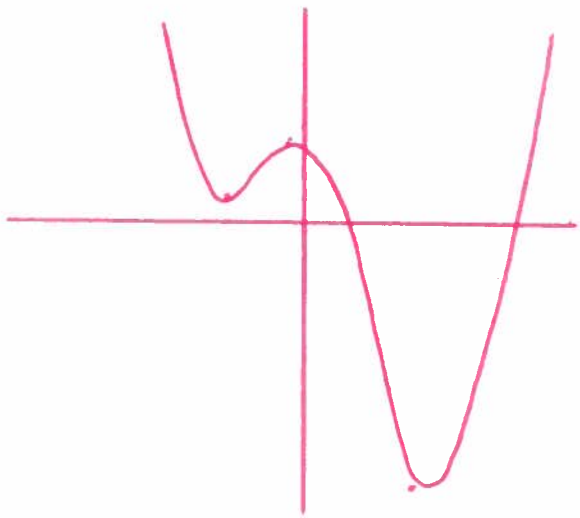
PreCalculus 2013-14
Fall Final --Part I
Dr. Quattrin
<u>Calculator allowed</u>

Name Solution Key

1. Given $f(x) = 8x^4 - 4x^3 - 20x^2 - 5x + 4$, sketch the complete graph and state the window used.



$x \in [-4.7, 4.7]$
$y \in [-23, 10]$

2. Find the Zeros and Extreme Points of $f(x) = 8x^4 - 4x^3 - 20x^2 - 5x + 4$

Zeros: (.337, 0) (1.909, 0)

Extremes: (-.863, .428)
(-.132, 4.323)
(1.370, -22.491)

3. The Eyjafjallajokull volcano in Iceland disrupted air travel during many of the spring and summer days of 2010; pilots were forced to change course in order to protect their jet engines from ash. One such pilot was obliged to zig and zag across the sky to ensure the safety of his passengers and crew. First, he flew 540 km along a bearing of 210°, then turned and flew 360 km along a bearing of 315°. Finally, he turned and flew 190 km along a bearing of 20°. Find the pilot's resultant distance and bearing.

$540\cos 210° \vec{i} + 540 \sin 210° \vec{j}$

$360 \cos 315° \vec{i} + 360 \sin 315° \vec{j}$

$190 \cos 20° \vec{i} + 190 \sin 20 \vec{j}$

$-34.554 \vec{i} - 459.575 \vec{j}$

$|\vec{v}| = \sqrt{34.554^2 + 459.575^2} = 460.872$ km

$\theta = -\cos^{-1} \frac{-34.554}{460.872} = -94.300°$

4. Find:

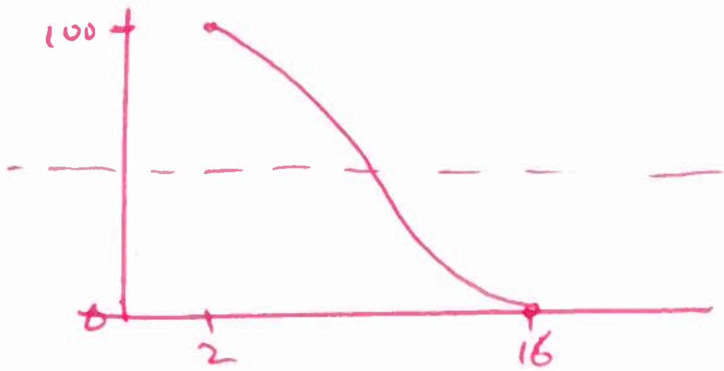
a. $\frac{d}{dx}\left[13x^7 - 21x^4 + 5x^2 - 3x + 4\right] = 91x^6 - 84x^3 + 10x - 3$

b. $\frac{d}{dx}\left[\frac{12}{x^7} - 9\sqrt[3]{x^4} + \sqrt{x^5} - \pi\right] = \frac{d}{dx}\left[12x^{-7} - 9x^{4/3} + x^{5/2} - \pi\right]$

$= -84x^{-8} - 12x^{1/3} + \frac{5}{2}x^{3/2}$

5. Suppose that your attention span in this class is a periodic function and is scaled on a 0 to 100 scale (0 means you are bored, 100 means you are paying attention, 50 is right in the middle). After 2 minutes of class, your attention is at its highest point. At 16 minutes into the class, you hit your lowest point (you are bored!!!).

a. Sketch this curve for a standard 50-minute class.



b. Write an equation for this sinusoid. $A = 50 + 50\cos\frac{\pi}{8}(t-2)$

c. Find what your attention span is at one minute before the bell rings (i.e. 49 minutes into the class)

$A = 50 + 50\cos\left[\frac{\pi}{8}(1-2)\right] = 96.194$

d. Find the first time your attention span is at 75 (you may use the calc – intersect function on your calculator)

