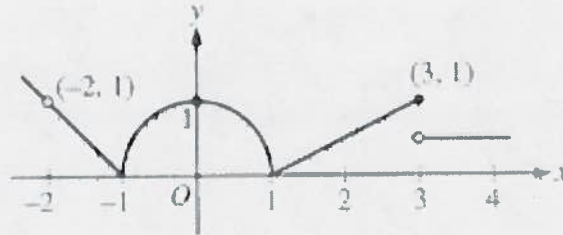


Multiple Choice (3 pts. each)

1. The graph of a function is shown below. For which of the following values of c does $\lim_{x \rightarrow c} f(x)$ not exist?



Graph of f

- (a) 0 only (b) 3 only (c) -2 only
(d) -2 and 3 only (e) -2, 0, and 3

2. The end behavior of $g(x) = \sqrt{\frac{x^3 - 4x}{x^2 - 9}}$

\sqrt{x} RIGHT UP
NONE LEFT

- (a) Up on both ends
(b) Up on the left and none on the right
(c) None on the left and none on the right
(d) None on the left and up on the right
(e) None on the left and $y = 2$ on the right

3. Let k and b be real numbers and let the function f be defined by

$$f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \leq 1 \\ 3x + b & \text{for } x > 1 \end{cases}$$

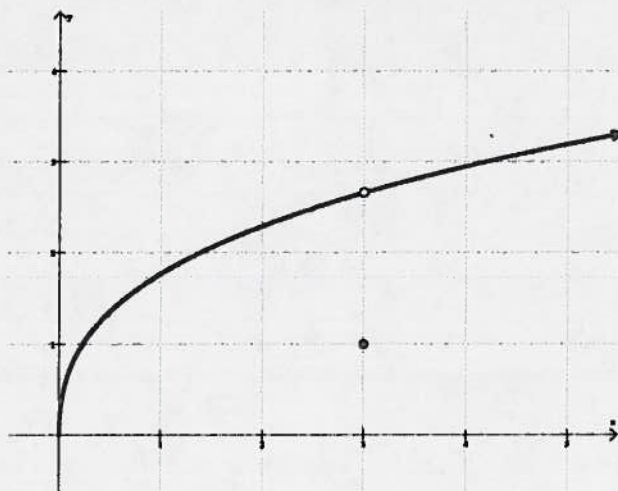
$f'(x) = \begin{cases} 2x + k & (1) \\ k = 1 \end{cases}$

If f is both continuous and differentiable at $x = 1$, then

- (a) $k = -1, b = -3$
- (b) $k = 1, b = 3$
- (c) $k = 1, b = 4$
- (d) $k = 1, b = -4$**
- (e) $k = -1, b = 6$

$x^2 + kx - 3 = 3x + b$
 $1 + 1 - 3 = 3 + b$
 $-4 = b$

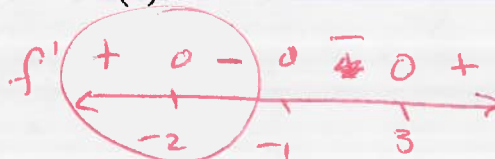
4. Use the graph of f below to select the **correct** answer from the choices below.



- (a) $\lim_{x \rightarrow 3} f(x) = f(3)$
- (b) f is not continuous at $x = 3$**
- (c) f is differentiable at $x = 3$
- (d) $f'(1) < f'(4)$
- (e) $\lim_{x \rightarrow 3} f(x)$ does not exist

5. If the derivative of the function f is $f'(x) = 3(x+2)(x+1)^2(x-3)^3$, then f has a local maximum at $x =$

- (a) -2 only**
- (b) -1 only
- (c) 3 only
- (d) -2 and 3
- (e) -1 and 3



6. Let f be defined by $f(x) = \begin{cases} 5x^2 + 9, & \text{if } x < -5 \\ 5x, & \text{if } x = -5 \\ Ax + 0, & \text{if } -5 < x \end{cases}$. Determine the value of A for which is continuous for all real x .

