PreCalculus H 2016--Quattrin
Trig Identities Group Test
Page 1

Name Solution Key

Score__________

For Problem 1-6, use:

(-4, -3) is on the terminal side of C and $180° \le C \le 270°$

$\sin C = \frac{-3}{5}$ $\cos C = \frac{-4}{5}$ $\tan C = \frac{3}{4}$

$\sec E = 17/8$ and $0° \le E \le 90°$; and

$\sin E = \frac{15}{17}$ $\cos E = \frac{8}{17}$ $\tan E = \frac{15}{8}$

$\cot Q = 12/5$ and $-180° \le Q \le -90°$

$\sin Q = \frac{-5}{13}$ $\cos Q = \frac{-12}{13}$ $\tan Q = \frac{5}{12}$

to find the exact values of:

1. Sin (C-E)

$\sin C \cos E - \cos C \sin E$

$\left(\frac{-3}{5}\right)\left(\frac{8}{17}\right) - \left(\frac{-4}{5}\right)\left(\frac{15}{17}\right)$

$\frac{-\cancel{48}\,24+60}{85} = \frac{36}{85}$

2. Cos 2C $= \cos^2 C - \sin^2 C$

$= \left(\frac{-4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2$

$= \frac{7}{25}$

3. Sec 2Q $= \frac{1}{\cos^2 Q - \sin^2 Q}$

$= \frac{1}{\left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2} = \frac{169}{119}$

4. Csc (C+E) $= \frac{1}{\sin C \cos E + \cos C \sin E}$

$\frac{1}{\left(\frac{-3}{5}\right)\left(\frac{8}{17}\right) + \left(\frac{-4}{5}\right)\left(\frac{15}{17}\right)} = \frac{85}{\cancel{-12}\,-84}$

5. Tan 2E $= \frac{2\tan E}{1-\tan^2 E}$

$= \frac{2\left(15/8\right)}{1-\left(15/8\right)^2} = \frac{15}{4} \cdot \frac{64}{-161}$

$= -\frac{240}{161}$

6. cot (E+Q)

$= \frac{1 - \tan E \tan Q}{\tan E + \tan Q}$

$= \frac{1 - \left(\frac{15}{8}\right)\left(\frac{5}{12}\right)}{\frac{15}{8} - \frac{5}{12}} = \cancel{\frac{96-75}{96}} \frac{96}{120-40}$

$= \frac{21}{220}$

7. Prove: $\dfrac{2\sin^2 w-5\cos w+1}{6\sin^2 w-5\cos w-2}=\dfrac{\cos w+3}{3\cos w+4}$

$$\frac{2(1-\cos^2 w)-5\cos w+1}{6(1-\cos^2 w)-5\cos w \mp 2}$$

$$\frac{-2\cos^2 w-5\cos w+3}{-6\cos^2 w-5\cos w+4}$$

$$=\frac{-\cancel{(2\cos w+1)}(\cos w+3)}{-\cancel{(2\cos w+1)}(3\cos w+4)}$$

9. Solve exactly for A: $\cos^4 A-\sin^4 A=1$

$$(\cos^2 A-\sin^2 A)\underbrace{(\cos^2 A+\sin^2 A)}_{1}$$

$$\cos 2A=1$$

$$2A=0\pm 2\pi n$$

$$\boxed{A=0\pm \pi n}$$

8. Prove:
$\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$

$$(\sin A\cos B+\cos A\sin B)\cdot(\sin A\cos B-\cos A\sin B)$$

$$=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B$$

$$=\sin^2 A(1-\sin^2 B)-(1-\sin^2 A)\sin^2 B$$

$$=\sin^2 A-\cancel{\sin^2 A\sin^2 B}-\sin^2 B+\cancel{\sin^2 A\sin^2 B}$$

$$=\sin^2 A-\sin^2 B$$

10. Solve exactly for $x\in[0,\pi)$:
$\dfrac{\tan x+\cot x}{\cot x-\tan x}=2$

$$\frac{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}=\frac{\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}}{\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}}$$

$$=\frac{1}{\cos^2 x-\sin^2 x}=2$$

$$\cos 2x=\frac{1}{2}$$

$$2x=\begin{cases}\pi/3\pm 2\pi n\\ -\pi/3\pm 2\pi n\end{cases}$$

$$x=\begin{cases}\pi/6\pm \pi n\\ -\pi/6\pm \pi n\end{cases}\qquad \boxed{x=\frac{\pi}{6},\ \frac{5\pi}{6}}$$

