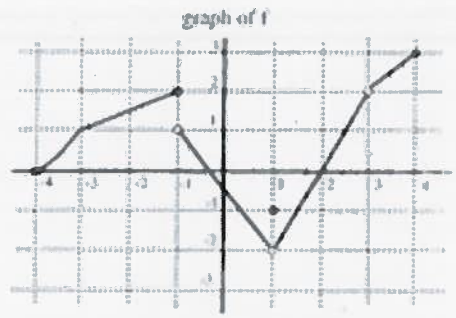
Honors PreCalculus '15-16
Piece Wise Defined Functions
Dr. Quattrin
Calculator allowed

Name: Solution Key

(-4, 4)

1. The function f is defined on the interval $x \in \cancel{[-5, 5]}$ and has the graph shown below.



For which of the following values is f not differentiable?

(a) -1 only (b) -1 and 1 only (c) -1, 1, and 3 only

(d) -4 only (e) -4, -1, 1, and 3

is okay

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2. The end behavior of $g(x)=\sqrt{\dfrac{x^2-4x}{x^2+9}}$

(a) Up on both ends
(b) $y=1$ on the left and none on the right
(c) None on the left and $y=1$ on the right
(d) None on the left and up on the right
(e) $y=1$ on both ends

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3. Let m and b be real numbers and let the function f be defined by

$$f(x)=\begin{cases} 3x^2+mx+5 & \text{for } x\le 1 \\ mx+b & \text{for } x>1 \end{cases}$$

$m+8=m+b$

$8=b$

If f is both continuous and differentiable at $x=1$, then

(a) $m=5,\ b=8$

(b) $m=5,\ b=-8$

(c) $m=-5,\ b=8$

(d) $m=-8,\ b=-5$

(e) None of these

$f'(x)=\begin{cases} 6x+m \\ m \end{cases}$

$x=1 \rightarrow 6+m=m$

$b=0$

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4. Let $f(x)=\begin{cases} -x+5, & \text{if } x\le -2 \\ x^2+1, & \text{if } -2\le x\le 1 \\ 2x^3, & \text{if } 1<x \end{cases}$. Which of the following statements is true about f?

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$1^2+1=2(1)^3$

I. f is continuous at $x=1$.

~~II.~~ f is differentiable at $x=1$.

~~III.~~ f has a local minimum at $x=-2$.

$f'=\begin{cases} -1 \\ 2x \\ 6x^2 \end{cases}$

(a) I only (b) II only (c) III only

(d) I and III only (e) II and III only

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5. If the derivative of the function f is $f'(x) = -3(x+2)(x+1)^2(x-3)^3$, then f has a local maximum at $x =$

f' $-$ 0 $+$ 0 $+$ 0 $-$

-2 -1 3

(a) -2 only (b) -1 only (c) 3 only (d) -2 and 3 (e) -1 and 3

$5(-2)^2+10 = -2A+10$

$A = -10 \to 30$

6. Let f be defined by $f(x) = \begin{cases} 5x^2+10, & \text{if } x < -2 \\ 30, & \text{if } x = -2 \\ Ax+10, & \text{if } -2 < x \end{cases}$. Determine the value of A for which is continuous for all real x.

(a) -6 (b) -2 (c) -1 (d) -10 (e) None of these

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7. The function f is continuos at the point $(c, f(c))$. Which of the following statements could be false?

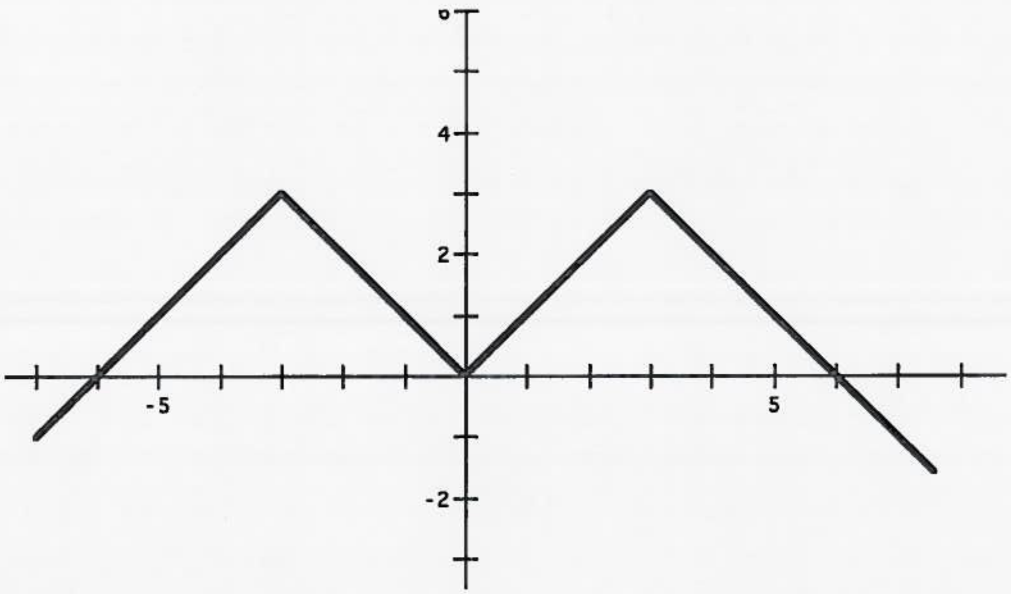
(a) $\lim_{x \to c} f(x)$ exists T

(b) $\lim_{x \to c} f(x) = f(c)$ T

(c) $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$ T

(d) $f'(c)$ exists

(e) None of these



8. The graph of the even function f (shown above) consists of four line segments. Which of the following statements about f is false?

