

Honors Precalculus '16  
 Fall Midterm  
 Part I--CALCULATOR ALLOWED

Name SOLUTION KEY

1. Which of the following is equivalent to  $\sin 2A - 2\sin A$ ?

a.  $(\sin A)(\sin A - 2)$

b.  $(2\sin A)(\sin A - 1)$

c.  $(2\sin A)(\sin A - 2)$

d.  $(2\sin A)(\cos A - 1)$

e.  $(\sin A)(\cos A - 2)$

$2\sin A \cos A - 2\sin A$   
 $2\sin A (\cos A - 1)$

2. The magnitude of  $\vec{v} = 7\vec{i} - \sqrt{6}\vec{j}$  is

a.  $\sqrt{55}$

b. 1

c. 13

d. 85

e.  $\sqrt{85}$

$\sqrt{7^2 + 6} = \sqrt{55}$

3. If  $\sec \theta = \frac{9}{2}$  and  $\sin \theta < 0$ , then  $\tan \theta = \frac{y}{x}$ .

a.  $-\sqrt{77}$

b.  $\frac{\sqrt{77}}{2}$

c.  $\frac{\sqrt{77}}{9}$

d.

$-\frac{\sqrt{77}}{2}$

e.  $-\frac{\sqrt{77}}{9}$

$r = 9$   
 $x = 2$   
 $y = \sqrt{9^2 - 2^2} = -\sqrt{77}$

4. What is  $\theta$  if  $\csc \theta = -2.353$  and  $\pi \leq \theta \leq \frac{3\pi}{2}$ ?

a. -0.439

b. 5.844

c. 3.581

d. -25.150

e. There is no such angle

$\sin^{-1}(1/-2.353) = \{-.436 \pm 2\pi\}$   
 $\{3.581 \pm 2\pi\}$

5. Find the equation of the following

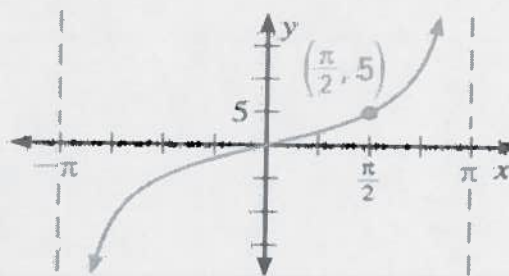
~~a.~~  $y = \frac{1}{2} \tan 5x$

**b.**  $y = 5 \tan \frac{1}{2} x$

~~c.~~  $y = \tan 5x$

~~d.~~  $y = 5 \tan x$

~~e.~~  $y = 5 \tan \frac{1}{8} x$



$$2\pi = P = \frac{\pi}{B} \Rightarrow \frac{1}{2} = B$$

6. For which of the following values in QI does  $(\sin x + \cos x)^2 = 1$ .

**a.** 0

b.  $\frac{\pi}{6}$

c.  $\frac{\pi}{4}$

d.  $\frac{\pi}{3}$

e. For all  $x$  where  $0 \leq x \leq \frac{\pi}{2}$

$$\sin^2 x + 2\sin x \cos x + \cos^2 x = 1$$

$$2\sin x \cos x = 0$$

$$\sin 2x = 0$$

$$2x = 0 \pm \pi n$$

$$x = 0 \pm \frac{\pi}{2} n$$

7. If  $\tan A = \frac{1}{2}$  and  $\tan B = 1$  then  $\tan(A+B) =$

a. 14.101

b. 0.026

**c.** 3.000

d. -0.333

e. 1.500

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{2} + 1}{1 - \frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$$

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 Part II--CALCULATOR ALLOWED  
 Round answers to three decimals.

Name SOLUTION KEY

1. If  $\cot A = \frac{1}{3}$  in QIII, find the other five trig values (exact values) and the approximate value of angle A.

