PreCalculus Honors 12-13
Advanced Curve Sketching Group Quiz
Page 1

Name_____

Directions: Round at 3 decimal places.
Show all work.

1. Find the Extremes of $y=\left(x-\frac{1}{2}x^2\right)e^{-x}$

$$\frac{dy}{dx}=\left(x-\tfrac{1}{2}x^2\right)e^{-x}(-1)+e^{-x}(1-x)$$

$$=\left(\tfrac{1}{2}x^2-2x+1\right)e^{-x}=0$$

$$x=\frac{2\pm\sqrt{4-4(1/2)(1)}}{2(1/2)}=\frac{2\pm\sqrt{2}}{1}=\begin{cases}\cancel{1.707}\ 3.414\\ \cancel{\pm 293}\ .586\end{cases}$$

~~(1.707~~, ~~±293)~~

(3.414, −.079)
(.586, −.231)

2. Find the Points of Inflection of $f(x)=x^3-x^2-8x+8$.

$$f''(x)=6x-2=0$$

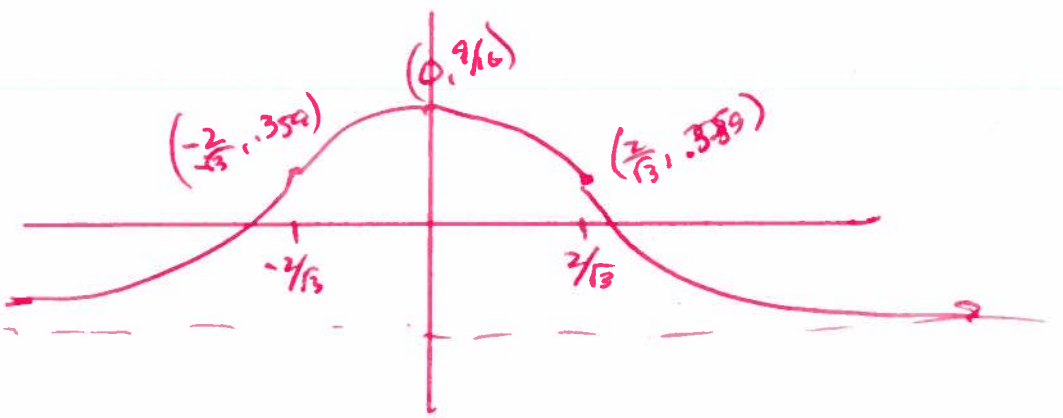
$$x=1/3$$

$(1/3, 5.259)$

3. Make a Key Trait Table for $f(x)=x^3-x^2-8x+8$.

x		$-4/3$		$1/3$		2		
y		14.519		5.259		-4		
y'	$+$	0	$-$	$-$	$-$	0	$+$	$+$
y''	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$
	∩		∪		∪		∪	

4. Sketch $y=\dfrac{9-x^2}{4x^2+16}$, labeling all the Key Traits.



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Name_____

Directions: Round at 3 decimal places.
Show all work.

1. Find the Extremes of $y=\sqrt{x^4-5x^2+4}$.

Domain: x^4-5x^2+4 $+\ 0\ -\ 0\ +\ \ 0\ -\ 0\ +$ on the number line $-2,\ -1,\ 1,\ 2$

$x\in(-\infty,-2]\cup[-1,1]\cup[2,\infty)$

$$\frac{dy}{dx}=\frac{4x^3-10x}{2(x^4-5x^2+4)^{1/2}}=0$$

$x=0,\ \pm\sqrt{\frac{5}{2}}$ (crossed out)

$(0,2)$
$(\pm 2,0)$
$(\pm 1,0)$

$\frac{dy}{dx}$ DNE: $x=\pm 2,\pm 1$

2. Find the Points of Inflection of $y=\left(x-\frac{1}{2}x^2\right)e^{-x}$.

$$\frac{dy}{dx}=\left(\frac{1}{2}x^2-2x+1\right)e^{-x}$$

$$\frac{d^2y}{dx^2}=\left(\frac{1}{2}x^2-2x+1\right)e^{-x}(-1)+e^{-x}(x-2)$$

$$=\left(-\frac{1}{2}x^2+3x-3\right)e^{-x}=0$$

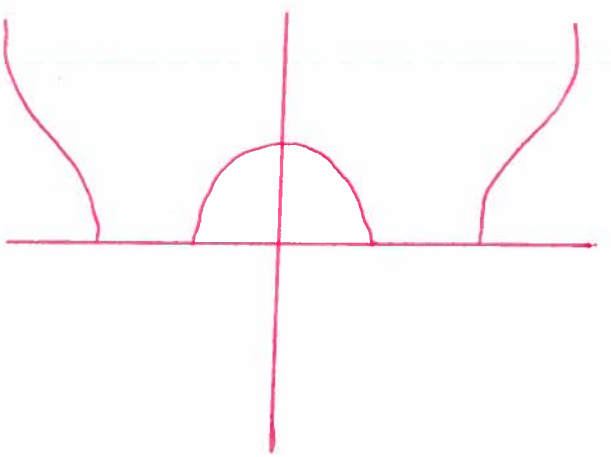
$$x=\frac{-3\pm\sqrt{9-4(-1/2)(-3)}}{2(-1/2)}=\begin{cases}1.268\\4.732\end{cases}$$

$(1.268,\ .131)$
$(4.732,\ -.057)$

3. Make a Key Trait Table (without Zeros) for $y = \dfrac{9 - x^2}{4x^2 + 16}$.

x		$-2/\sqrt{3}$		0		$2/\sqrt{3}$	
y		$.359$		$9/16$		$.359$	
y'	$+$	$+$	$+$	0	$-$	$-$	$-$
y''	$+$	0	$-$	$-$	$-$	0	$+$
	∪		∩		∩		∪

4. Sketch $y = \sqrt{x^4 - 5x^2 + 4}$, labeling all the Key Traits.



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Name_____

Directions: Round at 3 decimal places.
Show all work.

