

Honors PreCalculus
 Dr. Quattrin
 Limits and Derivatives Test
 CALCULATOR ALLOWED

Name: SOLUTION KEY

Score _____

Round to 3 decimal places. Show all work.

1. If $\lim_{x \rightarrow a} \frac{x^4 - a^4}{x^2 - a^2} = 16$, then $a =$

$\lim_{x \rightarrow a} \frac{(x^2 - a^2)(x^2 + a^2)}{x^2 - a^2} = 2a^2 = 16$
 $a^2 = 8$
 $a = 2\sqrt{2}$

- a) 2 **b) $2\sqrt{2}$** c) 4 d) $4\sqrt{2}$ e) 8

2. An equation of the line normal to the graph of $y = 7x^4 + 2x^3 + x^2 + 2x + 5$ at the point where $x = 0$ is

$\frac{dy}{dx} = 28x^3 + 6x^2 + 2x + 2$ $m_{TAN} = 2$
 $m_{\perp} = -\frac{1}{2}$
 $x = 0 \rightarrow y = 5$
 $y - 5 = -\frac{1}{2}x$

- a) $x + 2y = 10$** b) $2x + y = 10$ c) $5x + 5y = 2$
 d) $2x - y = -5$ e) $2x + y = -10$

3. If $f(x) = \sqrt{x^2 - 1}$, which of the following is equal to $f'(3)$?

~~(a)~~ $\lim_{x \rightarrow 3} \frac{\sqrt{(x+h)^2 - 1} - \sqrt{8}}{x-3}$

~~(b)~~ $\lim_{h \rightarrow 0} \frac{\sqrt{(x+h)^2 - 1} - \sqrt{x^2 - 1}}{h}$

~~(c)~~ $\lim_{h \rightarrow 0} \frac{\sqrt{(x+h)^2 - 1} - \sqrt{8}}{h}$

(d) $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 1} - \sqrt{8}}{x-3}$

(e) $\lim_{h \rightarrow 0} \frac{\sqrt{x^2 - 1} - \sqrt{8}}{x-3}$

$$\frac{dy}{dx} = 4x + 1 = -3 \rightarrow x = -1 \rightarrow y = 4$$

$$m = -3$$

4. If the line tangent to $y = 2x^2 + x + k$ is the line $3x + y = 1$, then $k =$

- a) 1 b) 2 **c) 3** d) 4 e) 5

$$4 = 2(-1)^2 + (-1) + k$$

$$3 = k$$

5. At what point on the graph of $y = \frac{1}{4}x^4$ is the tangent parallel to the line

$8x + 27y = 3$ $m = -\frac{8}{27}$ $\frac{dy}{dx} = x^3 = -\frac{8}{27} \rightarrow x = -\frac{2}{3}$ $y = \frac{1}{4} \left(-\frac{2}{3} \right)^4$

- ~~(a)~~ $\left(\frac{8}{27}, \frac{2}{3} \right)$ ~~(b)~~ $\left(\frac{2}{3}, \frac{4}{81} \right)$ (c) $\left(-\frac{2}{3}, -\frac{8}{27} \right)$

- ~~(d)~~ $\left(-\frac{3}{2}, -\frac{1}{2} \right)$ **(e)** $\left(-\frac{2}{3}, \frac{4}{81} \right)$

6. A particle moving in the xy -plane with its x -coordinate given by

$x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + \frac{1}{2}t^2 - 1$ and its y -coordinate given by $y(t) = \frac{1}{2}t^2 - t + 1$. When

the particle is moving up it is also

- (a)** moving right
 (b) moving left
 (c) at rest
 (d) cannot be determined
 (e) does not exist

$$v_x = t^3 + 2t^2 + t = t(t+1)^2$$

$$v_y = t - 1$$

MOVING UP $\Rightarrow t > 1$

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1. Use the equation of the line tangent to $y = 6x^3 - 3x^2 + 5x - 4$ at $x = 1$ to approximate $f(.9)$

$$y(1) = 4 \quad m = 18x^2 - 6x + 5$$

$$y - 4 = 17(x - 1) \quad m|_{x=1} = 17$$

$$\begin{aligned} f(.9) &\approx y(.9) = 17(.9 - 1) + 4 \\ &= -1.7 + 4 \\ &= 2.3 \end{aligned}$$

