

PreCalculus '14-15
Take-Home Midterm
Dr. Quattrin

Name: SOLUTION KEY

Score _____

CALCULATOR ALLOWED

Round to 3 decimal places. Show all work.

1. Find the equations of the lines tangent and normal to

$$y = \frac{1}{4}x^4 + \frac{1}{3}x^3 + 2x^2 + 2x - 7 \text{ at } x = 2.$$

$$x=2 \rightarrow y=11.667$$

$$\frac{dy}{dx} = x^3 + x^2 + 4x + 2$$

$$\left. \frac{dy}{dx} \right|_{x=2} = 8 + 4 + 8 + 2 = 22$$

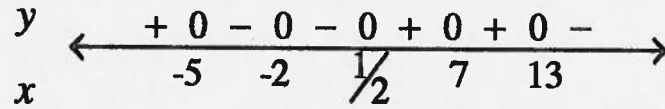
$$\text{TANGENT: } y - \left(11\frac{2}{3}\right) = 22(x - 2)$$

$$\text{NORMAL: } y - \left(11\frac{2}{3}\right) = \frac{-1}{22}(x - 2)$$

2. Find all zeros of $y = \frac{1}{4}x^4 + \frac{1}{3}x^3 + 2x^2 + 2x - 7$ on your calculator.

$$(-1.583, -7.528) \quad (-2.152, 0) \\ (1.261, 0)$$

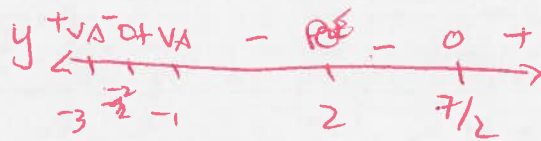
3. Find an inequality that has this sign pattern and solution:



and $x \in (-\infty, -5] \cup \{-2\} \cup [5, 13]$

$$\cancel{0} \leq (x+5)(x+2)^2(2x-1)(x-7)^2(x-13)$$

4. Solve this inequality: $0 \leq \frac{(x-2)(x+2)(2x-7)}{(x+3)(x-2)(x+1)}$.



$$x \in (-\infty, -3) \cup [-2, -1) \cup [7/2, \infty)$$

5. The extreme points of $y = \frac{1}{4}x^4 + \frac{1}{3}x^3 + 2x^2 + 2x - 7$ algebraically.

$$\frac{dy}{dx} = x^3 + x^2 + 4x + 2 = 0$$

$$(-.533, -7.528)$$

6. Find VAs, HA, POEs and zeros of $y = \frac{x^3 - 4x^2 - 16x + 64}{x^3 - 4x^2 + 4x - 16}$

Zeros: ~~(4, 0)~~

VAs: NONE

POEs: (4, 0)

EB: $y = 1$

$$= \frac{x^2(x-4) - 16(x-4)}{x^2(x-4) + 4(x-4)}$$

$$= \frac{(x^2 - 16)(x-4)}{(x^2 + 4)(x-4)}$$

$$(x^2 - 16)(x-4)$$

7. Find extreme points of $y = \frac{x^3 - 4x^2 - 16x + 64}{x^3 - 4x^2 + 4x - 16}$. Show the derivative and algebra supporting the points

