

PreCalculus '13-14

Name: Solution Key

Dr. Quattrin

Ln/Exponential Test-- CALCULATOR ALLOWED

Round to 3 decimal places.

Score \_\_\_\_\_

Show all work.

Formulas:  $S = P \left(1 + \frac{r}{n}\right)^{nt}$

$$S = \frac{P \left( \left(1 + \frac{r}{n}\right)^n - 1 \right)}{\frac{r}{n}}$$
$$L = \frac{P \left( 1 - \left(1 + \frac{r}{n}\right)^{-n} \right)}{\frac{r}{n}}$$

1. Suppose you can take out a 30 year loan for a \$775,000 house, at a fixed APR of 3.5% compounded monthly. What are your monthly payments and how much will you actually pay the bank?

$$775,000 = \frac{P \left( 1 - \left(1 + \frac{0.035}{12}\right)^{-360} \right)}{(0.035/12)}$$

$$P = \frac{2760.43}{3480.10} = 3480.10$$

$$\text{TOTAL} = 360 (3480.10) = \$1,252,834.68$$

2. Suppose you invest \$400 per month at 4.5% for 10 years, then invest the total at 10% compounded monthly for 20 years. How much will you have 30 years from now?

$$S = \frac{400 \left( \left(1 + \frac{0.045}{12}\right)^{120} - 1 \right)}{(0.045/12)} = 60,479.23$$

$$S = (60,479.23) \left(1 + \frac{0.10}{12}\right)^{240} = \$443,196.25$$

$$\begin{aligned}
 3a. \quad \frac{d}{dx} [5x^2 e^{-x} + 1] &= 5x^2 e^{-x} (-1) + e^{-x} (10x) \\
 &= 5x e^{-x} (2-x)
 \end{aligned}$$

$$3b. \quad D_x [(4x-3)^5 (2x^2+1)^{10}]$$

$$\begin{aligned}
 &(4x-3)^5 (10(2x^2+1)^9 (4x)) + (2x^2+1)^{10} 5(4x-3)^4 (4) \\
 &\quad 40x (4x-3)^5 (2x^2+1)^9 + 20(2x^2+1)^{10} (4x-3)^4 \\
 &= 20 (4x-3)^4 (2x^2+1)^9 [2x(4x-3) + (2x^2+1)] \\
 &= 20 (4x-3)^4 (2x^2+1)^9 (10x^2 - 6x + 1)
 \end{aligned}$$

4. Find the zeros and Domain of  $y = 8(2x - x^2)e^{-2x}$ . Show the supporting algebraic work.

$$\text{Zeros: } (0, 0), (2, 0)$$

Domain: ALL REALS

5. Find the critical values and extreme values of  $y = 8(2x - x^2)e^{-2x}$ . Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = 8(2x - x^2)e^{-2x}(-2) + 8e^{-2x}(2 - 2x)$$

$$= -16e^{-2x} [2x - x^2 + x - 1]$$

$$= -16e^{-2x} (-x^2 + 3x - 1) = 0$$

$$x = \frac{-3 \pm \sqrt{5}}{-2} = \begin{cases} 0.382 \\ 2.618 \end{cases}$$

$$\begin{aligned} u &= 8(2x - x^2) \\ &= 16x - 8x^2 \\ du &= 16 - 16x \\ v &= e^{-2x} \\ dv &= e^{-2x}(-2) \end{aligned}$$

$$y(0.382) = 2.303$$

$$y(2.618) = -0.069$$

6. Find the traits and sketch  $y = 8(2x - x^2)e^{-2x}$ .

Domain: All Reals

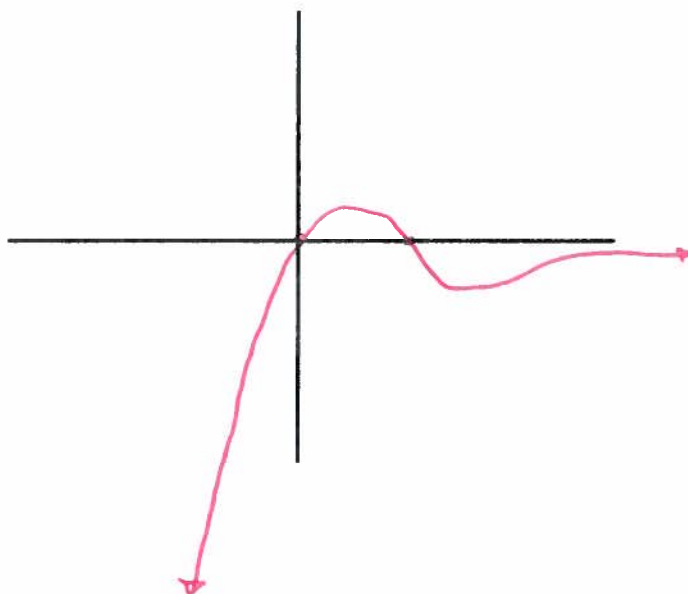
Range:  $y \in (-\infty, 2.303]$

Y-Int:  $(0, 0)$

End Behavior: RIGHT:  $y = 0$   
LEFT: DOWN

Zeros:  $(0, 0)$   $(2, 0)$

Extreme Points:  
 $(.382, 2.303)$   
 $(2.618, -.069)$



7. Find the zeros and Domain of  $y = (x^2)\sqrt{16-x^2}$ . Show the supporting algebraic work.

Domain:  $16-x^2 \geq 0 \Rightarrow x \in [-4, 4]$

Zeros:  $(0, 0)$   $(\pm 4, 0)$

8. Find the critical values and extreme values of  $y = (x^2)\sqrt{16-x^2}$ . Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = \cancel{2x} x^2 \left[ \frac{-1}{2} (16-x^2)^{-1/2} (-2x) \right] + (16-x^2)^{1/2} (2x)$$

$$= \frac{-x^3}{(16-x^2)^{1/2}} + 2x(16-x^2)^{1/2} = \frac{-x^3 + 2x(16-x^2)}{(16-x^2)^{1/2}}$$

$$= \frac{-3x^2 + 32x}{(16-x^2)^{1/2}}$$

i)  $\frac{dy}{dx} = 0 \Rightarrow x = 0, \pm \sqrt{\frac{32}{3}} \quad (\pm 3.266, 24.63)$

ii)  $\frac{dy}{dx}$  DNE  $\rightarrow (\pm 4, 0), \cancel{(0, 0)}$

9. Find the traits and sketch of  $y = (x^2)\sqrt{16-x^2}$ .

Domain:  $x \in [-4, 4]$

Y-Int:  $(0, 0)$

Zeros:  $(\pm 4, 0)$   $(0, 0)$

Range:  $y \in [0, 24.634]$

End Behavior: NONE

Extreme Points:  $(\pm 4, 0)$   $(0, 0)$   $(\pm 3.266, 24.634)$

