

PreCalculus Honors

Name: SOLUTION KEY

Dr. Quattrin

Rational Functions Test

CALCULATOR ALLOWED

Score \_\_\_\_\_

Round to 3 decimal places. Show all work.

1. What is  $f'(1)$  if  $f(x) = \frac{x^2 + 8}{7x}$ ?

$$f'(x) = \frac{7x(2x) - (x^2 + 8)(7)}{(7x)^2} = \frac{2x^2 - 8}{7x^2}$$

(a)  $-\frac{1}{49}$

(b)  $\frac{15}{49}$

(c)  $-1$

(d)  $\frac{15}{7}$

(e)  $\frac{16}{7}$

2. The equation of the line tangent to the graph of  $y = \frac{3x+4}{4x-3}$  at the point  $(1,7)$  is

(a)  $y = -25x + 32$

(b)  $y = 31x - 24$

(c)  $y = 7x$

(d)  $y = 5x + 12$

(e)  $y = 25x - 18$

$m = -25$

$y - 7 = -25(x - 1)$

3. A function is defined as  $g(x) = \frac{(x-4)^2}{x-7}$ . Which of the following is **false**?

(a)  $g(x)$  is increasing for  $x > 7$ .

(b)  $g(x)$  is decreasing on  $[4, 7]$ .

(c)  $g(x)$  has a local maximum at  $x = 4$ .

(d)  $g(x)$  has a horizontal asymptote at  $y = 1$ .

(e)  $g(x)$  has a vertical asymptote at  $x = 7$ .

4. Let  $f(x)$  and  $g(x)$  be differentiable functions. The table below gives the values of  $f(x)$  and  $g(x)$ , and their derivatives, at several values of  $x$ .

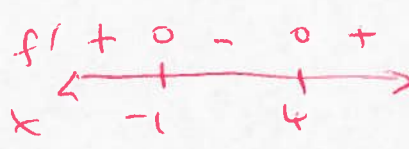
$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	-6
2	1	8	-5	7
3	7	-2	7	9

If  $h(x) = \frac{f(x)}{g(x)}$ , what is the value of  $h'(2)$ ?  $= \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2}$

- a) -4    b) -63    c) 51    **d)  $-\frac{47}{64}$**     e)  $-\frac{33}{64}$

5. Suppose  $f'(x) = \frac{(x+1)(x-4)^3}{(x^4+1)}$ . Which of the following statements must be true?

- I. The slope of the line tangent to  $y = f(x)$  at  $x = 2$  is -8.  
**II.**  $f(x)$  is increasing on  $x \in (1, 4)$   
**III.**  $f(x)$  has a minimum at  $x = 4$



- (a) I only    (b) II only    **(c) III only**    **(d) II and III only**    (e) I, II and III

6. An equation of the line tangent to the curve  $y = \frac{kx+8}{k+x}$  at  $x = -2$  is  $y = 2x + 4$ . What is the value of  $k$ ?

- a)  $-\frac{3}{4}$     b)  $-\frac{1}{2}$     c) 0    d)  $\frac{1}{2}$     **(e) 3/4**

$$\frac{(k+x)(k) - (kx+8)(1)}{(k+x)^2} = 2$$

$$\frac{(k-2)(k) - (-2k+8)}{(k-2)^2} = 2$$

$$k^2 - 8 = 2(k-2)^2 \quad u = k^2 - 8k + 8$$

7. If  $y = \frac{2x+3}{2-3x}$ , then  $\frac{dy}{dx} = \frac{(2-3x)(2) - (2x+3)(-3)}{2(2-3x)^2} =$

- a)  $\frac{-12x-2}{(3x+2)^2}$     b)  $\frac{-12x+2}{(3x+2)^2}$     c)  $\frac{13}{(3x+2)^2}$   
 d)  $\frac{-13}{(3x+2)^2}$     e)  $-\frac{2}{3}$

8.  $\lim_{x \rightarrow \infty} \frac{5x^3 - x^2 + 20x - 4}{5x^2 - 11x + 2}$

- a) 0    b) 1    c) -2    d) 5    e)  $\infty$

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1. Find asymptotes, POEs, and zeros of  $y = \frac{5x^3 - x^2 + 20x - 4}{5x^2 - 11x + 2}$ . Show the algebraic work to support the zeros. Find the extreme points graphically, but show the algebraic work to support the critical values.