$75 = 50 + 50\cos\frac{\pi}{8}(t-2)$

$\cos^{-1}\left[\frac{1}{2} = \cos\frac{\pi}{8}(t-2)\right]$

$\pm\frac{\pi}{3} \pm 2\pi n = \frac{\pi}{8}(t-2)$

$\pm\frac{8}{3} \pm 16n = t-2$

$t = \begin{cases} \frac{14}{3} \pm 16n \\ -\frac{2}{3} \pm 16n \end{cases}$

$t = \frac{14}{3}$ MINUTES

6. Use synthetic substitution to find all the exact zeros algebraically of $f(x)=3x^4-7x^3-25x^2+63x-18$.

-3	3	-7	-25	63	-18
		-9	48	-69	18
	3	-16	23	-6	0

2	3	-16	23	-6
		6	-20	6
	3	-10	3	0

$(x+3)(x-2)(3x^2-10x+3)$

$(x+3)(x-2)(3x-1)(x-3)$

$(-3,0)$
$(2,0)$
$(1/3,0)$
$(3,0)$

7. Use the equation of the line tangent to $y=6x^3-3x^2+5x-4$ at $x=-1$ to approximate $f(-0.9)$

$f(-1)=-18$

$f'=18x^2-6x+5$

$m=f'(-1)=29$

$y+18=29(x+1)$

$f(-.9)\approx y(-.9)=29(-.9+1)-18$

$=2.9-18$

$=15.1$

8. **Find:**

a. $\lim_{x \to -2} \frac{4x+8}{x^2+6x+8} = \lim_{x \to -2} \frac{4(x+2)}{(x+2)(x+4)}$

$= \lim_{x \to -2} \frac{4}{x+4} = 2$

b. $\lim_{x \to 4} \frac{2x^3-5x^2-17x+20}{x^3-4x^2-x+4}$

$$4 \,\rfloor\; 2 \quad -5 \quad -17 \quad 20$$
$$\qquad 8 \quad 12 \quad -20$$
$$2 \quad 3 \quad -5 \quad 0$$

$= \lim_{x \to 4} \frac{(x-4)(2x^2+3x-5)}{(x^2-1)(x-4)}$

$= \frac{2(4)^2+3(4)-(5)}{4^2-1}$

$= \frac{39}{15} = \frac{13}{5}$

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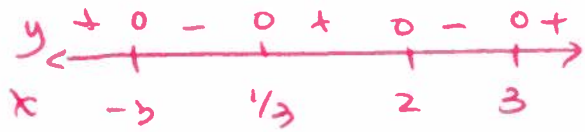
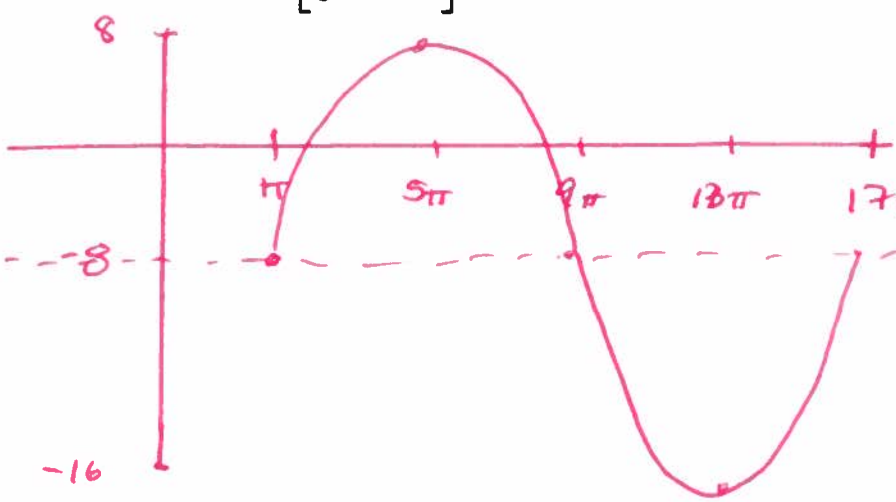
Name Solution Key

1. Show the sign pattern and solve $3x^4 - 7x^3 - 25x^2 + 63x - 18 < 0$. [Note that this is the same polynomial from #5 above]