Q III  
 $x = -1$   
 $y = -3$   
 $r = \sqrt{10}$

$\sin A = -3/\sqrt{10}$

$\csc A = -\sqrt{10}/3$

$\cos A = -1/\sqrt{10}$

$\sec A = -\sqrt{10}$

$\tan A = 1/3$

$\cot A = \frac{1}{3}$

$A = \underline{-1.893 \pm 2\pi n}$

2. Prove the following identity:  $\frac{\sin x - \sin^3 x}{\cos^3 x} = \tan x$

$$\frac{\sin x (1 - \sin^2 x)}{\cos^3 x} = \frac{\sin x \cos^2 x}{\cos^3 x} = \frac{\sin x}{\cos x}$$

$$\cos A = -12/13$$

$$\sin A = 5/13$$

$$\sin B = -15/17$$

$$\sin C = -11/61$$

$$\cos C = -60/61$$

3.  $(-12, 5)$  is on the terminal side of  $A$ ;  $\cos B = \frac{8}{17}$ ,  $\sin B < 0$ ,  $\tan C = \frac{11}{60}$  and  $-180^\circ \leq C \leq -90^\circ$ , find:

$$\tan B = -\frac{15}{8}$$

a.  $\sin(A-C) = \sin A \cos C - \cos A \sin C$

$$= \left(\frac{5}{13}\right)\left(\frac{-60}{61}\right) - \left(\frac{-12}{13}\right)\left(\frac{-11}{61}\right)$$

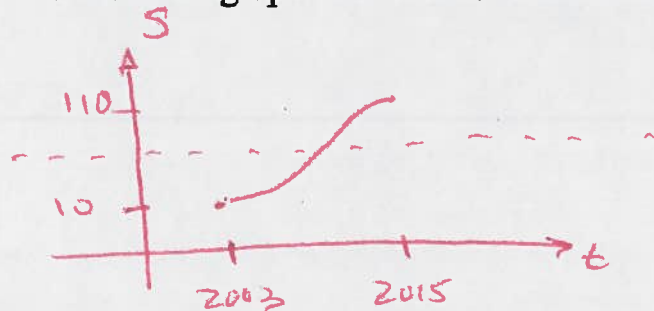
$$= \frac{-300 - 132}{793} = \frac{-432}{793}$$

b.  $\tan(2B) = \frac{2 \tan B}{1 - \tan^2 B} = \frac{2\left(-\frac{15}{8}\right)}{1 - \left(-\frac{15}{8}\right)^2} = \frac{-\frac{15}{4}}{1 - \frac{225}{64}}$

$$= \frac{-15}{4} \cdot \frac{-64}{161}$$
$$= \frac{+240}{161}$$

4. Astronomers have noticed that the number of visible sunspots varies sinusoidally with time. In 2003, there was a minimum of 10 sunspots and 12 years later the sunspots reached a maximum of 110.

a. Sketch the graph of this sinusoidal function



b. Write the particular equation expressing the number of visible sunspot over time.

$$S = 60 - 50 \cos\left[\frac{\pi}{12}(t - 2003)\right] \approx 108.296 \approx \boxed{108}$$

c. Predict the number of sunspot that will be visible in 2016.

$$S(2016) = 108.296 \approx \boxed{108}$$

d. How many years after 2003 did the amount of visible sunspot first reach 100.

$$100 = 60 - 50 \cos\left[\frac{\pi}{12}(t - 2003)\right]$$

$$-0.8 = \cos\left[\frac{\pi}{12}(t - 2003)\right]$$

$$\pm 2.498 \pm 2\pi n = \frac{\pi}{12}(t - 2003)$$

$$\left. \begin{array}{l} 9.9542 \pm 24n \\ -9.542 \pm 24n \end{array} \right\} = t - 2003$$

$$\left. \begin{array}{l} \cancel{20012 \pm 24n} \\ 2012.542 \pm 24n \\ 1993.458 \pm 24n \end{array} \right\} = t$$

