PreCalculus Honors
Exponential/Log Test
CALCULATOR ALLOWED
Directions: Round at 3 decimal places.

Name_____

Score_____.

1. If h is the function defined by $h(x)=\ln(e^x+x^e)$, what is the slope of the line tangent to h at $x=2$?

(A) 0.072

(B) 0.666

(C) 1.000

(D) 1.029

(E) 1.169

$h' = \frac{1}{e^x+x^e}(e^x+xe^{(x-1)})$

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2. Let f be the function given by $f(x)=3\ln(2x)$ and let g be a function given by $g(x)=x^3+2x$. At what value of x do the graphs of f and g have parallel tangent lines?

$=3\ln 2+3\ln x$

$f' = \frac{3}{x}$

$g' = 3x^2+2$

(a) -0.781 (b) -0.301 (c) 0.521

(d) 0.782 (e) 1.000

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3. If $f(x)=\frac{x}{\ln(3x)}$, then what is the interval of decreasing?

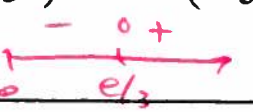
$f' = \frac{\ln(3x)(1)-\frac{x}{x}}{(\ln 3x)^2} =$

(a) (1, 3) (b) (1, 3e) (c) $\left(0,\frac{1}{3}\right)$ (d) $\left(0,\frac{1}{3}e\right)$ (e) $\left(1,\frac{1}{3}e\right)$

$\ln 3x = 1$

$3x = e$

$x = e/3$



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4. $\lim_{x\to 1}\frac{\ln x^2}{x^2-1}$ is

$\overset{L'H}{=} \lim_{x\to 1}\frac{2/x}{2x}$

(a) -1 (b) 0 (c) 1 (d) e (e) undefined

5. The equation of the line **normal** to $y=2x\sqrt{x^2+8}+2$ at $(0, 2)$ is

(a) $x-4\sqrt{2}y=-8\sqrt{2}$
(b) $x+4\sqrt{2}y=8\sqrt{2}$
(c) $4\sqrt{2}x+y=2$
(d) $4\sqrt{2}x-y=-2$
(e) $x+4\sqrt{2}y=2$

$\frac{dy}{dx}=2x\left(\frac{1}{2}(x^2+8)^{-1/2}(2x)\right)+(x^2+8)^{1/2}(2)$

$m_{TAN}=0+\frac{2\sqrt{8}}{\sqrt{8}}=4\sqrt{2}$

$m_{NORM}=-\frac{1}{4\sqrt{2}}$

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6. If $e^{x+y}=y$, then $\frac{dy}{dx}=$

$x+y=\ln y \qquad 1+\frac{dy}{dx}=\frac{1}{y}\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{y}{-y+1}$

(a) $\frac{e^{x+y}}{1-e^{x+y}}$ (b) $\frac{e^{x+y}}{1+e^{x+y}}$ (c) $\frac{e^{x+y}}{e^{x+y}-1}$

(d) e^{x+y} (e) $2e^{x+y}$

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7. Let $f(x)=\frac{e^x}{x}$ on $x\in(0,\infty)$. The minimum value attained by $f(x)$ is

(a) 1 (b) e (c) $\frac{1}{e}$ (d) $e-1$ (e) $\frac{1}{e^2}$

$f'(x)=\frac{xe^x-e^x(1)}{x^2}=0$

$e^x(x-1)=0$

$x=1 \qquad y=\frac{e}{1}$

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Name SOLUTION KEY

Formulas: $S = P\left(1+\frac{r}{n}\right)^{nt}$ $\quad S = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)}{\frac{r}{n}}$ $\quad A = \frac{P\left(1-\left(1+\frac{r}{n}\right)^{-nt}\right)}{\frac{r}{n}}$

1. Suppose you take a $345,000 loan at 10.5% fixed APR compounded monthly for 30 years.

a) What are the monthly payments?

$$345{,}000 = \frac{P\left(1-\left(1+\frac{.105}{12}\right)^{-360}\right)}{.105/12}$$

$$P = \$3155.85$$

b) How much will you actually pay the bank?

