PreCalculus Honors '15-16 Name: Solution Key
Spring Midterm Part I
Dr. Quattrin Score ____________
CALCULATOR ALLOWED
Round to 3 decimal places. Show all work.

1. Let f be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line to the graph of f at $x=3$ is used to find an approximation of a zero of f, that approximation is

0.4
0.5
2.6
3.4
5.5

$y - 2 = 5(x-3)$

$\cancel{f(0) = 5(0-3)}$ $y = 5x - 13 = 0$

$x = 13/5$

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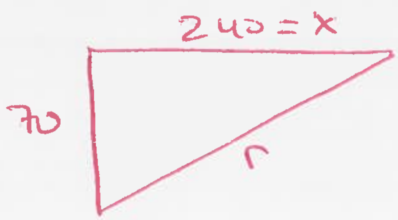
2. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

57.60
57.88
59.20
60.00
67.40



$x^2 + 70^2 = r^2$

$\cancel{2}x\frac{dx}{dt} = \cancel{2}r\frac{dr}{dt}$

$(240)(60) = 250\left(\frac{dr}{dt}\right)$

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3. An equation of the tangent line to curve $x^2+y^2=169$ at the point (5, -12) is

a) $5y-12x=-120$

b) $5x-12y=119$

c) $5x-12y=169$

d) $12x+5y=0$

e) $12x+5y=169$

$2x+2y\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-x}{y}$ $\quad m=\frac{5}{12}$

$5x-12y=25+144$

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4. Let m and b be real numbers and let $f(x)=\begin{cases}1+3bx+2x^2 & \text{if } x\le 1\\ mx+b & \text{if } 1<x\end{cases}$. If is continuous and differentiable at $x=1$, then

a) $m=1,\ b=1$
b) $m=1,\ b=-1$
c) $m=-1,\ b=1$
d) $m=-1,\ b=-1$
e) None of these

$x=1 \to 3b+3=m+b$

$2b+3=m$

$f'=\begin{cases}3b+4x\\ m\end{cases}$

$3b+4=m$

$3b+4=2b+3$

$b=-1$

$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$

5. Let $f(x)=\begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x=2 \end{cases}$. Which of the following statements about f are true?

I. f has a limit at $x=2$.

~~II.~~ f is continuous at $x=2$.

~~III.~~ f is differentiable at $x=2$.

a) I only b) II only c) III only

d) I and II only e) I, II, and III

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6. If $\frac{d}{dx}[f(x)]=g(x)$ and if $h(x)=x^2$, then $\frac{d}{dx}[f(h(x))]=$

$f'(h(x)) \cdot h'(x)$

$g'(h(x)) \cdot 2x$

$2x\, g'(x^2)$

a) $g(x^2)$
b) $2xg(x)$
c) $g'(x)$
d) $2xg(x^2)$
e) $x^2g(x^2)$

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7. Find the domain of the following function $f(x)=\dfrac{\sqrt{x+5}}{x^2-x-72}$.

$(x-9)(x+8)$

$x \geq -5$

$x \neq -8, 9$

a) $x \in (-\infty, -8) \cup (-8, -5) \cup (-5, 9) \cup (9, \infty)$ $g(x^2)$
b) $x \in [-5, 9) \cup (9, \infty)$
c) $x \in (0, \infty)$
d) $x \in (-\infty, -8) \cup [-5, 9) \cup (9, \infty)$
e) $x \in (-\infty, \infty)$

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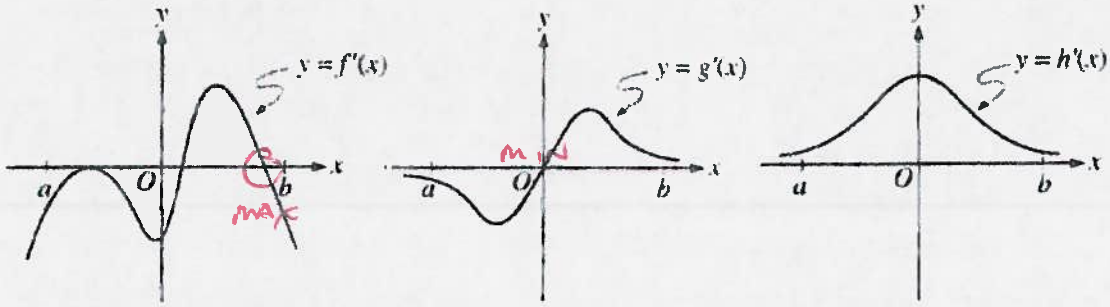
8. The graphs of the **derivatives** of the functions f, g, and h are shown below. Which of the functions f, g, or h have a relative maximum on the open interval ?



a) f only   b) g only   c) h only

d) f and g only   e) f, g, and h

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9. If g is a differentiable function such that $g(x)<0$ for all real numbers x and if $f'(x)=(x^2-4)g(x)$, which of the following is true?

f' − 0 + 0 −

−2   2

a) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
b) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
c) f has a relative minima at $x = -2$ and $x = 2$.
d) f has a relative maxima at $x = -2$ and $x = 2$.
e) It cannot be determined if f has any relative extremes.

