

PreCalculus Honors

Name: SOLUTION KEY

Dr. Quattrin

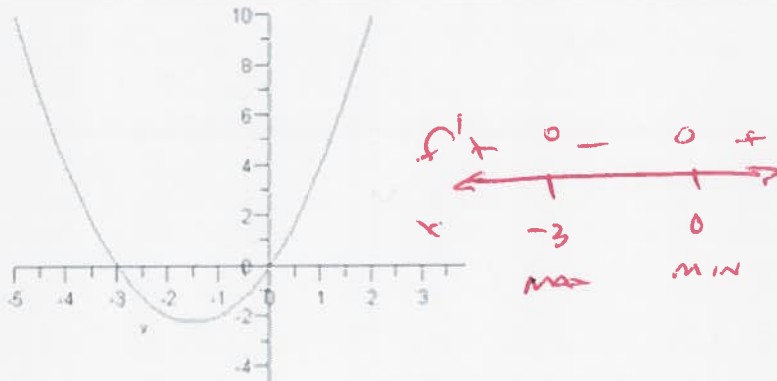
Polynomials Test

CALCULATOR ALLOWED

Score _____

Round to 3 decimal places. Show all work.

1. The derivative of f is graphed below. Give the value of x where f has a relative minimum point.



- a. -3 b. $-\frac{3}{2}$ **c. 0** d. 1 e. No such value

2. A farmer with 890 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

- a. 19,825.5 ft²** b. 19,825.5 ft² c. 19,801.5 ft²
d. 19,902.5 ft² e. 19,791.5 ft²

3. Find the point on the graph of $y = \sqrt[3]{x^2}$ between $(1, 1)$ and $(9, 4)$ at which the tangent to the graph has the same slope as the line through $(1, 1)$ and $(9, 4)$

- a. $(1, 1)$ b. $(2, \sqrt[3]{4})$ c. $(\frac{3}{4}, 2)$ d. $(\frac{64}{27}, \frac{16}{9})$ **e. None of the above**

$$m = \frac{4-1}{9-1} = \frac{3}{8}$$
$$\frac{dy}{dx} = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}} = \frac{3}{8}$$
$$x^{1/3} = \frac{9}{16}$$

4. Consider the function $f(x) = \frac{x^4}{2} - \frac{x^5}{10}$. $f'(x)$, the derivative of f , attains its maximum value at $x =$

$$f' = 2x^3 - \frac{1}{2}x^4$$

$$f'' = 6x^2 - 2x^3 = 2x^2(3-x)$$

$$2x^2(x-3) = 0$$

- a) 0 b) 3 c) 4 d) 5 **e) Never**

$$f'' \begin{matrix} + & 0 & + & 0 & + \\ \leftarrow & & \rightarrow & & \\ x & & 0 & & 3 \end{matrix} \text{ min}$$

$$x = 0, 1$$

5. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $x \in [-1, 2]$?

- a) -7 **b) $-\frac{3}{4}$** c) 0 d) 2 e) 6

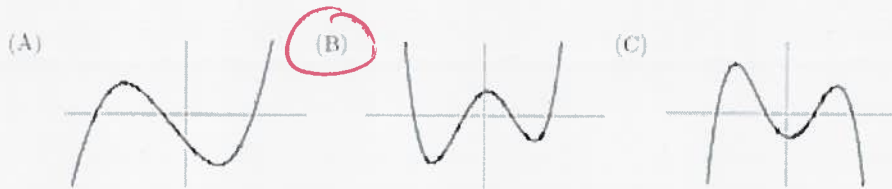
$$g(\frac{1}{2}) =$$

$$g' = 12x^2 + 6x - 6 = 0 \Rightarrow 2x^2 + x - 1 = 0 = (2x-1)(x+1)$$

$$f' \begin{matrix} + & 0 & - & 0 & + \\ \leftarrow & & \rightarrow & & \\ x & & -1 & & \frac{1}{2} \end{matrix} \text{ min}$$

6. If $f'(x) = (x-a)(x-b)(x-c)$ and $a < b < c$, then which of the following could be the graph of $f(x)$?

f' is cubic so f is quartic



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Show all work.

1. Find the zeros and extreme points of $y = -3x^3 + 2x^2 + 147x - 98$. Show the algebraic work to support the zeros and critical values.

a)

$$\begin{aligned} y &= \cancel{-3x^2(x-2)} + \cancel{49(x-2)} \\ &= -x^2(3x-2) + 49(3x-2) \\ &= (49-x^2)(3x-2) \\ \text{Zeros } &(\pm 7, 0) \quad (2/3, 0) \end{aligned}$$

b)

$$\frac{dy}{dx} = -9x^2 + 4x + 147 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4(-9)(147)}}{2(-9)} = \begin{cases} -3.825 \\ 4.270 \end{cases}$$

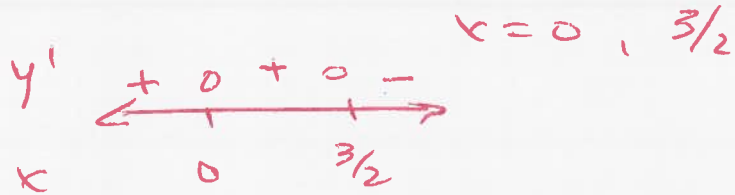
$$(-3.825, -463.127)$$

$$(4.270, 332.592)$$

2. Find the zeros and extreme points of $y = 2x^3 - x^4$ on $x \in [-1, 3]$. Show the derivative and algebra to support the critical values.