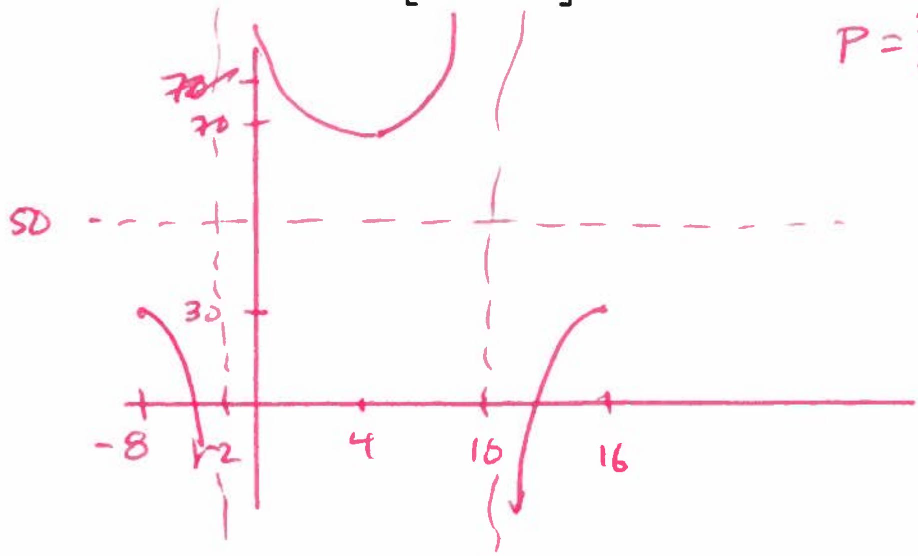
$x \in (-3, 1/3) \cup (2, 3)$

2. Graph $y = -8 + 16\sin\left[\frac{1}{8}(x - \pi)\right]$ over 1 cycle.



3. Graph $y = 50 - 20\sec\left[\frac{\pi}{12}(x+8)\right]$ over 1 cycle.

$P = \frac{2\pi}{\pi/12} = 24$



4. Given the following, find the exact values below:

$\cos\mu = \frac{3}{5},\ \sin\mu = -\frac{4}{5}$ $\qquad 270° < \mu < 360°$

$\cos\varphi = -\frac{8}{15}$ ~~15~~ 17$,\ \sin\varphi = -\frac{12}{15}$ ~~12~~ 15 / ~~15~~ 17 $\qquad 180° < \varphi < 270°$

$\sin(\mu - \varphi) = \sin\mu\cos\varphi - \cos\mu\sin\varphi = -\frac{4}{5}\left(\frac{-8}{17}\right) - \frac{3}{5}\left(\frac{-15}{17}\right) = \frac{32+45}{85}$

$= \frac{77}{85}$

$\cot 2\varphi = \frac{1 - \tan^2\varphi}{2\tan\varphi} = \frac{1 - \left(\frac{15}{8}\right)^2}{2\left(\frac{15}{8}\right)} = \frac{64-225}{16} \cdot \frac{1}{15} = -\frac{121}{240}$

$\cos\left(\frac{1}{2}\varphi\right) = -\sqrt{\frac{1}{2}(1+\cos\varphi)}$

$= -\sqrt{\frac{1}{2}\left(1 + \frac{-8}{17}\right)} = \frac{-3}{\sqrt{34}}$

5. Prove the identity: $\frac{\sec^2\theta - 6\tan\theta + 7}{\sec^2\theta - 5} = \frac{\tan\theta - 4}{\tan\theta + 2}$

$$\frac{\tan^2\theta + 1 - 6\tan\theta + 7}{\tan^2\theta + 1 - 5}$$

$$\frac{\tan^2\theta - 6\tan\theta + 8}{\tan^2\theta - 4}$$

$$\frac{(\tan\theta - 2)(\tan\theta - 4)}{(\tan\theta - 2)(\tan\theta + 2)}$$

6. Solve for $\omega \in [-45^\circ, 225^\circ)$: $\cos\omega(\sec\omega + 2\sin\omega) = \frac{1}{2}$

$$1 - 2\sin w \cos w = 1/2$$

$$-\sin 2w = -1/2$$

$$\sin 2w = 1/2$$

$$2w = \begin{cases} 30 \pm 360n \\ 150 \pm 360n \end{cases}$$

$$w = \begin{cases} 15 \pm 180n \\ 75 \pm 180n \end{cases}$$

$$w = \{15, 75\}$$

7. Find the exact value of the following:

a. $\sin^2 \frac{\pi}{3} - \cos^2 \frac{\pi}{4}$ $= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{3}{4} - \frac{1}{2}$

$= \frac{1}{4}$

b. $\frac{1}{\tan 135° + \sec 150°} + \frac{1}{\tan 45° + \sec 210°}$

$\frac{1}{-1 + \left(-\frac{\sqrt{3}}{2}\right)} + \frac{1}{1 + \left(-\frac{\sqrt{3}}{2}\right)}$

$\frac{1 - \frac{\sqrt{3}}{2} + \left(-1 - \frac{\sqrt{3}}{2}\right)}{-1 + \frac{3}{4}} = \frac{-\sqrt{3}}{-1/4} = 4\sqrt{3}$