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10. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

Given that $h(x)=f(x)\cdot g(x)$, $h'(8)=$ $f(8)\cdot g'(8)+g(8)\cdot f'(8)$

$6\cdot 4 \quad + \quad 2(-12)$

a) 0 b) 62 c) 12 d) -12 e) -48

PreCalculus Honors '15-16
Spring Midterm Part II
Dr. Quattrin
CALCULATOR ALLOWED
Round to 3 decimal places. Show all work.

Name:________________________

Score __________

1. $h(x)=\begin{cases} \dfrac{4-x^2}{4x^2-25}, & \text{if } -4\le x<-2 \\ 3, & \text{if } x=-2 \\ -\dfrac{1}{2}x+1, & \text{if } -2<x\le 2 \\ \sqrt{\dfrac{x-2}{4+x^2}}, & \text{if } 2<x \end{cases}$

i) Is $h(x)$ continuous at $x=2$? Why or why not?

i) $h(2)$ EXISTS

ii) $\lim_{x\to 2} h(x)$ ~~DNE~~ EXISTS

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}\left(-\frac{1}{2}x+1\right) = 0$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} \sqrt{\frac{x-2}{4+x^2}} = 0$

iii) $\lim_{x\to 2} h(x) = f(2)$ $\therefore$ CONTINUOUS

ii) Is it differentiable at $x=2$? Why or why not?

i) CONTINUOUS

ii) $\lim_{x\to 2^-} -\frac{1}{2} = -\frac{1}{2}$ EXISTS $\qquad \lim_{x\to 2^+} \frac{1}{2}\left(\frac{x-2}{4+x^2}\right)^{-1/2}\left(\frac{(4+x^2)(1)-(x-2)(2}{(x+4)^2}\right.$

DNE

$\therefore$ NOT DIFFERENTIABLE

2. **Find the Domain and Extreme points of** $g(x)=\sqrt{\dfrac{x^2-4x+4}{2x^3-5x^2+x+2}}$.

$$\begin{array}{r|rrrr} 2 & 2 & -5 & 1 & 2 \\ & & 4 & -2 & -2 \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

$$= \sqrt{\frac{(x-2)(x-2)}{(x-2)\,(2x^2-x-1)}}$$

$$(2x+1)(x-1)$$

a) $g'(x)$ $-$ VA $+$ VA $-$ POE $+$

x $-1/2$ 1 2

DOMAIN: $x \in \left(-\frac{1}{2}, 1\right) \cup (2, \infty)$

b) $$g(x) \approx \left(\frac{x-2}{2x^2-x-1}\right)^{1/2}$$

$$(4x^2-9x+2)$$

$$g'(x) = \frac{1}{2}\left(\frac{x-2}{2x^2-x-1}\right)^{-1/2}\left[\frac{(2x^2-x-1)(1)-(x-2)(4x-1)}{(2x^2-x-1)^2}\right]$$

$$= \frac{-2x^2+8x-3}{2(x-2)^{1/2}(2x^2\mp x-1)^{-3/2}}$$

i) $\frac{dy}{dx}=0 \rightarrow x = \frac{-8\pm\sqrt{64-24}}{2(-2)} = \begin{cases} .419 \\ 3.581 \end{cases}$

ii) $\frac{dy}{dx}$ DNE $\rightarrow x = \cancel{2}, -1/2, \cancel{1}$

iii) NO ENDPOINTS

$(.419, 1.217)$

$(3.581, .274)$

PreCalculus Honors '15-16
Spring Midterm Part III
Dr. Quattrin
NO CALCULATOR ALLOWED

Name: Solution Key

Score ________

1. Sketch $h(x)=\begin{cases} \dfrac{4-x^2}{4x^2-25}, & \text{if } -4\le x<-2 \\ 3, & \text{if } x=-2 \\ -\dfrac{1}{2}x+1, & \text{if } -2<x\le 2 \\ \sqrt{\dfrac{x-2}{4+x^2}}, & \text{if } 2<x \end{cases}$ and state the Traits listed.