Zeros $y = x^3(2-x)$ $(0,0)$ $(2,0)$

EXT: $\frac{dy}{dx} = 6x^2 - 4x^3 = 2x^2(3-2x) = 0$



$x=0$ IS NOT AN EXTREME

$(3/2, 1.688)$

$[-1, 3]$ $(3, -27)$

3. A farmer with 1400 feet of fencing is wants to enclose a rectangular area and then divide it into five pens with fencing parallel to the **horizontal** side of the rectangle. What is the largest possible total area of the five pens?

$A = xy$

$F = 6x + 2y = 1400$
 $y = 700 - 3x$

$= x(700 - 3x)$
 $= 700x - 3x^2$

$A = 4083\frac{1}{3}$

$A' = 700 - 6x$
 $x = \frac{700}{6} = 116\frac{2}{3}$

4a. Find the zeros, algebraically, of $y = x^4 - 3x^3 - 16x^2 + 48x$.

$$\begin{aligned} & \cancel{x^3(x-3)} \\ & x^3(x-3) - 16x(x-3) \\ & (x^3 - 16x)(x-3) \\ & x(x^2 - 16)(x-3) \\ & \Rightarrow (0, 0) (\pm 4, 0) (3, 0) \end{aligned}$$

4b. Find the extreme points, graphically, of $y = x^4 - 3x^3 - 16x^2 + 48x$. Show the derivative before using your calculator.

$$\frac{dy}{dx} = 4x^3 - 9x^2 - 32x + 48 = 0$$

$$x = -2.601, 1.299, 3.551$$

$$(-2.601, -134.534)$$

$$(1.299, 31.625)$$

$$(3.551, -66.34)$$

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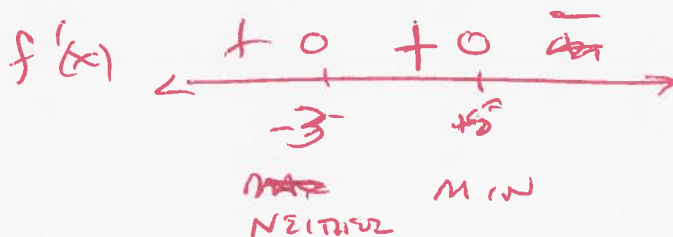
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Polynomials Test—CALCULATOR NOT ALLOWED

Show all work.

Score _____

5. Make a sign pattern for the function $f'(x)$ if f is increasing from $-\infty$ to -3 , increasing from -3 to 5 , and decreasing from 5 to ∞ . Be sure to label the sign pattern appropriately. Then, determine whether each critical value represents a max, a min, or neither. Explain how you know for each.



6. Find the traits and sketch $y = 2x^3 - x^4$ on $x \in [-1, 3]$.

Domain: $x \in [-1, 3]$

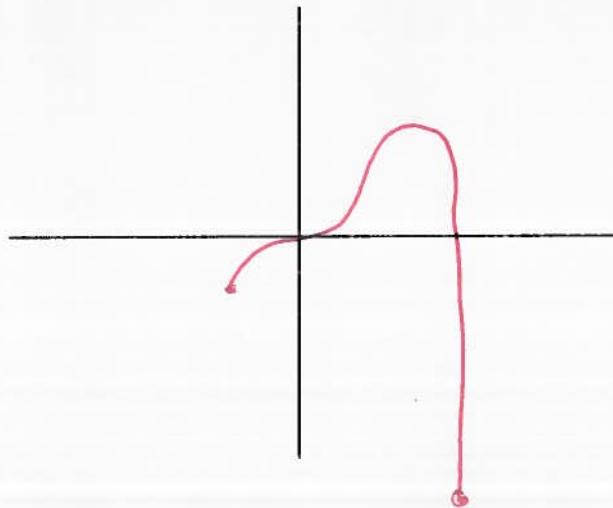
Range: $[-27, 1.688]$

Y-Int: $(0, 0)$

End Behavior: NONE

Zeros: $(0, 0)$ $(2, 0)$

Extreme Values: $-27, -3, 1.688$



7. Find the traits and sketch of $y = -3x^3 + 2x^2 + 147x - 98$.

Domain: All Reals

Range: All Reals

Y-Int: $(0, -98)$

End Behavior: LEFT UP; RIGHT DOWN

Zeros: $(\pm 7, 0)$ $(\frac{2}{3}, 0)$

Extreme Values:

$$y = -463.127, 332.592$$