$$\text{TOTAL} = 360(3155.85) = \$1{,}136{,}106.$$

2. Find the domain, VAs, and extreme values, algebraically, of $y = \ln(15 + 3x - 5x^2 - x^3)$.

Domain: $x \in (-\infty, -5) \cup (-\sqrt{3}, \sqrt{3})$ $3(5+x) - x^2(5+x)$

Zeros: $(-1.688, 0)$

$(1.688, 0)$

$(-5.045, 0)$

$15 + 3x - 5x^2 - x^3 = 1$

y-intercept: $(0, 15)$ OR $(0, 2.708)$

Extreme points: $(.277, 2.736)$

$$\frac{dy}{dx} = \frac{3 - 10x - 3x^2}{15 + 3x - 5x^2 - x^3}$$

i) $\frac{dy}{dx} = 0 \Rightarrow x = \frac{10 \pm \sqrt{100 + 36}}{-6} = \begin{cases} .277 \\ -3.614 \end{cases}$

ii) $\frac{dy}{dx}$ DNE $\Rightarrow x \neq -5, +\sqrt{3}$

3. Find the domain and extreme values, algebraically, of $y = x^3 e^{-2x}$.

Domain: All Reals

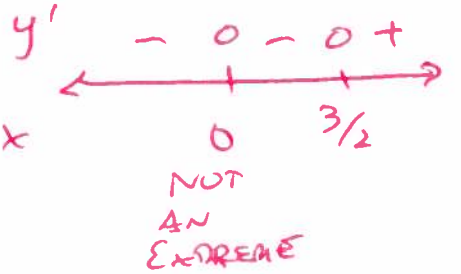
Extreme points: (1.5 , .168)

$$\frac{dy}{dx} = x^3 e^{-2x}(-2) + e^{-2x}(3x^2)$$

i) $= x^2 e^{-2x}(-2x + 3) = 0$

$x = 3/2, 0$ ~~(0,0)~~ $(3/2, .168)$

ii) ~~Also~~ Inverse



PreCalculus Honors
Exponential/Log Test
NO CALCULATOR

Name_____

Score_____.

Show all work.

4. $\frac{d}{dx}\left[Ln\sqrt[7]{e^{5x^4}}\right] = \frac{d}{dx}\left[\frac{5x^4}{7}\right] = \frac{20}{7}x^3$

5. $D_x\left[(4x-3)^5(5x^2+3)^6\right]$

$= (4x-3)^5 \cdot 6(5x^2+3)^5(10x) + (5x^2+3)^6(5(4x-3)^4(4))$

$= 20(4x-3)^4(5x^2+3)^5\left[3x(4x-3) + 5x^2+3\right]$

$= 20(4x-3)^4(5x^2+3)^5(17x^2-9x+3)$

6. Find all the traits and sketch $y = x^3 e^{-2x}$

Domain: All Reals

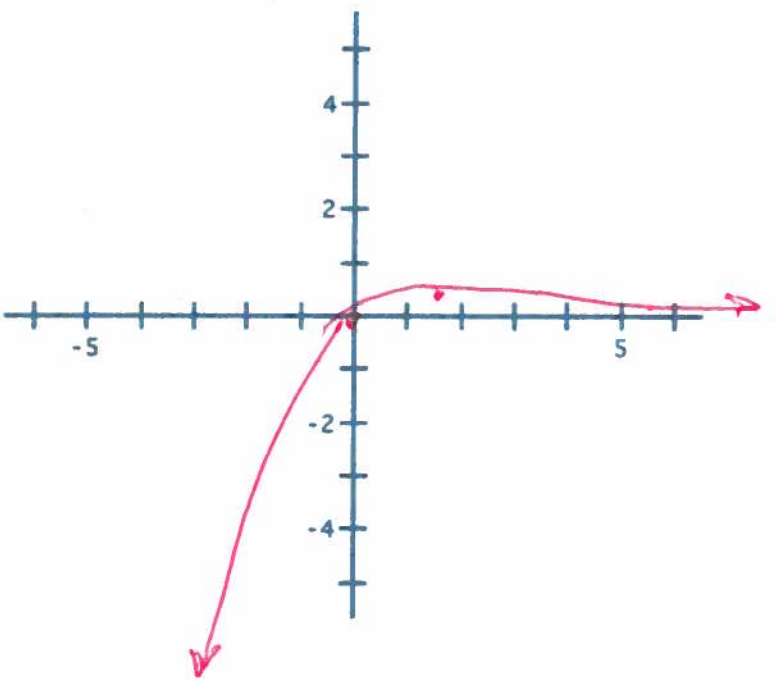
Range: $y \in (-\infty, .168]$

Zeros: $(0, 0)$

***y*-Intercepts:** $(0, 0)$

End Behavior: Left Down
Right $y = 0$

Extreme Points: $(1.5, .168)$



7. Find all the traits and sketch $y = \ln\left(15 + 3x - 5x^2 - x^3\right)$

Domain: $x \in (-\infty, -5) \cup (-\sqrt{3}, \sqrt{3})$ Range: ALL REALS

x-Intercepts: SEE # 2 y-Intercepts: $(0, \ln 15)$

End Behavior: LEFT UP
RIGHT NONE VAs: $x = -5, \pm\sqrt{3}$

Extremes Points:
$(.277, 2.736)$