Provide proof for the extreme points.

1 Domain: $x \in [-4, -\frac{5}{2}) \cup (-\frac{5}{2}, \infty)$

1 Range: $y \in (-\infty, -\frac{4}{13}] \cup [0, \infty)$

1 Zeros: $(2, 0)$

1 Y-int: $(0, 1)$

1 VAs: $x = -5/2$

1 EB (Left): NONE

1 EB (Right): $y = 0$

2 Discontinuities: $x = -5/2, -2$

3 x-values of non-differentiability: $x = -\frac{5}{2}, -2, 2$

3 Extreme Points: $(-2, 3), (2, 0), (-4, -\frac{4}{13})$

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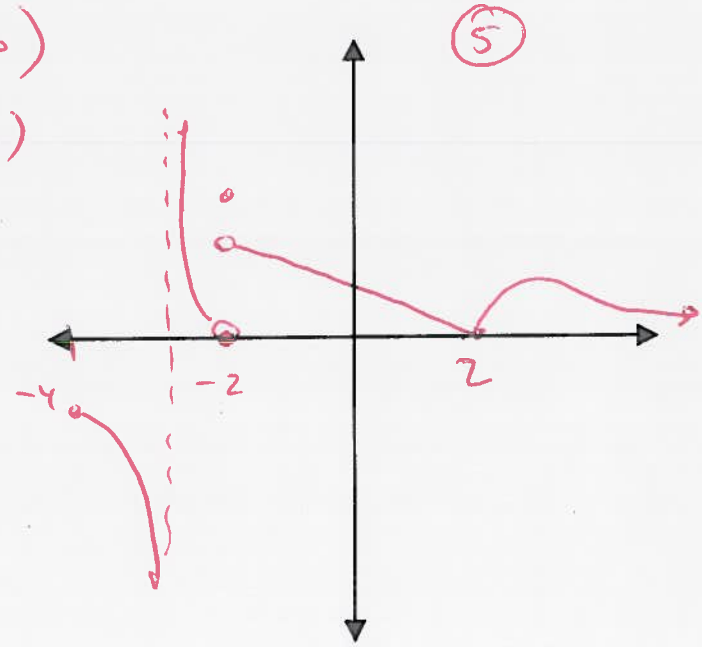
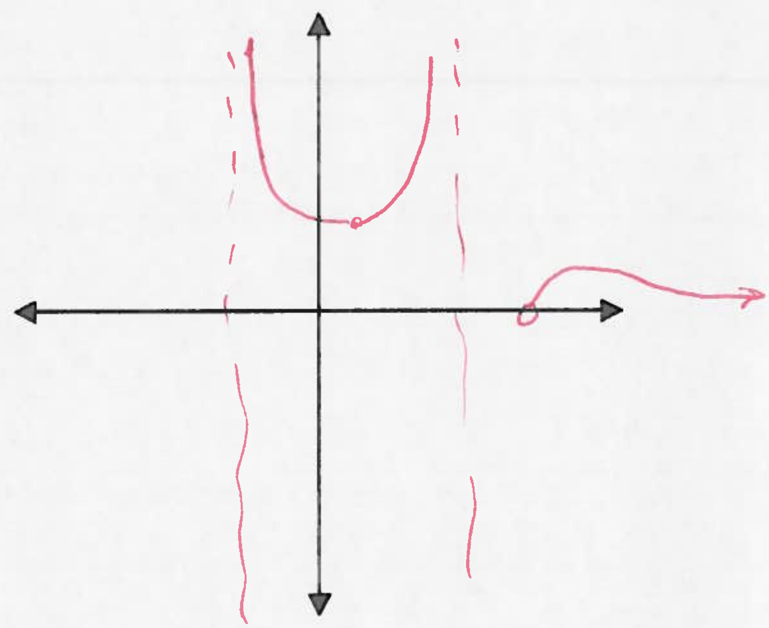


2. List all traits and sketch $g(x)=\sqrt{\dfrac{x^2-4x+4}{2x^3-5x^2+x+2}}$.

Domain: $x \in (-\frac{1}{2}, 1) \cup (2, \infty)$



Zeros: NONE

y-Int: $0, \sqrt{2}$

VAs: $x = -\frac{1}{2}, 1$

POEs: $(2, 0)$

EB: LEFT: NONE
RIGHT: $y=0$

Extreme Points: SEE #2

Range: $y \in (0, .274) \cup (1.217, \infty)$