PreCalculus '14-15
Trig Identities Group Test
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Name Solution Key

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For Problem 1-6, use:

(-7, 24) is on the terminal side of C and $\csc Q = -\frac{17}{8}$ in QIII

$\sin C = \frac{24}{25}$ $\cos C = \frac{-7}{25}$ $\tan C = \frac{-24}{7}$

$\sin Q = -\frac{8}{17}$ $\cos Q = \frac{-15}{17}$ $\tan Q = \frac{8}{15}$

to find the exact values of:

1. Sin (C+Q)

$\sin C \cos Q + \cos C \sin Q$

$\left(\frac{24}{25}\right)\left(\frac{-15}{17}\right) + \left(\frac{-7}{25}\right)\left(\frac{-8}{17}\right)$

$= \frac{-304}{425}$

2. Cos (C-Q)

$\cos C \cos Q + \sin C \sin Q$

$\left(\frac{-7}{25}\right)\left(\frac{-15}{17}\right) + \left(\frac{24}{25}\right)\left(\frac{-8}{17}\right)$

$\frac{-87}{425}$

3. $\tan(C+Q) = \dfrac{\frac{-24}{7} + \frac{8}{15}}{1 - \left(\frac{-24}{7}\right)\left(\frac{8}{15}\right)}$

$= \dfrac{-304/175}{367/175} = \dfrac{-304}{\cancel{367}\,297}$

4. sec (2C) $= \dfrac{1}{\cos^2 C - \sin^2 C}$

$= \dfrac{1}{\left(\frac{-7}{25}\right)^2 - \left(\frac{24}{25}\right)^2} =$

$= -\frac{625}{527}$

5. Cot (2C) $= \dfrac{1 - \left(\frac{-24}{7}\right)^2}{2(24/7)}$

$= -527/49 \cdot \frac{7}{48} = \frac{-527}{336}$

6. $\csc 2C = \dfrac{1}{2\sin C \cos C}$

$= \dfrac{1}{2\left(\frac{24}{25}\right)\left(\frac{-7}{25}\right)} = -\frac{625}{336}$

7. Prove: $\dfrac{\sec^2\theta-6\tan\theta+7}{\sec^2\theta-5}=\dfrac{\tan\theta-4}{\tan\theta+2}$

$$\frac{\tan^2\theta+1-6\tan\theta+7}{\tan^2\theta+1-5}$$

$$\frac{\tan^2\theta-6\tan\theta+8}{\tan^2\theta-4}$$

$$\frac{\cancel{(\tan\theta-2)}(\tan\theta-4)}{\cancel{(\tan\theta-2)}(\tan\theta+2)}$$

9. Solve exactly for $\varnothing\in[-180°, 180°)$:

$\frac{16}{8}\sin\varnothing\cos\varnothing=\frac{4}{8}$

$$\sin 2\varnothing = 1/2$$

$$2\varnothing=\begin{cases}30\pm 360n\\150\pm 360n\end{cases}$$

$$\varnothing=\begin{cases}15\pm 180n\\75\pm 180n\end{cases}$$

$$\boxed{\varnothing=15, -165, 75, -105}$$

8. Prove:

$\sin(A+30°)+\cos(A+60°)=\cos A$

$$\sin A\cos 30°+\cos A\sin 30°+\cos A\cos 60°-\sin A\sin 60°$$

$$\cancel{\frac{\sqrt{3}}{2}\sin A}+\frac{1}{2}\cos A+\frac{1}{2}\cos A-\cancel{\frac{\sqrt{3}}{2}\sin A}=$$

$$1\cos A$$

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For Problem 1-6, use:

(-7, 24) is on the terminal side of C and
$\csc Q = -\frac{17}{8}$ in QIII

to find the exact values of:

1. Sin (2Q) $= 2\sin Q \cos Q$

$= 2\left(-\frac{8}{17}\right)\left(\frac{-15}{17}\right)$

$= +\frac{240}{289}$

2. Cos (2Q) $= \cos^2 Q - \sin^2 Q$

$\left(\frac{-15}{17}\right)^2 - \left(\frac{-8}{17}\right)^2$

$\frac{161}{289}$

3. $\tan(2Q)$ $= \frac{2\tan Q}{1-\tan^2 Q}$

$= \frac{2(8/15)}{1-\left(\frac{8}{15}\right)^2} = \frac{16}{\cancel{15}} \cdot \frac{\cancel{225}^{15}}{161}$

$= \frac{240}{161}$

4. sec (C+Q) $= \frac{1}{\cos C \cos Q - \sin C \sin Q}$

$\frac{1}{\left(\frac{-7}{25}\right)\left(\frac{-15}{17}\right) - \left(\frac{24}{25}\right)\left(\frac{-8}{17}\right)}$