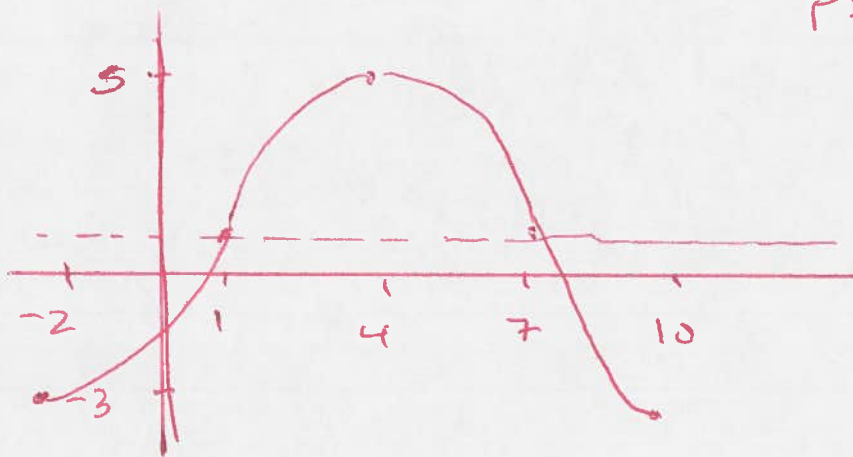
HAPPENED IN 2012

$\therefore$  9 YEARS

Round to 3 decimal places. Show all work.

Multiple Choice (3 pts. each)

5. Sketch a graph of the function  $y = 1 - 4\cos\left[\frac{\pi}{6}(x+2)\right]$ .



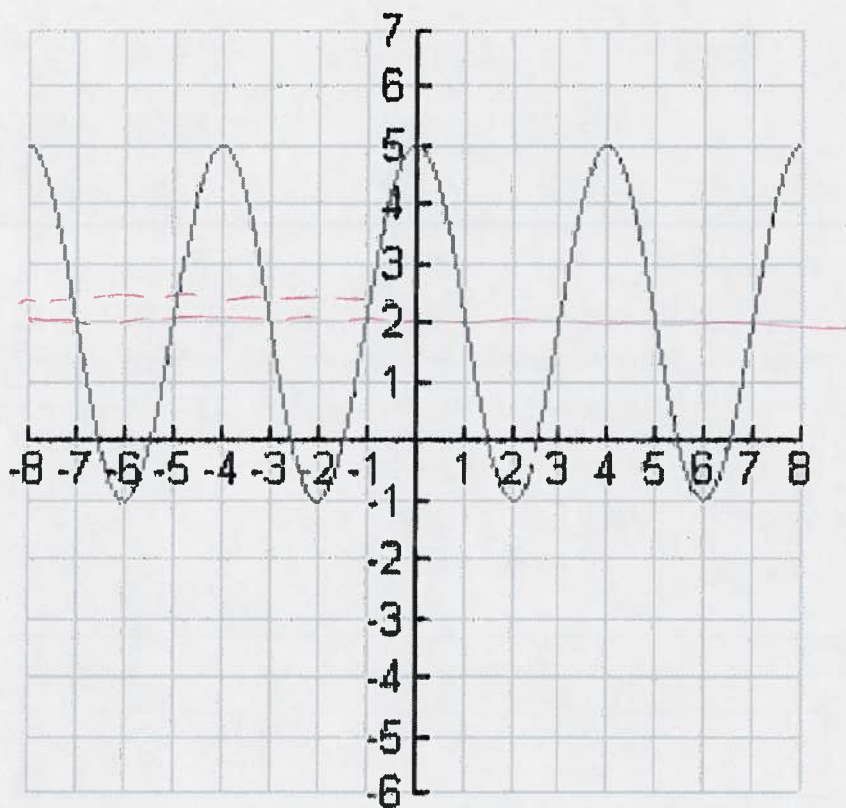
$$P = \frac{2\pi}{\pi/6} = 12$$

6. Find the exact value of the following:

(a)  $\cos\frac{\pi}{8}\cos\frac{5\pi}{8} + \sin\frac{\pi}{8}\sin\frac{5\pi}{8} = \cos\left(\frac{\pi}{8} - \frac{5\pi}{8}\right) = \cos\frac{-\pi}{2} = 0$

(b)  $\cot\frac{2\pi}{3} - \sin\frac{3\pi}{4} = \frac{-1}{\sqrt{3}} - \frac{1}{\sqrt{2}} = \frac{-\sqrt{2} - \sqrt{3}}{\sqrt{6}}$

7. Find one cosine function and one sine function for the graph below.



$$\frac{2\pi}{4} = B$$

$$y = 2 + 3 \cos \left[ \frac{\pi}{2} (x - 0) \right]$$

$$y = 2 + 3 \sin \left[ \frac{\pi}{2} (x + 1) \right]$$