

PreCalculus '14-15

Name: Solution Key

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Exponential Test-- CALCULATOR ALLOWED

Round to 3 decimal places.

Score _____

Show all work.

Formulas:
$$S = \frac{P \left(\left(1 + \frac{r}{n} \right)^n - 1 \right)}{\frac{r}{n}}$$

$$L = \frac{P \left(1 - \left(1 + \frac{r}{n} \right)^{-n} \right)}{\frac{r}{n}}$$

1. Suppose you can take out a 30-year loan for a \$645,000 house, at a fixed APR of 4.25% compounded monthly. What are your monthly payments and how much will you actually pay the bank?

$$645000 = \frac{P \left(1 - \left(1 + \frac{.0425}{12} \right)^{-360} \right)}{\left(\frac{.0425}{12} \right)}$$

$$P = \$3173.01$$

$$\text{TOTAL} = 360 P = 1,142,284.80$$

2. Suppose that, when you graduate from college, you must begin to pay off your \$120,000 student loan. If the loan was at 5% compounded monthly and you can make \$1000 payments per month, how long will it take to pay off the loan?

$$120000 = \frac{1000 \left(1 - \left(1 + \frac{.05}{12} \right)^{-12t} \right)}{\frac{.05}{12}}$$

$$.5 = 1 - \left(1 + \frac{.05}{12} \right)^{-12t}$$

$$-.5 = - \left(1 + \frac{.05}{12} \right)^{-12t}$$

$$.5 = \left(1 + \frac{.05}{12} \right)^{-12t}$$

$$\ln .5 = -12t \ln \left(1 + \frac{.05}{12} \right)$$

$$-166.70 = -12t$$

$$t = 13.892 \text{ years}$$

$$\begin{aligned}
 3a. \quad \frac{d}{dx}[7x^2 e^{-3x}] &= 7x^2 e^{-3x}(-3) + e^{-3x}(14x) \\
 &= 7x e^{-3x}[-3x + 2]
 \end{aligned}$$

$$3b. \quad D_x[(4x^5 - 2)^6 (3x^3 + 7)^5]$$

$$(4x^5 - 2)^6 \cdot 5(3x^3 + 7)^4 (12x^2) + (3x^3 + 7)^5 \cdot 6(4x^5 - 2)^5 (20x^4)$$

$$15x^2 (4x^5 - 2)^5 (3x^3 + 7)^4 [3(4x^5 - 2) + 8x^2(3x^3 + 7)]$$

$$30x^2 (4x^5 - 2)^5 (3x^3 + 7)^4 \left[\frac{18}{3}x^5 + \frac{28}{3}x^2 - \frac{2}{3} \right]$$

4. Find the zeros and Domain of $y = (3x - x^2)e^x$. Show the supporting algebraic work.

DOMAIN: $x \in \text{ALL REALS}$

Zeros: ~~$x =$~~ $(0, 0), (3, 0)$

$$3x - x^2 = 0$$

$$x(3 - x) = 0$$

$$x = 0, 3$$

5. Find the critical values and extreme values of $y = (3x - x^2)e^x$. Show the derivative and algebra to support the critical values.

$$\begin{aligned} \frac{dy}{dx} &= (3x - x^2)e^x + e^x(3 - 2x) \\ &= e^x(-x^2 + x + 3) = 0 \end{aligned}$$

$$x = \frac{-1 \pm \sqrt{1 + 12}}{2(1)} = \begin{cases} -1.303 \\ 2.303 \end{cases}$$

$$(-1.303, -1.523)$$

$$(2.303, 16.059)$$

6. Find the traits and sketch $y = (3x - x^2)e^x$.

Domain: All Reals

Range: $y \leq 16.059$

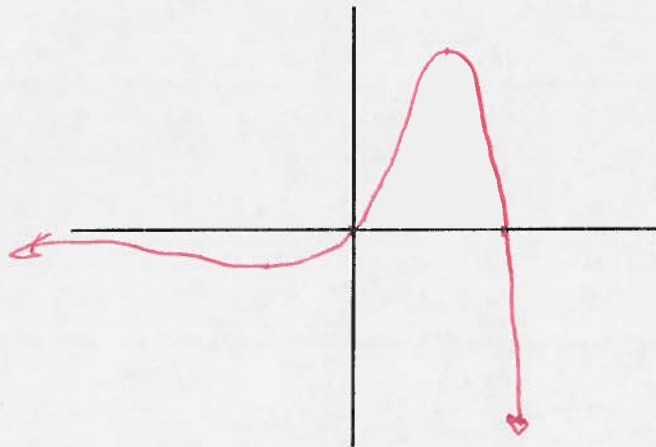
Y-Int: $(0, 0)$

End Behavior: Left = $y = 0$

Zeros: $(0, 0)$ $(3, 0)$

End Behavior: Right = Down

Extreme Points: $(-1.303, -1.523)$
 $(2.303, 16.059)$



7. Find the zeros and Domain of $y = -3x\sqrt{36-x^2}$. Show the supporting algebraic work.

$$\text{Zeros } (0,0) (\pm 6,0)$$

$$\text{Domain } 36-x^2 \begin{array}{c} - \quad 0 \quad + \quad 0 \quad - \\ \leftarrow \quad \quad \quad \rightarrow \\ x \quad -6 \quad 6 \end{array}$$

$$x \in [-6, 6]$$

8. Find the critical values and extreme values of $y = -3x\sqrt{36-x^2}$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = -3x \left[\frac{1}{2} (36-x^2)^{-1/2} (-2x) \right] + \sqrt{36-x^2} (-3)$$

$$= \frac{3x^2}{\sqrt{36-x^2}} - \frac{3\sqrt{36-x^2}\sqrt{36-x^2}}{\sqrt{36-x^2}}$$

$$= \frac{3x^2 - (08 - 3x^2)}{\sqrt{36-x^2}} = \frac{6x^2 - 108}{\sqrt{36-x^2}} = 0$$

$$i) \frac{dy}{dx} = 0 \Rightarrow 6x^2 - 108 = 0 \quad x^2 = 18 \quad x = \pm 3\sqrt{2} \quad (3\sqrt{2}, -54)$$

$$(-3\sqrt{2}, 54)$$

$$ii) \frac{dy}{dx} \text{ DNE} \Rightarrow x = \pm 6$$

$$(\pm 6, 0)$$

9. Find the traits and sketch of $y = -3x\sqrt{36-x^2}$.

Domain: $x \in [-6, 6]$

Range: $y \in [-54, 54]$

Y-Int: $(0, 0)$

End Behavior: Left= NONE

Zeros: $(0, 0)$ $(\pm 6, 0)$

End Behavior: Right= NONE

Extreme Points: $(\pm 6, 0)$

$(3\sqrt{2}, -54)$ $(-3\sqrt{2}, 54)$

