


Directions: Round to 3 decimal places. Show all work.

1. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

$$f' = 4x^3 + 2x = 2x(x^2 + 1)$$


- a) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$ b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ c) $(-\infty, 0)$
d) $(0, \infty)$ e) $\left(-\infty, \frac{1}{\sqrt{2}}\right)$

2. What is the area of the largest rectangle with lower base on the x -axis and upper vertices on the curve $y = 12 - x^2$?

- a) 8 b) 12 c) 16 d) 32 e) 48



$$A = lw$$
$$= 2xy$$

$$A = 2x(12 - x^2) = 24x - 2x^3$$

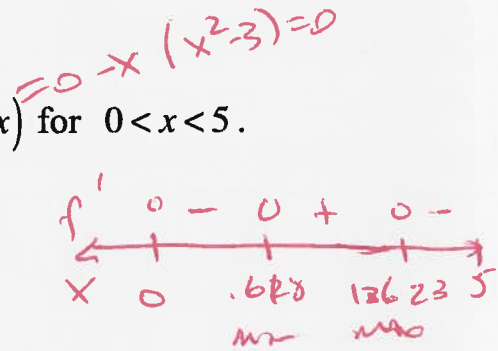
$$\frac{dA}{dx} = 24 - 6x^2 = 0$$
$$x = \pm 2$$

$$A = 24(2) - 2(2)^3 = 24 - 16 = 8$$

3. The derivative of f is given by $f'(x) = e^x(-x^3 + 3x)$ for $0 < x < 5$.

At what value of x does $f(x)$ have a local minimum?

- a) 0 b) 0.618 c) 1.623
d) 5 e) For no value of x



4. Find the equation of the line tangent to the graph of $y = 7x - x^2$ at the point where $f'(x) = 3$

- a) $y = 5x - 10$
b) $y = 3x + 4$
c) $y = 3x + 8$
d) $y = 3x - 10$
e) $y = 3x - 16$

$f' = 7 - 2x^2 = 3$
 $4 = 2x^2$
 $2 = x^2$
 $x = \pm\sqrt{2}$

$y - 10 = 3(x - 2)$
 $y = 3x - 6 + 10$
 $y = 3x + 4$

5. Find all the solutions to $\csc x = 2.4$

$$x = \sin^{-1}(1/2.4)$$

a) $x = .429 \pm 2\pi n$

b) $x = 2.712 \pm 2\pi n$

c) $x = 1.414 \pm 2\pi n$

d) $x = .429 \pm 2\pi n$ and $x = 1.414 \pm 2\pi n$

e) $x = .429 \pm 2\pi n$ and $x = 2.712 \pm 2\pi n$

6. $\lim_{x \rightarrow 0} \frac{5x^4 + 8x^2}{3x^4 - 16x^2} = \lim_{x \rightarrow 0} \frac{x^2(5x^2 + 8)}{x^2(3x^2 - 16)} = \frac{8}{-16}$

a) $-1/2$

b) 0

c) 1

d) $5/3$

e) dne

7. The graph of $f(x) = \sec x$ can be obtained from the graph of $y = \sec x$ by applying, in order, a horizontal stretch by a factor of 2, a vertical stretch by a factor of 3, and a vertical shift down 3 units. The equation of f is.