$$5x^2 + 9 = Ax + 9$$

- (a) -4 (b) -5 (c) 5 (d) -25 (e) -15

$$125 + 9 = -5A + 9$$

$$A = -25$$

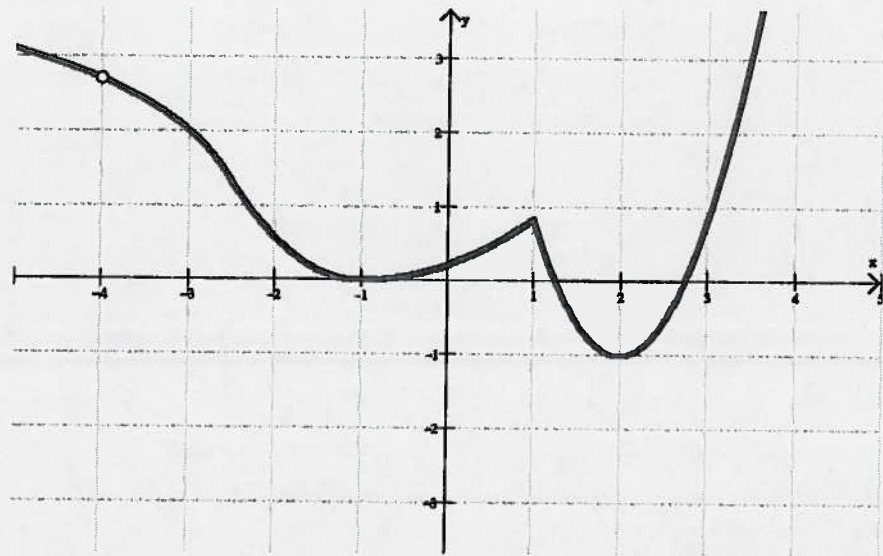
7. Let g be defined by $g(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{for } x \neq 2 \\ 1 & \text{for } x = 2 \end{cases}$.

Which of the following statements must be true?

- I. g has a limit at $x = 2$
 II. g is continuous at $x = 2$
 III. g is differentiable at $x = 2$

- (a) I only (b) II only (c) III only
 (d) I and II only (e) I, II, and III

8. At what point is the graph below not differentiable?



(a) -4

(b) -1

~~(c) 1~~

(d) 2

(e) -4 and 1

Free Response (10 pts. each)

1. Given by $f(x) = \begin{cases} \frac{x^2-3}{x-\sqrt{3}} & \text{for } x \neq \sqrt{3} \\ 1 & \text{for } x = \sqrt{3} \end{cases}$,

- a) Is the function f continuous at $x = \sqrt{3}$?
- b) If not, is the discontinuity removable or essential?
- c) Is the function f differentiable at $x = \sqrt{3}$?

a) i) $f(\sqrt{3})$ EXISTS. [= 1]

ii) $\lim_{x \rightarrow \sqrt{3}} \frac{x^2-3}{x-\sqrt{3}}$ EXISTS [= $2\sqrt{3}$]

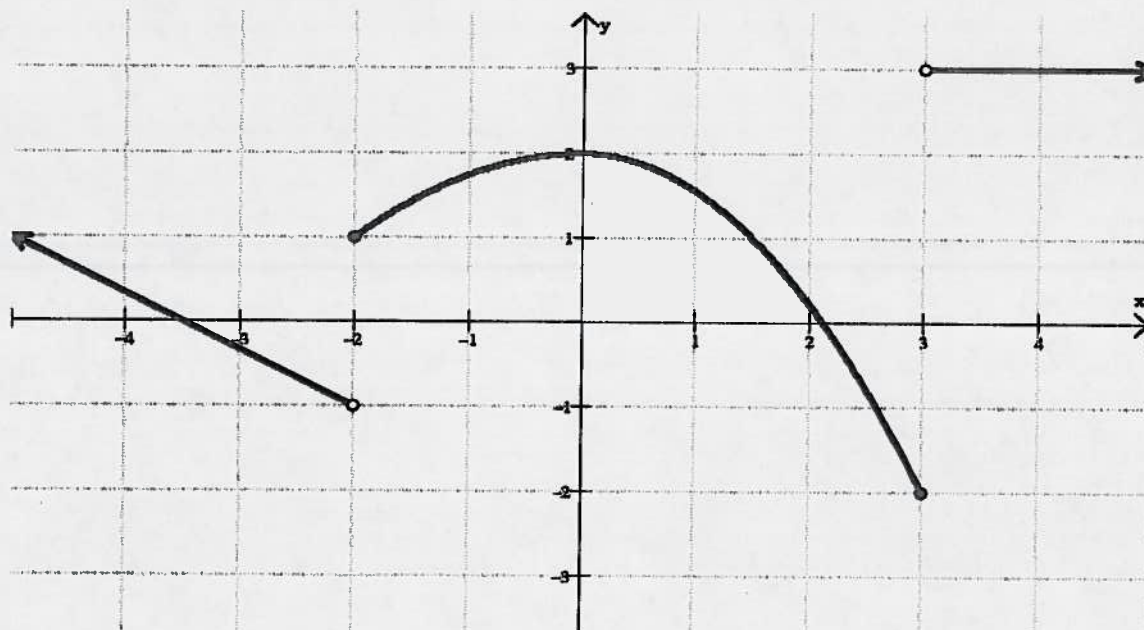
iii) $\lim_{x \rightarrow \sqrt{3}} f(x) \neq f(\sqrt{3})$

