Honors PreCalculus
Dr. Quattrin
Limits and Derivatives Test
CALCULATOR ALLOWED
Round to 3 decimal places. Show all work.

Name: Solution Key

Score ___________

1. If $\lim\limits_{x\to a}\frac{x^4-a^4}{x^2-a^2}=16$, then $a=$

$\lim\limits_{x\to a}\frac{(x^2-a^2)(x^2+a^2)}{x^2-a^2}=2a^2=16$

$a^2=8$

$a=2\sqrt{2}$

a) 2 b) $2\sqrt{2}$ c) 4 d) $4\sqrt{2}$ e) 8

2. An equation of the line normal to the graph of $y=7x^4+2x^3+x^2+2x+5$ at the point where $x=0$ is

$\frac{dy}{dx}=28x^3+6x^2+2x+2$ $m_{TAN}=2$

$m_{NORM}=$

$x=0\Rightarrow y=5$

$y-5=\frac{-1}{2}x$

a) $x+2y=10$ b) $2x+y=10$ c) $5x+5y=2$

d) $2x-y=-5$ e) $2x+y=-10$

3. If $f(x)=\sqrt{x^2-1}$, which of the following is equal to $f'(3)$?

(a) $\lim\limits_{x\to3}\frac{\sqrt{(x+h)^2-1}-\sqrt{8}}{x-3}$

(b) $\lim\limits_{h\to0}\frac{\sqrt{(x+h)^2-1}-\sqrt{x^2-1}}{h}$

(c) $\lim\limits_{h\to0}\frac{\sqrt{(x+h)^2-1}-\sqrt{8}}{h}$

(d) $\lim\limits_{x\to3}\frac{\sqrt{x^2-1}-\sqrt{8}}{x-3}$

(e) $\lim\limits_{h\to0}\frac{\sqrt{x^2-1}-\sqrt{8}}{x-3}$

$\frac{dy}{dx} = 4x+1 = -3 \Rightarrow x = -1 \rightarrow y = 4$

$m = -3$

4. If the line tangent to $y = 2x^2 + x + k$ is the line $3x + y = 1$, then $k =$

a) 1 b) 2 c) 3 d) 4 e) 5

$4 = 2(-1)^2 + (-1) + k$

$3 = k$

5. At what point on the graph of $y = \frac{1}{4}x^4$ is the tangent parallel to the line $8x + 27y = 3$ $m = \frac{-8}{27}$ $\frac{dy}{dx} = x^3 = \frac{-8}{27} \Rightarrow x = \frac{-2}{3}$ $y = \frac{1}{4}\left(\frac{-2}{3}\right)^4$

~~(a)~~ $\left(\frac{8}{27}, \frac{2}{3}\right)$ ~~(b)~~ $\left(\frac{2}{3}, \frac{4}{81}\right)$ (c) $\left(-\frac{2}{3}, -\frac{8}{27}\right)$

~~(d)~~ $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (e) $\left(-\frac{2}{3}, \frac{4}{81}\right)$

6. A particle moving in the xy-plane with its x-coordinate given by $x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + \frac{1}{2}t^2 - 1$ and its y-coordinate given by $y(t) = \frac{1}{2}t^2 - t + 1$. When the particle is moving up it is also

(a) moving right
(b) moving left
(c) at rest
(d) cannot be determined
(e) does not exist

$V_x = t^3 + 2t^2 + t = t(t+1)^2$

$V_y = t - 1$

MOVING UP $\Rightarrow t > 1$

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1. Use the equation of the line tangent to $y = 6x^3 - 3x^2 + 5x - 4$ at $x = 1$ to approximate $f(.9)$

$y(1) = 4$ $\quad m = 18x^2 - 6x + 5$

$y - 4 = 17(x - 1)$ $\quad m|_{x=1} = 17$

$f(.9) \approx y(.9) = 17(.9 - 1) + 4$

$= -1.7 + 4$

$= 2.3$

2. The motion of a particle is described by $x(t) = t^3 - 6t^2 + 9t + 1$.

a) When the particle is stopped?
b) Which direction it is moving at $t = 3$?
c) Where is it when $t = 3$?
d) Find $a(3)$.

$v = 3t^2 - 12t + 9$

$a = 6t - 12$

a) $v = 0 \Rightarrow t^2 - 4x + 3 = 0$
$(t - 3)(t - 1) = 0$ $\quad t = 1, 3$

b) $v(3) = 0 \therefore$ NEITHER

c) $x(3) = 3^3 - 6(3)^2 + 9(3) + 1 = 1$

d) $a(3) = 6$

3. A particle's position $\langle x(t), y(t)\rangle$ at time t is described by $\langle t^3-6t^2+9t+1, -t^2+6t+2\rangle$. When is the particle moving right and down?

$v \ \langle 3t^2-12t+9,\ -2t+6\rangle$

$3(t^2-4t+3)$

v_x + 0 − 0 +

t 1 3

v_y + 0 −

t 3

$t \geq 3$ or $t \in [3, \infty)$

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3. Set up, but do not solve, the limit definition of the derivative for
$y = 6x^4 - 2x^3 + \pi^2 + 4x$

$$\lim_{h \to 0} \frac{[6(x+h)^4 - 2(x+h)^3 + \pi^2 + 4(x+h)] - [6x^4 - 2x^3 + \pi^2 + 4x]}{h}$$

4. Use the Power Rule to find:

a) $\frac{d}{dx}\left[4x^5 - x^3 + \pi x + 421\right]$

$20x^4 - 3x^2 + \pi$

b) $D_x\left[\sqrt[4]{x^7} - \frac{6}{x^5} - \sqrt[3]{x} + \pi^2 x\right]$

$\frac{7}{4}x^{3/4} + 30x^{-6} - \frac{1}{3}x^{-2/3} + \pi^2$

5. Evaluate the following limits:

(a) $\lim_{x\to 1} \frac{x^3-1}{x^2+3x-4}$

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+4)}$$

$$= \frac{3}{5}$$

(b) $\lim_{x\to 9} \frac{x^3-9x^2-5x+45}{x^2-11x+18}$

$$\lim_{x\to 9} \frac{x^2\cancel{(x-9)}-5\cancel{(x-9)}}{\cancel{(x-9)}(x-2)}$$

$$= \frac{81-5}{9-2} = \frac{76}{7}$$