a) $f(x) = -3 + 3\sec 2x$

$A = 3$

b) $f(x) = -3 + 3\sec \frac{x}{2}$

$K = -3$

$B = \frac{1}{2}$

~~c) $f(x) = 3 - 3\sec 2x$~~

$h = 0$

~~d) $f(x) = 3 - 3\sec \frac{x}{2}$~~

e) None of the above

8. The amplitude of the graph of $y = 3 + 2\cos(x - \pi)$

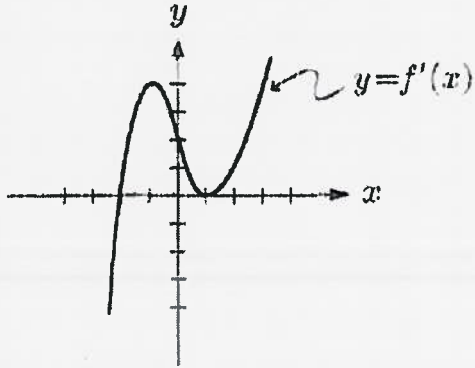
a) $\frac{\pi}{2}$

b) π

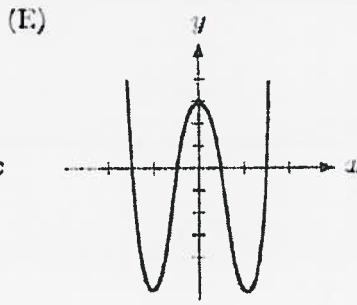
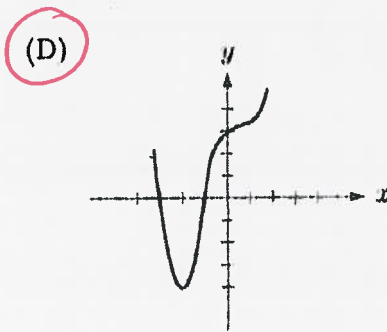
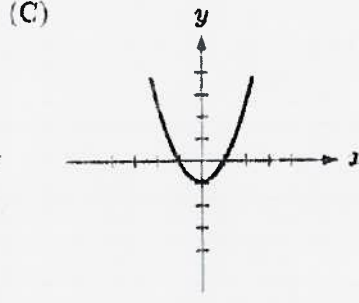
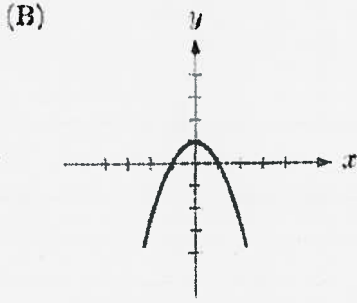
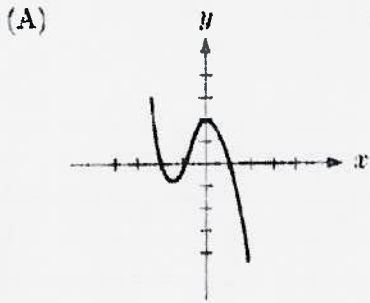
c) 2

d) 3

e) 2π



9. The graph of the derivative of f is shown above. Which of the following could be the graph of f ?



f' is cubic so f is 4^{th} .

At $x=1$, f' has a "BOUNCER", so f has a point but not an extreme.

1. Solve $\sin\left(\frac{\pi}{3}-x\right)+1=\sin\left(x+\frac{\pi}{3}\right)$ exactly for $x \in [0, 4\pi]$

$$\sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x + 1 = \sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3}$$

$$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x + 1 = \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$$

$$1 = \sin x$$

$$x = \frac{\pi}{2} \pm 2\pi n$$

$$x = \left\{ \frac{\pi}{2}, \frac{5\pi}{2} \right\}$$

2. $D_x \left[8x^7 - 3x^2 + \frac{1}{x^7} - \frac{2}{\sqrt{x^4}} + \pi^4 \right]$

$$= 56x^6 - 6x - 7x^{-8} + \frac{8}{9}x^{-13/9}$$

3. Homer Simpson has accidentally stolen a nuclear submarine. If he travels at 27 miles on a bearing of 18° , then turns and goes 57 miles at a bearing of 85° , and finally travels 300 miles at 270° , what is his total displacement and direction?

$$27 \cos 18^\circ \vec{i} + 27 \sin 18^\circ \vec{j}$$

$$57 \cos 85^\circ \vec{i} + 57 \sin 85^\circ \vec{j}$$

$$300 \cos 270^\circ \vec{i} + 300 \sin 270^\circ \vec{j}$$

$$30.646 \vec{i} - 234.873 \vec{j}$$

$$|\vec{r}| = \sqrt{30.646^2 + 234.873^2}$$

$$= 236.864$$

$$\theta = -\cos^{-1} \left(\frac{30.646}{236.864} \right) = -82.566^\circ$$

4a. Find the zeros, algebraically, of $y = x^4 - 5x^3 - 15x^2 + 45x + 54$.

$$\begin{array}{r|rrrrr} -3 & 1 & -5 & -15 & 45 & 54 \\ & & -1 & 6 & 9 & -54 \\ \hline & 1 & -6 & -9 & 54 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -6 & -9 & 54 \\ & & 3 & -9 & -54 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

$$\begin{aligned} & (6, 0) \\ & (\pm 3, 0) \\ & (-1, 0) \end{aligned}$$

$$\begin{aligned} & (x+1)(x-3)(x^2-3x-18) \\ & (x+1)(x-3)(x+3)(x-6) \end{aligned}$$

4b. Find the extreme points, graphically, of $y = x^4 - 5x^3 - 15x^2 + 45x + 54$. Show the derivative before using your calculator.

$$\frac{dy}{dx} = 4x^3 - 15x^2 - 30x + 45$$

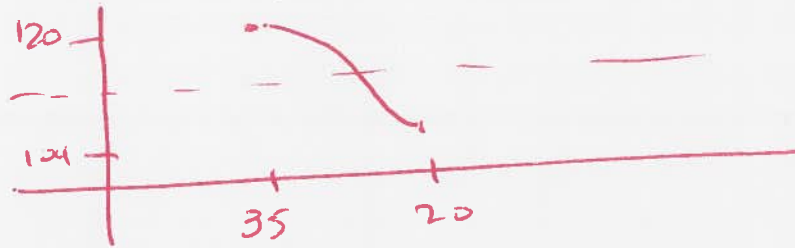
$$(-2.155, -41.029)$$

$$(1.083, 80.166)$$

$$(4.822, -97.743)$$

5. Researchers find an extra-terrestrial being. In studying it, they find that its body temperature varies sinusoidally with time. 35 minutes after they start timing, the temperature is at its highest, which is 120°C . 20 minutes after it has reached its maximum, the temperature hits its minimum, which is 104°C .

