{ converges} \quad \sum_{n=0}^{\infty} A(r)^n \text{ converges to } \frac{A}{1-r} \text{ if } |r| < 1$$