\therefore NOT CONTINUOUS

b) REMOVABLE

c) NOT CONTINUOUS \therefore NOT DIFFERENTIABLE

2. Given the graph of $f(x)$ below, find the values of the following:



a. $\lim_{x \rightarrow -2^-} f(x) = -1$

b. $\lim_{x \rightarrow -2^+} f(x) = 1$

c. $\lim_{x \rightarrow 2} f(x) = \text{DNE}$

d. $f(-2) = 1$

e. $\lim_{x \rightarrow 0^+} f(x) = 2$

f. $\lim_{x \rightarrow 0^-} f(x) = 2$

g. $\lim_{x \rightarrow 0} f(x) = 2$

h. $f(0) = 2$

i. $\lim_{x \rightarrow 3^+} f(x) = \text{DNE}$

j. $\lim_{x \rightarrow 3^-} f(x) = -2$

k. $\lim_{x \rightarrow 3} f(x) = \text{DNE}$

l. $f(3) = -2$

m. $\lim_{x \rightarrow 4^+} f(x) = 3$

n. $\lim_{x \rightarrow 4^-} f(x) = 3$

3. Is the function g , given by $g(x) = \begin{cases} x^2 - 2x + 1, & \text{if } x > -1 \\ 2, & \text{if } x = -1 \\ 3 - x, & \text{if } x < -1 \end{cases}$, continuous at

$x = -1$? differentiable at $x = -1$?

CONTINUOUS

i) $f(-1)$ EXISTS. [= 4]

ii) $\lim_{x \rightarrow -1} f(x)$ EXISTS [= 4]

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1} 3 - x = 4$$

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1} x^2 - 2x + 1 = 4$$

iii) $\lim_{x \rightarrow -1} f(x) = f(-1)$

\therefore CONTINUOUS

DIFFERENTIABLE

i) $f(x)$ IS CONTINUOUS @ $x = -1$

$$f'(x) = \begin{cases} 2x - 2 & \text{IF } x > -1 \\ -1 & \text{IF } x < -1 \end{cases}$$

ii) $\lim_{x \rightarrow -1^-} f'(x) \neq \lim_{x \rightarrow -1^+} f'(x)$

BOTH EXIST

iii) $\lim_{x \rightarrow -1^-} f(x) \neq \lim_{x \rightarrow -1^+} f(x)$

\therefore NOT DIFFERENTIABLE

4. Sketch $g(x) = \begin{cases} x^2 - 2x + 1, & \text{if } x > -1 \\ 4, & \text{if } x = -1 \\ 3 - x, & \text{if } x < -1 \end{cases}$ and state the Traits listed. Provide proof for the extreme points.

Domain: $x \in \text{All Reals}$

Range: $y \in (0, \infty)$

Zeros: NONE

Y-int: $(0, 1)$

VAs: NONE

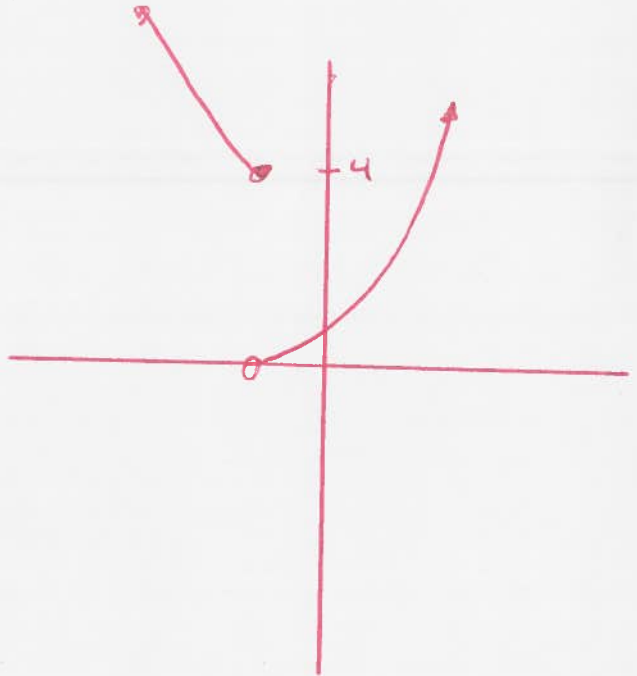
EB (Left): DOWN

EB (Right): UP

Extreme Points: $(-1, 4)$

(i) TRANSITION POINT

(ii) $x = -1$



CONTINUITY AT ~~TRANSITION POINT: NO~~ ? CONTINUOUS EVERYWHERE EXCEPT AT $x = -1$

DIFFERENTIABILITY AT ~~TRANSITION POINT: NO~~

DIFFERENTIABLE EVERYWHERE EXCEPT AT $x = -1$