

PreCalculus Honors

Name: SOLUTION KEY

Dr. Quattrin

Radical Test

CALCULATOR ALLOWED

Score _____

Round to 3 decimal places. Show all work.

1. Let $f(x)$ be the function with $f(1)=2$ and $f'(x)=\sqrt{x^2+3}$. Using the tangent line approximation to the graph of $f(x)$ at $x=1$, estimate $f(1.02)$.

- a. 2.01 b. 2.02 c. 2.03 **d. 2.04** e. 2.05

$$y - 2 = 2(x - 1)$$

$$y = 2 + 2(0.02)$$

2. Use Implicit Differentiation to find the points on $x^3 - y^2 + x^2 = 0$ has vertical tangent lines.

- (a) (0, 0) only
(b) (-1, 0) only
(c) $(1, \sqrt{2})$ only
(d) (-1, 0) and (0, 0)
(e) The tangent line is never vertical

$$3x^2 - 2y \frac{dy}{dx} + 2x = 0$$

$$\frac{dy}{dx} = \frac{3x^2 + 2x}{2y} = \text{DNE}$$

$$y = 0$$

$$x^3 + 0 + x^2 = 0$$

$$x^2(x+1) = 0$$

3. $\lim_{x \rightarrow \infty} \frac{3x^5 + 3x^4 + 2x^3 + x^2 + 1}{4x^5 - 9x^4 + 4x^3 + 15} =$

- (a) 0 **(b) $\frac{3}{4}$** (c) $\frac{4}{3}$ (d) 3 (e) DNE

4. The y-intercept of the tangent line to the curve $y = \sqrt{x+3}$ at the point (1, 2) is

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{5}{4}$ **(e) $\frac{7}{4}$**

$$y' = \frac{1}{2}(x+3)^{-1/2}$$

$$y - 2 = \frac{1}{4}(x - 1)$$

$$y - 2 = \frac{1}{4}x - \frac{1}{4}$$

$$y = \frac{1}{4}x + \frac{7}{4}$$

5. The line normal to $3x^2 + y^2 + 4y = 5$ is horizontal at
- $6x + 2y \frac{dy}{dx} + 4 \frac{dy}{dx} = 0 \quad \frac{dy}{dx} = \frac{-6x}{2y+4}$ ~~the~~ $y = -2$
- (a) $x = -3$ (b) $x = -2$ (c) $x = 0$ (d) $x = 2$ (e) $x = 3$ ~~379~~ ~~x=3~~

6. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

| x | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 2 | 4 | -2 | 8 | 1 |
| 4 | 8 | 8 | 4 | 3 |
| 8 | 6 | -12 | 2 | 4 |

- Given that $h(x) = g(f(x))$, $h'(4) = g'(f(4)) \cdot (f'(4))$
 $g'(8) \cdot (8) =$
 (a) 32 (b) 10 (c) -6 (d) 24 (e) 16

7. Find the maximum value of $f(x) = 2x^3 + 3x^2 - 12x + 4$ on the closed interval $[0, 2]$.

- $f(0) = 4$ $f(2) = 8$ ~~$f(x) = 2x^3 + 3x^2 - 12x + 4$~~
 $f' = 6x^2 + 6x - 12 = 6(x+2)(x-1)$ $f(1) = -4$
- a) -3 (b) 0 (c) 2 (d) 4 (e) 8

8. At what approximate rate (in cubic meters per minute) is the volume of a sphere changing at the instant when the surface area is 3 square meters and the radius is increasing at the rate of $1/5$ meters per minute?

- (a) 1.228 (b) 1.905 (c) 0.649 (d) 0.600 (e) 0.62016

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Show all work.

1. Find the zeros, critical values, and extreme values of $y = -\sqrt{-3x^3 + 5x^2 + 48x - 80}$. Show the algebraic work, including sign patterns where necessary, to support the zeros and critical values.

Zeros: $(\pm 4, 0)$ $(\frac{5}{3}, 0)$

$$y^2 = (-3x + 5)(x^2 - 16)$$

$$x = +\frac{5}{3}, \pm 4$$

Critical Values: $\pm 4, \frac{5}{3}, 2.931$

$$f'(x) = \frac{-9x^2 + 10x + 48}{-2(-3x^3 + 5x^2 + 48x - 80)^{1/2}}$$

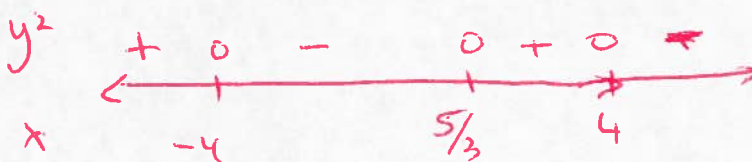
Extreme Values: $0, -5.301$

$$i) f'(x) = 0$$

$$x = -1.8, 0, 2.931$$

ii) $f'(x)$ DNE: $(\pm 4, 0), (\frac{5}{3}, 0)$

iii) NO END POINTS



2. Find the zeros, VAs, POEs, critical values, and extreme values of $y = \sqrt{\frac{x-2}{x^3-4x}}$. Show the derivative and algebra to support the critical values.