T (a) $\lim_{x\to 0} f(x)-f(0)=0$ (b) $\lim_{x\to 0}\frac{f(x)-f(0)}{x}=0$ $= m|_x = 0$ (c) $\lim_{x\to 0}\frac{f(x)-f(-x)}{2x}=\text{dne}$

(d) $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=1$ (e) $\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}=\text{dne}$

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1. $$h(x)=\begin{cases} \dfrac{x^2-4}{x^2-9}, & if \quad x\le -2 \\ \sqrt{4-x^2}, & if \quad -2<x<2 \\ 3, & if \quad x=2 \\ x, & if \quad 2<x\le 4 \end{cases}$$

i) Is $h(x)$ continuous at $x=-2$? Why or why not?

i) $h(-2)$ EXISTS

ii) $\lim\limits_{x\to -2^-} h(x) = \lim\limits_{x\to -2} \dfrac{x^2-4}{x^2-9} = 0$

$\lim\limits_{x\to -2^+} h(x) = \lim\limits_{x\to -2} \sqrt{4-x^2} = 0$ $\therefore \lim\limits_{x\to -2} h(x)$ EXISTS

iii) $\lim\limits_{x\to -2} h(x) = 0 = h(-2)$

CONTINUOUS

ii) Is it differentiable at $x=-2$? Why or why not?

$$h' @ x=-2 = \begin{cases} \dfrac{(x^2-9)(2x)-(x^2-4)(2x)}{(x^2-9)^2} = \dfrac{10x}{(x^2-9)^2} \\ \frac{1}{2}(4-x^2)^{-1/2}(-2x) = \dfrac{-x}{(4-x^2)^{1/2}} \end{cases}$$

i) CONTINUOUS

ii) $\lim\limits_{x\to -2^+} h'(x)$ DNE NOT DIFFERENTIABLE ~~CONTINUOUS~~

2. $h(x)=\begin{cases} \dfrac{x^2-4}{x^2-9}, & if \quad x \le -2 \\ \sqrt{4-x^2}, & if \quad -2<x<2 \\ 3, & if \quad x=2 \\ x, & if \quad 2<x\le 4 \end{cases}$

i) Is $h(x)$ continuous at $x=2$? Why or why not?

i) $h(2)$ EXISTS

ii) $\lim\limits_{x\to 2^-} h(x) = \lim\limits_{x\to 2} \sqrt{4-x^2} = 0$

$\lim\limits_{x\to 2^+} h(x) = \lim\limits_{x\to 2} x = 2$

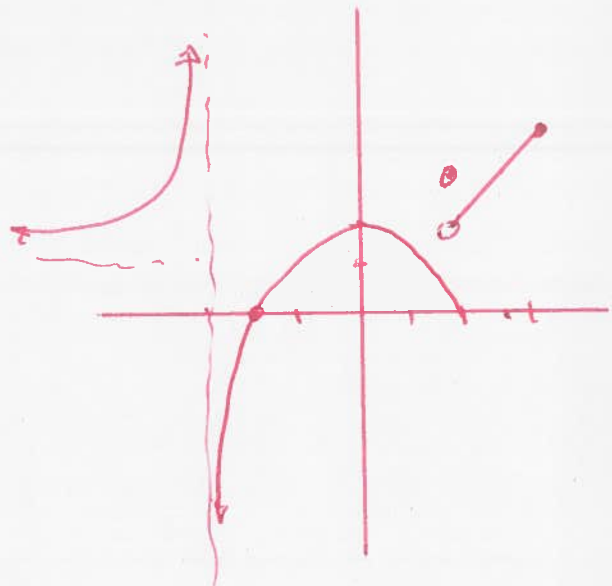
$\lim\limits_{x\to 2} h(x)$ DNE

$\therefore$ DISCONTINUOUS

ii) Is it differentiable at $x=2$? Why or why not?

NOT DIFFERENTIABLE BECAUSE NOT CONTINUOUS

3. Sketch $h(x)=\begin{cases} \dfrac{x^2-4}{x^2-9}, & if \quad x\le -2 \\ \sqrt{4-x^2}, & if \quad -2<x<2 \\ 3, & if \quad x=2 \\ x, & if \quad 2<x\le 4 \end{cases}$ and state the Traits listed. Provide proof for the extreme points.



Domain: $x \in (-\infty, -3) \cup (-3, 4]$

Range: $y \in$ ALL REALS

Zeros: $(-2, 0)$

Y-int: $(0, 2)$

VAs: $x = -3$

EB (Left): $y = 1$

EB (Right): NONE

Continuity: EVERYWHERE EXCEPT $x=-3$ & $x=2$

Differentiability: EVERYWHERE EXCEPT $x=-3, 2, 4$

Extreme Points: $(4,4)$ $(2,3)$

i) NONE

ii) $\frac{dy}{dx}$ DNE : $x=2 \rightarrow (2,3)$

iii) ENDPOINT: $x=4 \rightarrow (4,4)$