PreCalculus
Fall Take Home Midterm 2014

Name Solution Key

Directions: Round at 3 decimal places.
Show all work.

1. Find the approximate trig value for the following:

$\tan(18°)$ = .325

$\sin(193°)$ = −.225

$\sec 312°$ = 1.494

2. Find ALL approximate values in **degrees** for the following:

$\sin^{-1}(-0.461) = \begin{cases} -27.452 \pm 360n \\ 207.452 \pm 360n \end{cases}$

$\sec^{-1}(-1.415) = \{ \pm 134.968 \pm 360n$

$\cot^{-1} 1.45 = \begin{cases} 34.592 \pm 360n \\ 214.592 \pm 360n \end{cases}$

3. Identify the Quadrant and reference angle of each of these:

a. −465° Q III θ_{ref} = 75°

b. 1732° Q IV θ_{ref} = 68°

c. 614° Q III θ_{ref} = 74°

d. 1272° Q III θ_{ref} = 12°

4. The S.I. soccer team is playing a game. S.I.'s winning shot started with a 10-foot pass at $40°$ and a final shot of 25 feet at $287°$. How far from the original pass was the goal and at what direction?

$$10\cos 40\vec{i} + 10\sin 40\vec{j}$$
$$25\cos 287\vec{i} + 25\sin 287\vec{j}$$

$$14.970\vec{i} - 17.480\vec{j}$$

$$|\vec{r}| = \sqrt{14.970^2 + 17.480^2} = 23.014$$

$$\theta = -\cos^{-1}\left(\frac{14.970}{23.014}\right) = -49.423$$

5. If $\vec{s} = -12\vec{i} + 5\vec{j}$ and $\vec{r} = 5\vec{i} + 18\vec{j}$, find:

a. $4\vec{s} - \vec{r}$ $= -53\vec{i} + 2\vec{j}$

b. $|5\vec{s} + 2\vec{r}| = \sqrt{6201} = 78.873$

c. The unit vector in the direction $\vec{s}$

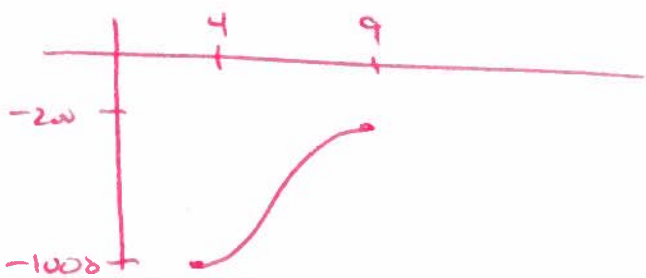
$$\frac{-12}{13}\vec{i} + \frac{5}{13}\vec{j}$$

6. Assume that you are aboard a submarine, submerged in the Pacific Ocean. You make contact with an enemy destroyer and need to keep track of it, but also need to avoid it, so you order the sub to start "porpoising," or going deeper and shallower in the water in such a way that the depth of the sub varies sinusoidally with time. After 4 minutes, you are at your deepest, at –1000 meters, and 5 minutes after that, you reach your shallowest depth, at –200 meters.

a. Sketch a graph.

b. Write an appropriate equation to represent this situation.

c. What is the first positive time when your depth is -550 meters?



$$d = -600 + 400\cos\frac{\pi}{5}(t-9)$$

c) $$-550 = -600 + 4\cos\frac{\pi}{5}(t-9)$$

$$\frac{1}{8} = \cos\frac{\pi}{5}(t-9)$$

$$\pm 1.445 \pm 2\pi n = \frac{\pi}{5}(t-9)$$

$$\pm 2.301 \pm 10n = t-9$$

$$\left.\begin{array}{l} 11.301 \pm 10n \\ 6.699 \pm 10n \end{array}\right\} = t$$

$$\boxed{t = 1.301}$$

7. (2, -7) is on the terminal side of A. Find the six exact trig values:

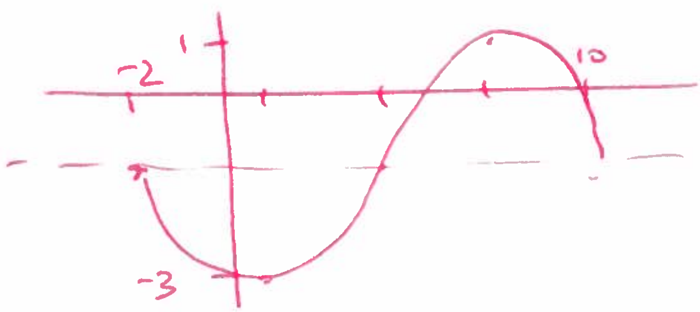
Sin A= $-7/\sqrt{53}$ Cos A= $2/\sqrt{53}$