1. Find the Extremes of $y=\frac{9-x^2}{4x^2+16}$

$y' = \frac{(4x^2+16)(-2x) - (9-x^2)(8x)}{(4x^2+16)^2}$

$= \frac{-8x^3 - 32x + 8x^3 - 72x}{(4x^2+16)^2} = \frac{-104x}{(4x^2+16)^2} = 0$

$x = 0$

$(0, 9/16)$

2. Find the Points of Inflection of $y=\sqrt{x^4-5x^2+4}$.

$\frac{dy}{dx} = \frac{2x^3-5x}{(x^4-5x^2+4)^{1/2}}$

$\frac{d^2y}{dx^2} = \frac{(x^4-5x^2+4)^{1/2}(6x^2-5) - (2x^3-5x)\left[\frac{2x^3-5x}{(x^4-5x^2+4)^{1/2}}\right]}{(x^4-5x^2+4)^{1}}$

$= \frac{(x^4-5x^2+4)(6x^2-5) - (2x^3-5x)^2}{(x^4-5x^2+4)^{3/2}} = 0$

$x = \pm 2.187$

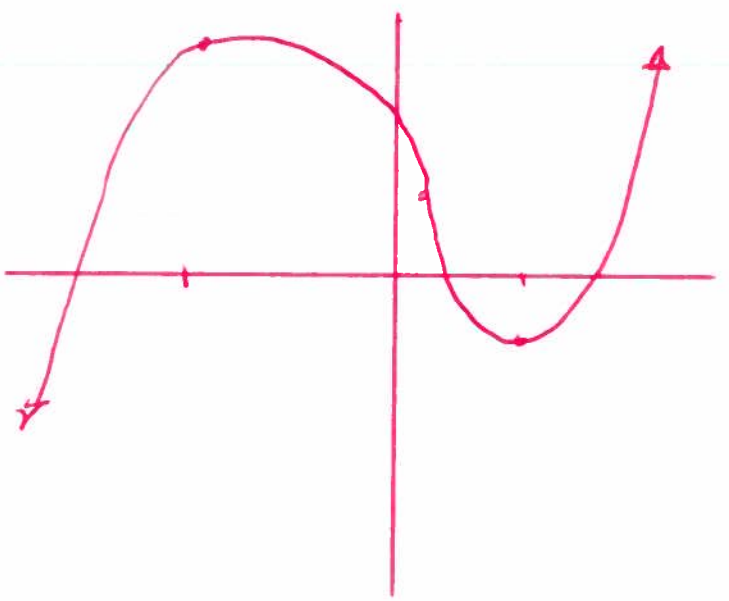
NO SOLUTIONS IN DOMAIN

$\frac{d^2y}{dx^2}$ Always $(\pm 2.187, 1.721)$

3. Make a Key Trait Table (without Zeros) for $y=\left(x-\frac{1}{2}x^2\right)e^{-x}$.

x		.586		1.268		3.414		4.732	
y		.231		.131		-.079		-.057	
y'	+	0	−	−	−	0	+	+	+
y''	−	−	~~0~~	~~0~~	+	0	+	0	−
	∩	Max	∩	∩	∪	MIN	∪		∩

4. Sketch $f(x)=x^3-x^2-8x+8$, labeling all the Key Traits.



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Name_____

Directions: Round at 3 decimal places.
Show all work.

1. Find the Extremes of $f(x)=x^3-x^2-8x+8$.

$f'(x) = 3x^2-2x-8=0$

$(3x+4)(x-2)=0$

$x=2, \cancel{2/3}\ -4/3$

$(2, -4)$

$(-4/3, 14.519)$

2. Find the Points of Inflection of $y=\dfrac{9-x^2}{4x^2+16}$.

$y' = \dfrac{-104x}{(4x^2+16)^2}$ $\quad y'' = \dfrac{(4x^2+16)^2(-104) - (-104x)(2(4x^2+16)^1(8x)}{(4x^2+16)^4}$

$= \dfrac{\cancel{(4x^2+16)}(-104)[4x^2+16-16x^2)}{\cancel{(4x^2+16)^4}}$

$= \dfrac{-104[-12x^2+16]}{(4x^2+16)^3} = 0$

$x = \cancel{\pm\frac{4}{\sqrt{12}}} \pm \frac{2}{\sqrt{3}}$

$(\pm\frac{2}{\sqrt{3}}, 1.359)$

3. Make a Key Trait Table (without Zeros) for $y=\sqrt{x^4-5x^2+4}$.

x		-2.187		-2		-1		0		1		2	
y		1.721		0		0				0		0	
y'	$-$	$-$	$-$	DNE		DNE	$+$	0	$-$	DNE		DNE	$+$
y''	$+$	0	$-$			$-$	$-$	$-$	$-$	$-$			$-$
	∪	POI	∩	MIN	∪	MIN	∩	MAX	∩	MIN	∪	MIN	∩

4. Sketch $y=\left(x-\frac{1}{2}x^2\right)e^{-x}$, labeling all the Key Traits.