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For Problem 1-6, use:

(-4, -3) is on the terminal side of C and
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$\sec E = 17/8$ and $0° \le E \le 90°$; and

$\cot Q = 12/5$ and $-180° \le Q \le -90°$

to find the exact values of:

1. cos (C-E)

$\cos C \cos E + \sin C \sin E$

$\left(\frac{-4}{5}\right)\left(\frac{8}{17}\right) + \left(\frac{-3}{5}\right)\left(\frac{15}{17}\right)$

$\frac{-32-45}{85} = \frac{-77}{85}$

2. Sin 2E $= 2\sin E \cos E$

$= 2\left(\frac{15}{17}\right)\left(\frac{8}{17}\right)$

$= \frac{240}{289}$

3. Sec (Q+E) $= \frac{1}{\cos Q \cos E - \sin Q \sin E}$

$= \frac{1}{\left(\frac{-12}{13}\right)\left(\frac{8}{17}\right) - \left(\frac{-5}{13}\right)\left(\frac{15}{17}\right)} = \frac{221}{-21}$

4. Tan (C+E) $= \frac{\tan C + \tan E}{1 - \tan C \tan E}$

$= \frac{3/4 + 15/8}{1 - \frac{3}{4}(15/8)} = \frac{24+60}{32} \cdot \frac{32}{32-45}$

$= \frac{84}{-13}$

5. Cot 2C $= \frac{1-\tan^2 C}{2\tan C}$

$= \frac{1-(3/4)^2}{2(3/4)} = \frac{+7}{16} \cdot \frac{2}{3}$

$= \frac{+7}{24}$

6. Csc 2C $= \frac{1}{2\sin C \cos C}$

$= \frac{1}{2(-3/5)(-4/5)} = \frac{25}{24}$

7.

$$\frac{\cos(A+2B)\cos(B)+\sin(A+2B)\sin(B)}{\sin A\cos B+\cos A\sin B} = \frac{1-\tan A\tan B}{\tan A+\tan B}$$

$$\frac{\cos((A+2B)-B)}{\sin(A+B)}$$

$$\frac{\cos(A+B)}{\sin(A+B)}$$

$$= \cot(A+B)$$

$$= \frac{1}{\tan(A+B)} =$$

8. Solve for x:

$$\sec\left(x-\frac{\pi}{4}\right) = 2+2\sec\left(x-\frac{\pi}{4}\right)$$

$$-2 = \sec\left(x-\frac{\pi}{4}\right)$$

$$\cos\left(x-\frac{\pi}{4}\right) = -\frac{1}{2}$$

$$x-\frac{\pi}{4} = \begin{cases} 2\pi/3 \pm 2\pi n \\ -2\pi/3 \pm 2\pi n \end{cases}$$

$$x = \begin{cases} \frac{11\pi}{12} \pm 2\pi n \\ \frac{5\pi}{12} \pm 2\pi n \end{cases}$$

9. Prove: $\tan x\sin 2x - 2\cos^2 x = -2\cos 2x$

$$\frac{\sin x}{\cancel{\cos x}} \cdot 2\sin x\cancel{\cos x}$$

$$2\sin^2 A - 2\cos^2 x$$

$$-2(\cos^2 x - \sin^2 x)$$

$$= -2\cos 2x$$

$$\cancel{\cos 2x = -1}$$

$$\cancel{2x = \pm\pi \pm 2\pi n}$$

$$\cancel{x = \pm\frac{\pi}{2} \pm \pi n}$$

10. Solve for $A \in (-2\pi, 2\pi)$:

$$\left(\sin\frac{1}{2}A - \cos\frac{1}{2}A\right)^2 = \frac{1}{2}$$

$$\sin^2\tfrac{1}{2}A - 2\sin\tfrac{1}{2}A\cos\tfrac{1}{2}A + \cos^2\tfrac{1}{2}A$$

$$1 - \sin 2\left(\tfrac{1}{2}A\right) = 1/2$$

$$\sin A = 1/2$$

$$A = \begin{cases} \pi/6 \pm 2\pi n \\ 5\pi/6 \pm 2\pi n \end{cases}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{11\pi}{6}, -\frac{7\pi}{6}$$