Tan A= $-7/2$ Cot A= $-2/7$

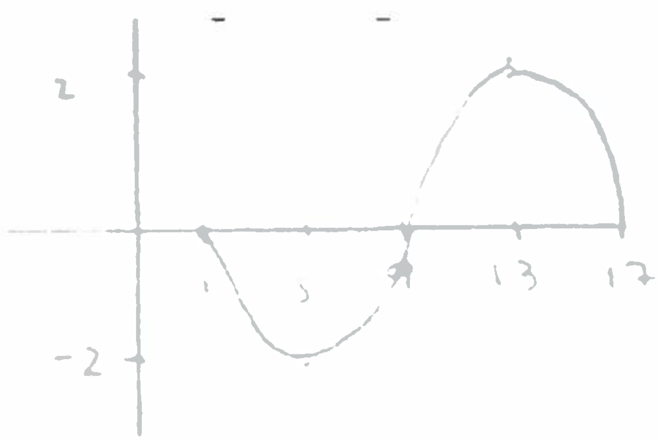
Sec A= $\sqrt{53}/2$ Csc A= $-\sqrt{53}/7$

A= $-74.054 \pm 360n$

8. Sketch one cycle of $y = -1 - 2\sin\left[\frac{\pi}{6}(x+2)\right]$



9. Find one cosine and one sine equation for this graph:



$y = -2\sin\frac{\pi}{8}(x-1)$

$y = -2\cos\frac{\pi}{8}(x-5)$

10. $\tan B = \dfrac{15}{8}$ in Quadrant III. Find the other five exact trig values:

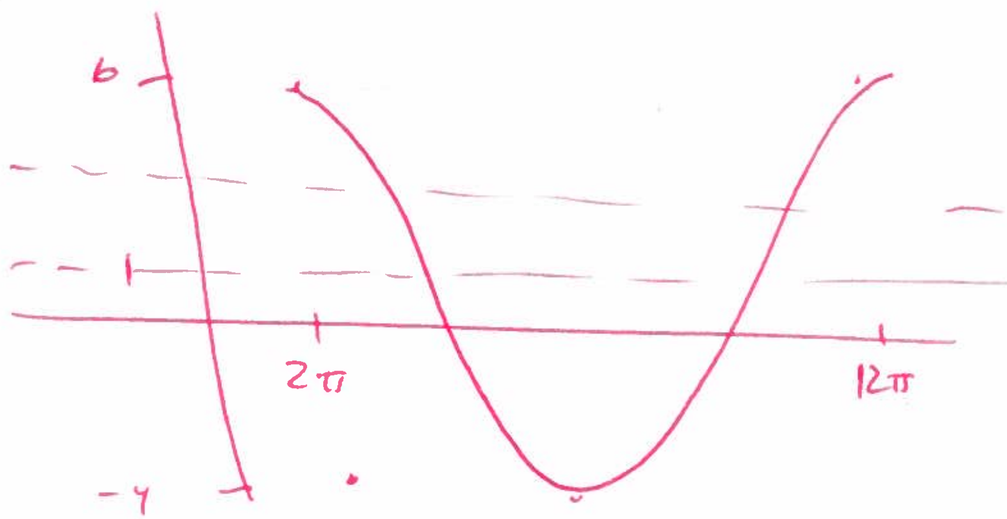
sin B= $-15/17$ cos B = $-8/17$

$\tan B = \dfrac{15}{8}$ cot B = $8/15$

$\sec B =$ $-17/8$ csc B = $-17/15$

B=______________

11. Sketch one cycle of $y = 1 + 5\cos\left[\frac{1}{5}(x - 2\pi)\right]$



12. **Prove:** $(1-\sin y)(\sec y+\tan y)=\cos y$

$$\left(1-\sin y\right)\left(\frac{1}{\cos y}+\frac{\sin y}{\cos y}\right)$$

$$\left(1-\sin y\right)\left(\frac{1+\sin y}{\cos y}\right)$$

$$\frac{1-\sin^2 y}{\cos y}$$

$$\frac{\cos^2 y}{\cos y} = \cos y$$

13. **Prove:** $\dfrac{\cos\phi-5}{\cos\phi-1}=\dfrac{\sin^2\phi+4\cos\phi+4}{\sin^2\phi}$

$$= \frac{1-\cos^2\phi+4\cos\phi+4}{1-\cos^2\phi}$$

$$= \frac{\cos^2\phi-4\cos\phi-5}{\cos^2\phi-1}$$

$$= \frac{(\cos\phi-5)\cancel{(\cos\phi+1)}}{(\cos\phi-1)\cancel{(\cos\phi+1)}}$$