PreCalculus Honors
Dr. Quattrin
Limits and Derivatives Test
CALCULATOR ALLOWED
Round to 3 decimal places. Show all work.

Name: Solution Key

Score ___________

1. $\lim\limits_{x \to 2} \dfrac{x^2+2x-8}{x^2-4} = \lim\limits_{x \to 2} \dfrac{(x-2)(x+4)}{(x-2)(x+2)} = \dfrac{6}{4}$

(a) -2 (b) -1.5 (c) 10 (d) 1.5 (e) 2.5

2. The slope of the line tangent to $y = x^3 - 4x$ at $(-1, 3)$ is

$\dfrac{dy}{dx} = 3x^2 - 4$

$m = 3(-1)^2 - 4$

(a) 3 (b) 23 (c) -1 (d) -5 (e) -3

3. What is the equation of the line tangent to $y = x^4 + 2x^2$ at the point where $f'(x) = 1$?

~~(a)~~ $y = 8x - 5$ (b) $y = x + 7$ (c) $y = x + .763$

(d) $y = x - .122$ (e) $y = x - 2.146$

$m = 1$

$\dfrac{dy}{dx} = 4x^3 + 4x = -1$

$x = .237$

$y = .115$

$y - .115 = (x - .237)$

$y = x - .122$

4. At what point on the graph of $y=\frac{1}{3}x^3$ is the tangent parallel to the line $2x-8y=3$ $m = 1/4$

(a) $\left(\frac{1}{2},-\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},\frac{1}{8}\right)$ (c) $\left(\frac{1}{2},\frac{1}{24}\right)$

(d) $\left(1,-\frac{1}{2}\right)$ (e) $(2, 2)$

$\frac{dy}{dx} = x^2 = \frac{1}{4}$

$x = \pm 1/2$

$y = 1/24$

5. If $f(x)=\sqrt{x^2-4}$, which of the following is equal to $f'(2)$?

I. $\lim\limits_{x\to 2}\frac{\sqrt{x^2-4}-\sqrt{8}}{x-2}$ II. $\lim\limits_{x\to 2}\frac{\sqrt{x^2-4}}{x-2}$

III. $\lim\limits_{h\to 0}\frac{\sqrt{(2+h)^2-2}-\sqrt{2^2-4}}{h}$

a. I only b. II only c. III only

d. II and III only e. I, II and III

6. A particle moves in the xy-plane so that its coordinates at time t are $x=t^2$ and $y=4+t^3$. At $t=1$, the acceleration vector is

(a) $\langle 2,-3\rangle$ (b) $\langle 2,-6\rangle$ (c) $\langle 1,6\rangle$

(d) $\langle 2,6\rangle$ (e) $\langle 1,-2\rangle$

$v = \langle 2t, 3t^2\rangle$

$\vec{a} = \cancel{\langle 2t, 3t^2\rangle}\ \langle 2, 6t\rangle$

$a(1) = \langle 2, 6\rangle$

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1. A particle's position $\langle x(t), y(t)\rangle$ at time t is described by $\langle t^3-6t^2+9t+1, -t^2+6t+2\rangle$. When is the particle moving right and down?

$x'(t) = 3t^2 - 12t - 9$

x': + 0 − 0 +
t: 1 3

$y'(t) = -2t + 6$

y': + + 0 −
t: 3

$t \in (3, \infty)$

2. The motion of a particle is described by $x(t)=t^3-6t^2+9t+1$.

a) When the particle is stopped?
b) Which direction it is moving at $t=3$?
c) Where is it when $t=3$?
d) Find $a(3)$.

a) $v(t) = 3t^2 - 12t + 9 = 0$

$t = 1, 3$

b) $v(3) = 0$ NEITHER

c) $x(3) = 1$

d) $a(t) = 6t - 12$

$a(3) = 6$

PreCalculus
Limits and Derivatives Test v4
NO CALCULATOR ALLOWED

Name: Solution Key

3. Set up, but do not solve, the limit definition of the derivative for
$y = 6x^4 - 2x^3 + \pi^2 + 4x$

$$\lim_{h \to 0} \frac{\left[6(x+h)^4 - 2(x+h)^3 + \pi^2 + 4(x+h)\right] - \left[6x^4 - 2x^3 + \pi^2 + 4x\right]}{h}$$

4. $D_x\left[\sqrt[4]{x^3} + \frac{9}{x^2} - 2\sqrt[4]{x^9} - \pi^2\right]$

$$= D_x\left[x^{3/4} + 9x^{-2} - 2x^{9/4} - \pi^2\right]$$

$$= \frac{3}{4}x^{-1/4} - 18x^{-3} - \frac{9}{2}x^{5/4}$$

5. Evaluate the following limits:

(a) $\lim_{x \to 1} \frac{x^3 - 1}{x^2 + 3x - 4} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+4)}$

$= \lim_{x \to 1} \frac{x^2+x+1}{x+4} = \frac{3}{5}$

(b) $\lim_{x \to -3} \frac{x^2 + 7x + 12}{x^2 - 8} = 0$

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