2. The motion of a particle is described by $x(t) = t^3 - 6t^2 + 9t + 1$.

a) When the particle is stopped?

b) Which direction it is moving at $t = 3$?

c) Where is it when $t = 3$?

d) Find $a(3)$.

$$v = 3t^2 - 12t + 9$$

$$a = 6t - 12$$

$$\begin{aligned} \text{a) } v = 0 &\rightarrow t^2 - 4t + 3 = 0 \\ &(t - 3)(t - 1) = 0 \quad t = 1, 3 \end{aligned}$$

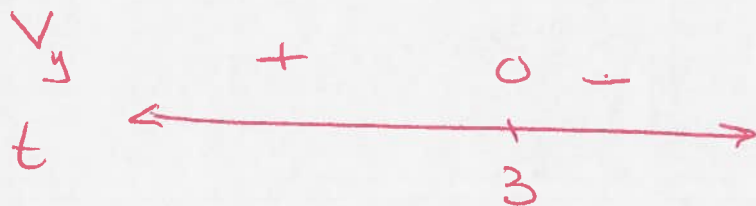
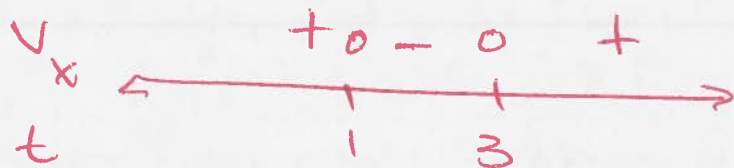
$$\text{b) } v(3) = 0 \therefore \text{NEITHER}$$

$$\text{c) } x(3) = 3^3 - 6(3)^2 + 9(3) + 1 = 1$$

$$\text{d) } a(3) = 6$$

3. A particle's position $\langle x(t), y(t) \rangle$ at time t is described by $\langle t^3 - 6t^2 + 9t + 1, -t^2 + 6t + 2 \rangle$. When is the particle moving right and down?

$$v \left\langle \frac{3t^2 - 12t + 9}{3(t^2 - 4t + 3)}, -2t + 6 \right\rangle$$



$$t \geq 3 \quad \text{or} \quad t \in [3, \infty)$$

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3. Set up, but do not solve, the limit definition of the derivative for
 $y = 6x^4 - 2x^3 + \pi^2 + 4x$

$$\lim_{h \rightarrow 0} \frac{[6(x+h)^4 - 2(x+h)^3 + \pi^2 + 4(x+h)] - [6x^4 - 2x^3 + \pi^2 + 4x]}{h}$$

4. Use the Power Rule to find:

a) $\frac{d}{dx} [4x^5 - x^3 + \pi x + 421]$

$$20x^4 - 3x^2 + \pi$$

b) $D_x \left[\sqrt[4]{x^7} - \frac{6}{x^5} - \sqrt[3]{x} + \pi^2 x \right]$

$$\frac{7}{4} x^{3/4} + 30x^{-6} - \frac{1}{3} x^{-2/3} + \pi^2$$

5. Evaluate the following limits:

(a) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 + 3x - 4}$

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+4)} \\ = \frac{3}{5} \end{aligned}$$

(b) $\lim_{x \rightarrow 9} \frac{x^3 - 9x^2 - 5x + 45}{x^2 - 11x + 18}$

$$\begin{aligned} \lim_{x \rightarrow 9} \frac{x^2(x-9) - 5(x-9)}{(x-9)(x-2)} \\ = \frac{81 - 5}{9 - 2} = \frac{76}{7} \end{aligned}$$