Zeros: NONE

VAs: $x=0, -2$

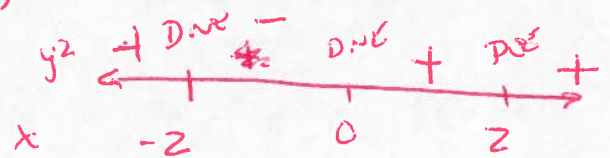
Critical Values: NONE

POEs: $(2, \frac{1}{\sqrt{8}})$

Extreme Values: NONE

$$y = \sqrt{\frac{x-2}{x(x^2-4)}} \approx \sqrt{\frac{1}{x(x+2)}} = (x^2+2x)^{-1/2}$$

$$\frac{dy}{dx} = -\frac{1}{2} (x^2+2x)^{-3/2} (2x+2)$$



$$\begin{aligned} \text{i) } \frac{dy}{dx} = 0 &\rightarrow x = -1 \rightarrow y \text{ DNE} \\ \text{ii) } \frac{dy}{dx} \text{ DNE} &\rightarrow x = 0, -2 \rightarrow y \text{ DNE} \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{i) } \frac{dy}{dx} = 0 \\ \text{ii) } \frac{dy}{dx} \text{ DNE} \end{aligned}} \right\} N$$

iii) NO ARBITRARY DOMAIN.

3a. $\frac{d}{dx}[(7x^2-2x)^{17}]$

$$17(7x^2-2x)^{16}(14x-2)$$

3b. $\frac{d}{dx}[\sqrt[5]{5x^2-10x+1}]$

$$= \frac{1}{5} (5x^2-10x+1)^{4/5} (10x-10)$$

$$= \frac{2x-2}{(5x^2-10x+1)^{1/5}}$$

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Polynomials Test—CALCULATOR NOT ALLOWED

Show all work.

Score _____

4. Find the traits and sketch $y = -\sqrt{-3x^3 + 5x^2 + 48x - 80}$.

Domain: $x \in (-\infty, -4] \cup [5/3, 4]$ Range: $y \in (-\infty, 0]$

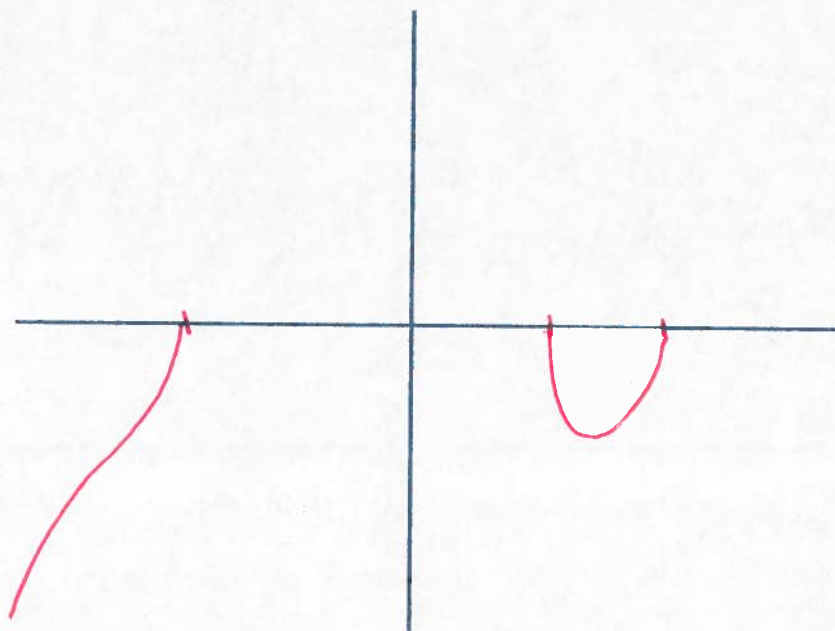
Y-Int: NONE

Zeros: $(-4, 0)$, $(5/3, 0)$

Extreme Points: $(-4, 0)$, $(5/3, 0)$, $(2.931, -5.351)$

End Behavior (Right): ~~NONE~~

End Behavior (Left): ~~NONE~~
DOWN



5. List the traits and sketch of $y = \sqrt{\frac{x-2}{x^3-4x}}$ on $x \in [-4, 4]$.

Domain: $x \in [-4, -2) \cup (0, 2) \cup (2, \infty)$

Range: $y \in [\frac{1}{\sqrt{24}}, \infty)$

Y-Int: NONE

VAs: $x = 0, -2$

Zeros: NONE

Extreme Points: $(4, \frac{1}{\sqrt{24}})$

End Behavior (Right): NONE

POE $(-4, \frac{1}{8})$

End Behavior (Left): NONE

POE $(2, \frac{1}{8})$