$$y \approx \frac{x^2 + 16}{x^2 + 4}$$

$$\frac{dy}{dx} = \frac{(x^2 + 4)(2x) - (x^2 - 16)(2x)}{(x^2 + 4)^2}$$

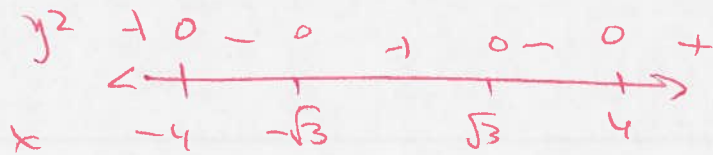
$$= \frac{40x}{(x^2 + 4)^2} = 0$$

$$x = 0$$

$$(0, -4)$$

8. Find all the zeros and domain of $y = \sqrt{x^4 - 19x^2 + 48}$

$$(x^2 - 16)(x^2 - 3)$$



$$x \in \cancel{[-4, \sqrt{3}]}$$

$$(-\infty, -4] \cup [-\sqrt{3}, \sqrt{3}] \cup [4, \infty)$$

9. Find $\frac{dy}{dx}$ if $y = \sqrt{x^4 - 19x^2 + 48}$.

$$\frac{dy}{dx} = \frac{1}{2} (x^4 - 19x^2 + 48)^{-1/2} (4x^3 - 38x)$$

$$= \frac{2x^3 - \cancel{38} \cdot 19x}{\sqrt{x^4 - 19x^2 + 48}}$$

10. List all the traits and sketch $y = x^3 - x^2 - 4x + 4 = (x^2 - 4)(x - 1)$

Domain: ALL REALS

Range: ALL REALS

X-Intercepts: $(-2, 0)$ $(1, 0)$

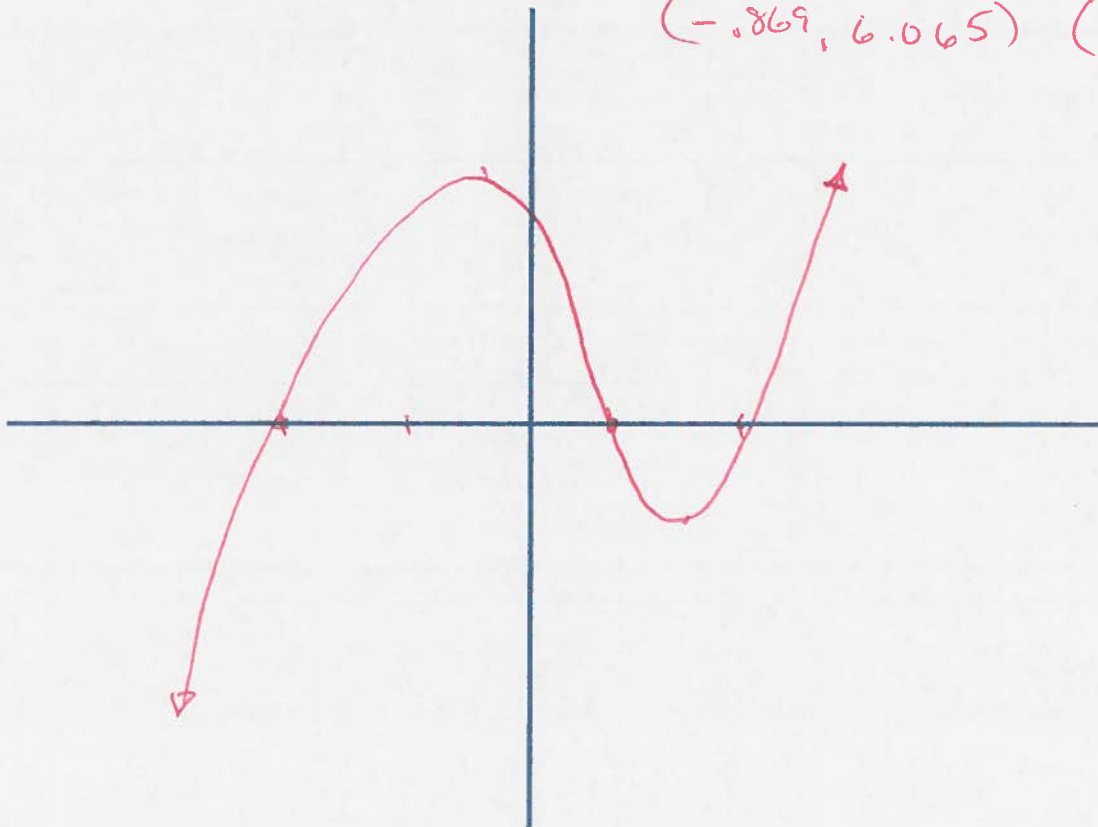
Y-Intercept: $(0, 4)$

