

PreCalculus '13-14

Dr. Quattrin

Rational Functions Test-- CALCULATOR ALLOWED

Round to 3 decimal places.

Show all work.

Name: SOLUTION KEY

Score 80

1. Find the Zeros, POEs, and VAs of $y = \frac{2x^2 - 5x + 3}{2x^3 - 11x^2 - 5x + 12}$. Show the supporting algebraic work.

Zeros (1, 0)

VA: $x = 4/3, -1$

POE $(3/2, 2/5)$

$$\frac{(2x-3)(x-1)}{(2x-3)(x+1)(3x-4)}$$

$$\text{POE @ } x = 3/2 \rightarrow y = \frac{3/2 - 1}{(3/2 + 1)(3(3/2) - 4)} = \frac{1/2}{(5/2)(1/2)} = \frac{1/2}{5/4} = \frac{2}{5}$$

2. Find the critical values and extreme values of $y = \frac{2x^2 - 5x + 3}{2x^3 - 11x^2 - 5x + 12}$. Show the derivative and algebra to support the critical values.

$$\frac{d}{dx} \left[\frac{x-1}{3x^2-x-4} \right] = \frac{(3x^2-x-4)(1) - (x-1)(6x-1)}{(3x^2-x-4)^2}$$

$$= \frac{3x^2 - x - 4 - 6x^2 + 5x - 1}{(3x^2 - x - 4)^2}$$

$$= \frac{-3x^2 - 6x - 5}{(3x^2 - x - 4)^2} = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4(-3)(-5)}}{2(-3)} = \text{NO SOLUTION}$$

SO NO EXT POINTS.

$$\begin{aligned}
 3. \quad \frac{d}{dx} \left[\frac{x^2 - 3x - 18}{9 - x^2} \right] &= \frac{d}{dx} \left[\frac{(x-6)(x+3)}{(3-x)(3+x)} \right] \text{ NOT } (x+3)(x-3) \\
 &= \frac{(3-x)(1) - (x-6)(-1)}{(3-x)^2} \\
 &= \frac{-3}{(3-x)^2}
 \end{aligned}$$

4. Find the zeros, VAs, and POEs of $y = \frac{x^3 + 4x^2 - 12x}{x^2 - 4x}$. Show the supporting algebraic work.

Zeros: $(-2, 0)$ $(6, 0)$

VAs: $x = 4$

POE $(0, 3)$

$$\begin{aligned}
 &= \frac{x(x^2 + 4x - 12)}{x(x-4)} \\
 &= \frac{x(x-2)(x+6)}{x(x-4)}
 \end{aligned}$$

5. Find the end behavior, critical values and extreme values of

$y = \frac{x^3 + 4x^2 - 12x}{x^2 - 4x}$. Show the derivative and algebra to support the critical values.

$$\text{EB: } \begin{array}{r} x+8 \\ x-4 \overline{) x^2+4x-12} \\ \underline{-(x^2-4x)} \\ 8x-12 \end{array} \quad y = x + 8$$

$$\begin{aligned} \text{CV/EV: } \frac{dy}{dx} &= \frac{(x-4)(2x+4) - (x^2+4x-12)(1)}{(x-4)^2} \\ &= \frac{x^2 - 8x - 4}{(x-4)^2} = 0 \end{aligned}$$

$$x = \frac{8 \pm \sqrt{8^2 - 4(1)(-4)}}{2(1)} = \begin{cases} 8.472 \\ -0.472 \end{cases}$$

$$y(-0.472) = 3.056$$

$$y(8.472) = 20.944$$

6. Write an equation of a rational function that has x -intercepts at $(2, 0)$, VA at $x = -1$, a POE at $x = -3$, and a HA at $y = \frac{1}{3}$.

$$y = \frac{(x-2)(x+3)}{3(x+1)(x+3)}$$

7. Find the traits and sketch $y = \frac{2x^2 - 5x + 3}{2x^3 - 11x^2 - 5x + 12}$.

Domain: $x \neq \frac{4}{3}, -1, \frac{3}{2}$

Y-Int: $(0, \frac{4}{3})$

Zeros: $(1, 0)$

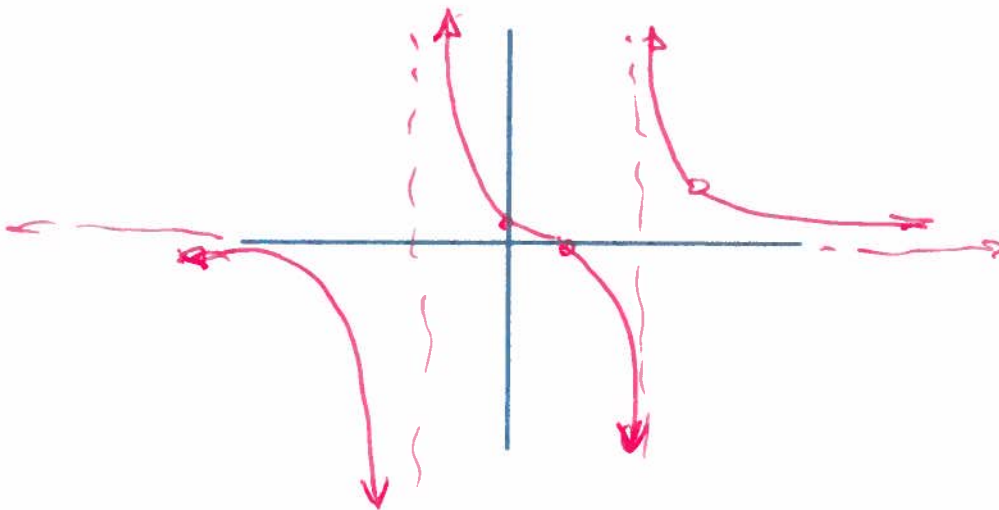
Range: ALL REALS

Vas: $x = \frac{4}{3}, -1$

End Behavior: $y = 0$

POEs: $(\frac{3}{2}, \frac{2}{5})$

Extreme Points: NONE



8. Find the traits and sketch of $y = \frac{x^3 + 4x^2 - 12x}{x^2 - 4x}$.

Domain: $x \neq 0, 4$

Y-Int: ~~8~~, NONE

Zeros: $(-2, 0)$ $(6, 0)$

Range: $y \in (-\infty, 3.056] \cup [20.944, \infty)$

Vas: $x = 4$

End Behavior: $y = x + 8$

POEs: $(0, 3)$

Extreme Points: $(-0.472, 3.056)$
 $(8.472, 20.944)$