$$y = \frac{(x^2 + 4)(5x - 1)}{(5x - 1)(x - 2)}$$

$$\text{Ext Pts: } (-0.828, -1.657) \\ (4.828, 9.657)$$

Zeros: NONE

VA:  $x = 2$

POE  $(1/5, -2.244)$

$$\frac{dy}{dx} = \frac{(x-2)(2x) - (x^2+4)(1)}{(x-2)^2} = 0$$

SA

$$\begin{array}{r} 2x + 4 \\ x - 2 \overline{) x^2 + 4} \\ \underline{-(x^2 - 2x)} \\ 2x \end{array}$$

$$x^2 - 4x - 4 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 16}}{2}$$

2.  $\frac{d}{dx} \left[ \frac{2x^2 - x - 3}{3 + 2x - x^2} \right]$

$$= \frac{d}{dx} \left[ \frac{(2x-3)(x+1)}{(3-x)(x+1)} \right]$$

$$= \frac{d}{dx} \left[ \frac{2x-3}{3-x} \right] = \frac{(3-x)(2) - (2x-3)(-1)}{(3-x)^2}$$

$$= \frac{3}{(3-x)^2}$$

3. Find all asymptotes, zeros, POEs, and Extreme Points of  $y = \frac{16-x^2}{x^2-4}$ . Show the derivative and algebra to support the critical values.

$$\text{HA: } y = -1$$

$$\text{VA: } x = \pm 2$$

$$\text{Zeros } (\pm 4, 0)$$

POE: NONE

$$\frac{dy}{dx} = \frac{(x^2-4)(-2x) - (16-x^2)(2x)}{(x^2-4)^2} = 0$$

$$= \frac{8x - 32x}{(x^2-4)^2} = 0$$

$$x = 0$$

$$y = +4$$

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Rational Functions Test – NO CALCULATOR ALLOWED

Show all work.

4. Write an equation of a rational function that has  $x$ -intercepts at  $(-3, 0)$ , VA at  $x = 5$ , a POE at  $x = -2$ , and a HA at  $y = \frac{6}{5}$ .

$$y = \frac{6(x+3)(x+2)}{5(x-5)(x+2)}$$

5. Find the traits and sketch  $x^2y + x^2 - 4y - 16 = 0 \Rightarrow y = \frac{16 - x^2}{x^2 - 4}$

Domain:  $x \neq \pm 2$

Range:  $y \in (-\infty, -4] \cup [-1, \infty)$

Y-Int:  $(0, -4)$

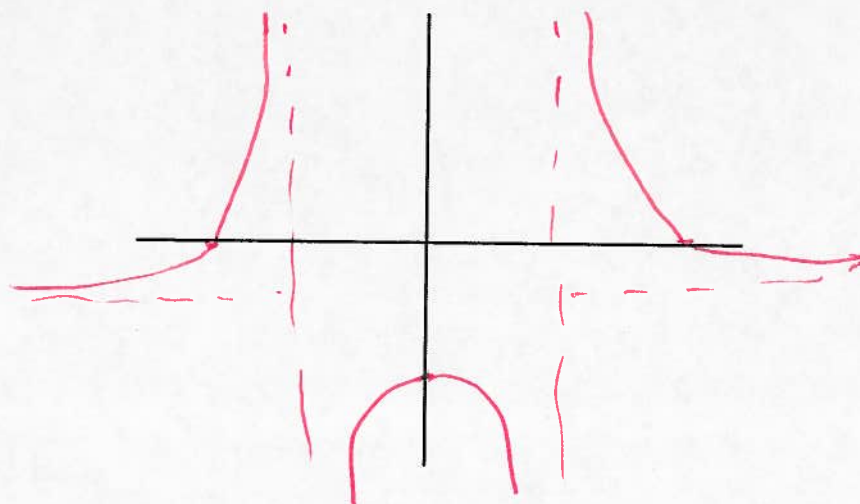
End Behavior:  $y = -1$

Vas:  $x = \pm 2$

POEs: NONE

Zeros:  $(\pm 4, 0)$

Extreme Values:  $(0, -4)$





6. Find the traits and sketch of  $y = \frac{5x^3 - x^2 + 20x - 4}{5x^2 - 11x + 2}$  on  $x \in [0, 7]$ .

$x \in [0, 1/5) \cup (1/5, 2) \cup (2, 7]$

Domain:  ~~$x \in [0, 2) \cup (2, 7]$~~

Range:  $y \in (-\infty, -\frac{41}{2.244}) \cup (-2.244, -2] \cup [9, \infty)$

Y-Int:  $(0, -2)$

End Behavior: NONE

Zeros: NONE

Extreme Values:  $(4.828, 4.657)$

POEs:  $(1/5, -2.244)$

VAs:  $x = 2$   
 $(7, 53/5)$