(a) Sketch a graph of the temperature as a function of time.



(b) Write a sinusoidal equation that describes the temperature y in terms of t .

$$T = 112 + 8 \cos \frac{\pi}{20} (t - 35)$$

(c) What was the temperature when they started timing?

$$117.657^{\circ}\text{C}$$

(d) What are the first three times that the temperature is 111°C ?

$$111 = 112 + 8 \cos \frac{\pi}{20} (t - 35)$$

$$-\frac{1}{8} = \cos \frac{\pi}{20} (t - 35)$$

$$\frac{\pi}{20} (t - 35) = \begin{cases} 1.696 \pm 2\pi n \\ -1.696 \pm 2\pi n \end{cases}$$

$$t - 35 = \begin{cases} 10.798 \pm 40n \\ -10.798 \pm 40n \end{cases}$$

$$t = \begin{cases} 45.798 \pm 40n \\ 24.202 \pm 40n \end{cases}$$

$$t = 5.798, 24.202, 45.798$$

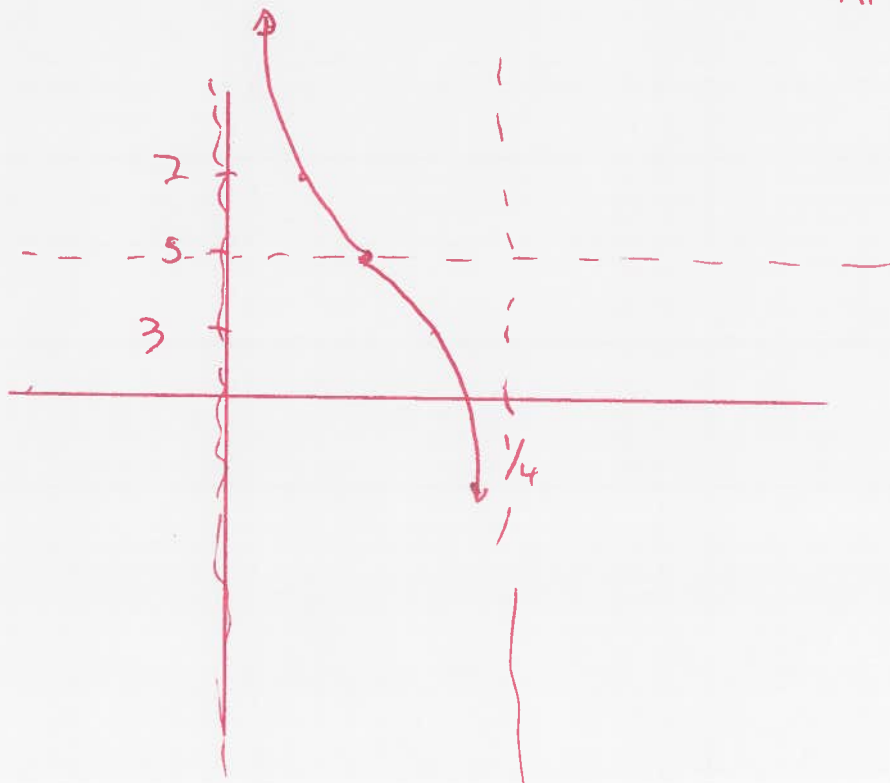
6. Find an inequality that has this sign pattern and solution:

$$\begin{array}{c} y \\ \leftarrow \begin{array}{cccccc} + & 0 & - & 0 & - & 0 & + & 0 & - \\ -5 & -2 & \frac{1}{2} & 7 & 13 \end{array} \rightarrow \\ x \end{array} \text{ and } x \in (-5, -2) \cup \left(-2, \frac{1}{2}\right) \cup (13, \infty)$$

$$-(x+5)(x+2)^2(2x-1)(x-7)^2(x-13) < 0$$

7. Sketch the primary cycle of $y = 5 + 2 \cot(4\pi x)$

$$P = \frac{\pi}{4\pi} = \frac{1}{4}$$



$$\begin{aligned}
 8. \quad \text{Prove } 1 + \sin(2x) &= \frac{\sec^2 x + 2 \sin x \sec x}{1 + \tan^2 x} = \frac{\cancel{520x} (520x + 2510x)}{520^2 x} \\
 &= \frac{520x + 2510x}{\cancel{520x}} \\
 &= \cos x (\sec x + 2 \sin x) \\
 &= 1 + 2 \sin x \cos x \\
 &= 1 + \sin 2x
 \end{aligned}$$

9. Find the traits and sketch of $y = x^4 - 5x^3 - 15x^2 + 45x + 54$.

Domain: ALL REALS

Range: $y \in [-97.743, \infty)$

Y-Int: (0, 54)

End Behavior: LEFT UP
RIGHT UP

Zeros: $(-3, 0)$ $(-1, 0)$ $(6, 0)$

Extreme Values:

$y = -41.029, 80.166,$
 -97.743