$\frac{425}{297}$

5. Cot (C-Q) $= \frac{1+\tan C \tan Q}{\tan C - \tan Q}$

$= \frac{1+\left(\frac{-24}{7}\right)\left(\frac{8}{15}\right)}{\frac{-24}{7} - \frac{8}{15}} = \frac{-87/105}{-416/105}$

$= \frac{87}{416}$

6. $\csc(Q+C)$ $= \frac{1}{\sin Q \cos C + \cos Q \sin C}$

$= \frac{1}{\left(\frac{-8}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{-15}{17}\right)\left(\frac{24}{25}\right)}$

$= \frac{\cancel{425}}{} \; \frac{425}{-304}$

7. Prove: $\sqrt{2}\sin\left(\frac{\pi}{4}-x\right)=\cos x-\sin x$

$\sqrt{2}\left(\sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x\right)$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)$

$\cos x - \sin x$

9. Prove: $\frac{1}{\tan x-\sec x}+\frac{1}{\tan x+\sec x}=\frac{-2}{\cot x}$

$\frac{\tan x + \cancel{\sec x} + \tan x - \cancel{\sec x}}{\tan^2 x - \sec^2 x}$

$\frac{2\tan x}{-1}$

$\frac{-2}{\cot x}$

8. Solve for $\theta\in[0°, 360°]$: $\sin\theta=\sin 2\theta$

$= 2\sin\theta\cos\theta$

$\sin\theta - 2\sin\theta\cos\theta = 0$

$\sin\theta = (1 - 2\cos\theta) = 0$

$\sin\theta = 0 \qquad \cos\theta = 1/2$

$\theta = 0 \pm 180n \qquad \theta = \pm 60 \pm 360n$

$x = 0, 180, 360, 60, 300$

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For Problem 1-6, use:

(-7, 24) is on the terminal side of C and
$\csc Q = -\frac{17}{8}$ in QIII

to find the exact values of:

1. Sin (2C) $= 2\sin C\cos C$

$= 2\left(\frac{24}{25}\right)\left(\frac{-7}{25}\right)$

$= -\frac{336}{625}$

2. Cos (2C) $= \cos^2 C - \sin^2 C$

$= \left(\frac{24}{25}\right)^2 - \left(\frac{-7}{25}\right)^2$

$= -\frac{527}{625}$

3. $\tan(2C)$ $= \frac{2\tan C}{1-\tan^2 C}$

$= \frac{2\left(\frac{-24}{7}\right)}{1-\left(\frac{-24}{7}\right)^2} = -\frac{48}{7} \cdot \frac{49}{-527}$

$= +\frac{336}{527}$

4. sec (Q-C) $= \frac{1}{\cos Q\cos C + \sin Q\sin C}$

$= \frac{1}{\left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{-8}{17}\right)\left(\frac{24}{25}\right)} = -\frac{425}{87}$

5. Cot (Q-C) $= \frac{1 + \tan Q\tan C}{\tan Q - \tan C}$

$= \frac{1 + \left(\frac{8}{15}\right)\left(\frac{-24}{7}\right)}{\frac{8}{15} - \left(\frac{-24}{7}\right)} = \frac{\frac{-87}{105}}{\frac{416}{105}}$

$= \cancel{\frac{-17}{686}}$ $\quad -87/416$

6. $\csc(Q-C)$ $= \frac{1}{\sin Q\cos C - \cos Q\sin C}$

$= \frac{1}{\left(\frac{24}{25}\right)\left(\frac{-15}{17}\right) - \left(\frac{-7}{25}\right)\left(\frac{-8}{17}\right)}$

$= \frac{425}{416}$

7. Prove:

$$\cot^2\phi - 1 = \cos 2\phi \cot^2\phi + \cos 2\phi$$

$$\frac{1}{\tan^2\phi} - 1 = \cos 2\phi\,(\cot^2\phi + 1)$$

$$\frac{1-\tan^2\phi}{\tan^2\phi} = \cos 2\phi \csc^2\phi$$

$$\frac{1-\frac{\sin^2\phi}{\cos^2\phi}}{\frac{\sin^2\phi}{\cos^2\phi}} = (\cos^2\phi - \sin^2\phi)\left(\frac{1}{\sin^2\phi}\right)$$

$$\frac{\cos^2\phi - \sin^2\phi}{\sin^2\phi}$$

8. Solve for $\emptyset \in [0°, 180°)$:

$$\tan\emptyset + \tan 2\emptyset = 1 - \tan\emptyset \tan 2\emptyset$$

$$\frac{\tan\emptyset + \tan 2\emptyset}{1 - \tan\emptyset \tan 2\emptyset} = 1$$