End Behavior: ↘ ↗

Extremes: $3x^2 - 2x - 4 = 0$

$$x = \frac{2 \pm \sqrt{52}}{6}$$

$(-0.869, 6.065)$ $(1.535, -0.879)$



11. List all the traits and sketch $y = \frac{x^3 - 4x^2 - 16x + 64}{x^3 - 4x^2 + 4x - 16}$

Domain: $x \neq 4$

Range: $y \in [-4, 1)$

X-Intercepts: $(-4, 0)$

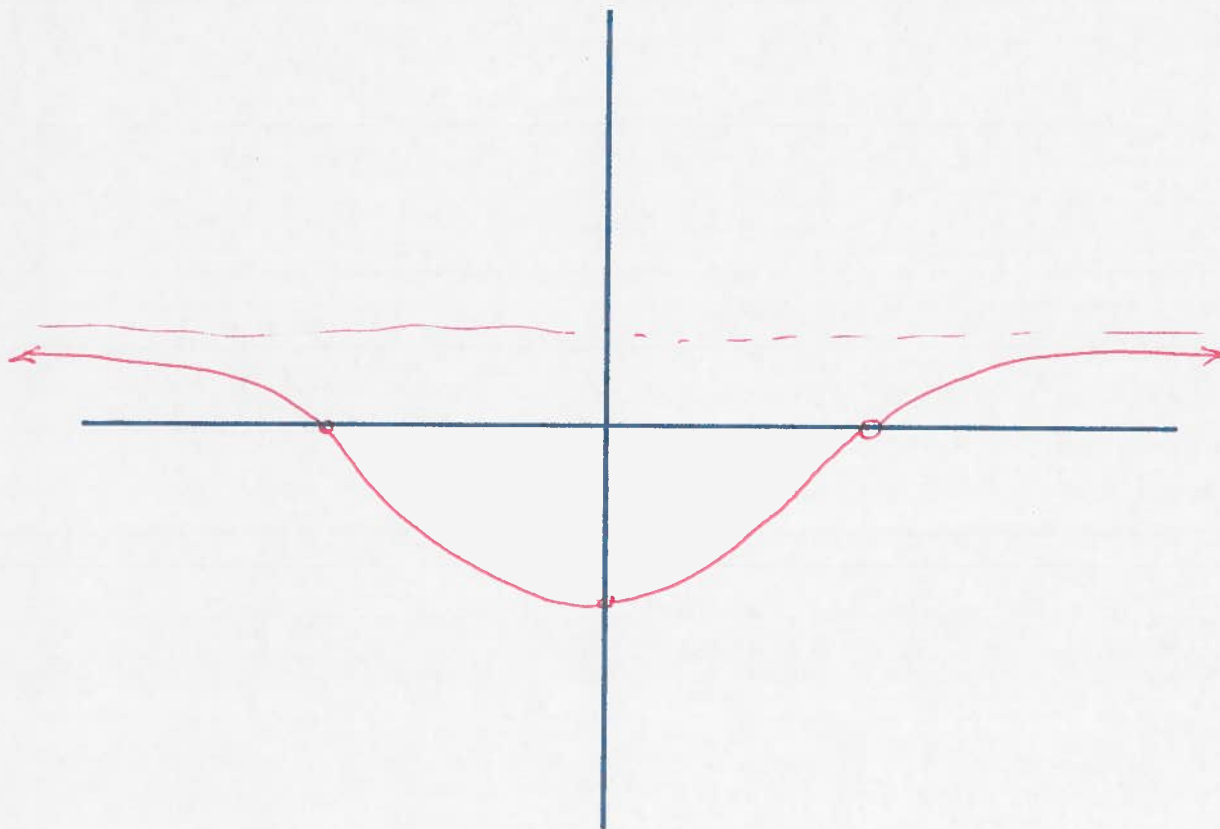
Y-Intercepts: $(0, -4)$

VAs: NONE

POEs: ~~none~~ $(4, 0)$

End Behavior: $y = 1$

Extremes: $(0, -4)$



EC. List all the traits and sketch $y = \sqrt{x^4 - 19x^2 + 48}$ on

Domain: $x \in (-\infty, -4] \cup [-\sqrt{3}, \sqrt{3}] \cup [4, \infty)$ Range: $y \in [0, \infty)$

X-Intercepts: $(\pm 4, 0)$ $(\pm \sqrt{3}, 0)$ Y-Intercepts: $0, \sqrt{48}$

VAs: NONE POEs: NONE

End Behavior: $\nearrow \nearrow$ Extremes: $(\pm 4, 0)$ $(\pm \sqrt{3}, 0)$ $(0, \sqrt{48})$