$$\tan(\emptyset + 2\emptyset) = 1$$

$$\tan 3\emptyset = 1$$

$$3\emptyset = 45° \pm 180n$$

$$\emptyset = 15° \pm 60n$$

$$\emptyset = 15°, 75, 135$$

9. Solve exactly for $x \in [0, \pi)$:

$$-\cos^2 4x = \sin^2 4x - 2\sin\left(4x + \frac{\pi}{2}\right)$$

$$0 = \underbrace{\sin^2 4x + \cos^2 4x}_{=1} - 2\sin\left(4x + \frac{\pi}{2}\right)$$

$$\sin\left(4x + \frac{\pi}{2}\right) = \frac{1}{2}$$

$$4x + \frac{\pi}{2} = \begin{cases} \pi/6 \pm 2\pi n \\ 5\pi/6 \pm 2\pi n \end{cases}$$

$$4x = \begin{cases} -\frac{\pi}{3} \pm 2\pi n \\ \frac{2\pi}{3} \pm 2\pi n \end{cases}$$

$$x = \begin{cases} \frac{-\pi}{12} \pm \frac{\pi}{2} n \\ \frac{2\pi}{6} \pm \frac{\pi}{2} n \end{cases}$$

$$x = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{\pi}{6}, \frac{7\pi}{6}$$

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For Problem 1-6, use:

(-7, 24) is on the terminal side of C and

$\csc Q = -\frac{17}{8}$ in QIII

to find the exact values of:

1. Sin (C-Q) $= \sin C \cos Q - \cos C \sin Q$

$= \left(\frac{24}{25}\right)\left(\frac{-15}{17}\right) - \left(\frac{-7}{25}\right)\left(\frac{-8}{17}\right)$

$= \frac{-416}{425}$

2. Cos (Q-C) $= \cos Q \cos C + \sin Q \sin C$

$= \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{-8}{17}\right)\left(\frac{24}{25}\right)$

$= -\frac{87}{425}$

3. $\tan(C-Q)$ $= \frac{\tan C - \tan Q}{1 + \tan C \tan Q}$

$= \frac{-\frac{24}{7} - \frac{8}{15}}{1 + \left(-\frac{24}{7}\right)\left(\frac{8}{15}\right)} = \frac{\frac{416}{105}}{\frac{-87}{105}} = \frac{-416}{87}$

4. sec (2Q) $= \frac{1}{\cos^2 Q - \sin^2 Q}$

$= \frac{1}{\left(\frac{-15}{17}\right)^2 - \left(\frac{-8}{17}\right)^2} = \frac{289}{161}$

5. Cot (2Q) $= \frac{1 - \tan^2 Q}{2 \tan Q}$

$= \frac{1 - \left(\frac{8}{15}\right)^2}{2(8/15)} = \frac{161}{225} \cdot \frac{15}{16}$

$= \frac{161}{240}$

6. $\csc(2Q)$ $= \frac{1}{2 \sin Q \cos Q}$

$= \frac{1}{2\left(\frac{-8}{17}\right)\left(\frac{-15}{17}\right)} = \frac{289}{240}$

7. Solve for $\theta \in [0°, 360°]$:

$\cos\theta \cos 20° - \sin\theta \sin 20° = \frac{1}{\sqrt{2}}$

$\cos(\theta + 20°) = \frac{1}{\sqrt{2}}$

$\theta + 20° = \left\{ \pm 45 \pm 360n \right.$

$\theta = \begin{cases} 25 \pm 360n \\ -65 \pm 360n \end{cases}$

$\theta = 25, 295$

8. Solve for $x \in [-180°, 180°]$:

$\tan^2 x + 3\sec x + 3 = 0$

$\sec^2 x - 1 + 3\sec x + 3 = 0$

$\sec^2 x + 3\sec x + 2 = 0$

$(\sec x + 2)(\sec x + 1)$

$\sec x = -2$ $\sec x = -1$

$\cos x = -\frac{1}{2}$ $\cos x = -1$

$x = \pm 120 \pm 360n$ $x = 180 \pm 360n$

$x = \pm 120, \pm 180$

9. Prove: $\frac{\cos x}{1 - \cos x} = \cot x \csc x + \cot^2 x$

$= \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} + \frac{\cos^2 x}{\sin^2 x}$

$= \frac{\cos x (1 + \cos x)}{\sin^2 x}$

$= \frac{\cos x (1 + \cos x)}{1 - \cos^2 x}$

$= \frac{\cos x \cancel{(1 + \cos x)}}{\cancel{(1 + \cos x)}(1 - \cos x)}$