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For Problem 1-6, use:

(-4, -3) is on the terminal side of C and
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$\sec E = 17/8$ and $0° \le E \le 90°$; and

$\cot Q = 12/5$ and $-180° \le Q \le -90°$

to find the exact values of:

1. Tan (Q-E) $= \dfrac{\tan Q - \tan E}{1 + \tan Q \tan E}$

$= \dfrac{5/12 - 15/8}{1 + \left(\frac{5}{12}\right)\left(\frac{15}{8}\right)} = \dfrac{-140}{96} \cdot \dfrac{96}{171}$

$= \dfrac{-140}{171}$

2. Cot 2Q $= \dfrac{1 - \tan^2 Q}{2 \tan Q}$

$= \dfrac{1 - (5/12)^2}{2(5/12)}$

$= \dfrac{(144-25)}{144} \cdot \dfrac{6}{5} = \dfrac{119}{120}$

3. Cos (C+Q) $= \cos C \cos Q - \sin C \sin Q$

$= \left(\frac{-4}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right)$

$= \dfrac{48-15}{65} = \dfrac{33}{65}$

4. Sec (Q+C) $= \dfrac{1}{\cos Q \cos C - \sin Q \sin C}$

$= \dfrac{1}{\left(\frac{-12}{13}\right)\left(\frac{-4}{5}\right) - \left(\frac{+5}{13}\right)\left(\frac{-3}{5}\right)}$

$= \dfrac{65}{33}$

5. Csc 2E $= \dfrac{1}{2 \sin E \cos E}$

$= \dfrac{1}{2\left(\frac{15}{17}\right)\left(\frac{8}{17}\right)} = \dfrac{289}{240}$

6. Sin 2C $= 2 \sin C \cos C$

$= 2\left(\frac{-3}{5}\right)\left(\frac{-4}{5}\right) = \dfrac{24}{25}$

7. Prove: $\frac{2\cos^4\phi - \sin^4\phi}{2\sin\phi\cos\phi} = \frac{1-\tan^2\phi}{\tan\phi}$

$$\frac{2(\cos^2\phi - \sin^2\phi)(\cos^2\phi + \sin^2\phi)}{2\sin\phi\cos\phi}$$

$$\frac{2\cos 2\phi}{\sin 2\phi}$$

$$2\cot 2\phi$$

$$\cancel{2}\left(\frac{1-\tan^2\phi}{\cancel{2}\tan\phi}\right)$$

9. Solve exactly for $x \in [0, \pi)$:
$\tan 3x - \tan x + \tan 3x \tan x = -1$

$$\tan 3x - \tan x = -1(1 + \tan 3x \tan x)$$

$$\frac{\tan 3x - \tan x}{1 + \tan 3x \tan x} = -1$$

$$\tan 2x = -1$$

$$2x = \frac{3\pi}{4} \pm \pi n$$

$$x = \frac{3\pi}{8} \pm \frac{\pi}{2} n$$

$$x = \frac{3\pi}{8}, \frac{7\pi}{8}$$

8. Prove: $2\cot f = \cot\frac{f}{2} - \tan\frac{f}{2}$

$$2\cot 2w = \cot w - \tan w$$

$$\frac{1-\tan^2 w}{\tan w} = \frac{1}{\tan w} - \tan w$$

$$= \frac{1-\tan^2 w}{\tan w}$$

10. Solve exactly for x: $\dfrac{1-\tan x\tan\frac{\pi}{12}}{\tan x + \tan\frac{\pi}{12}} = \sqrt{3}$

$$\cot\left(x + \frac{\pi}{12}\right) = \sqrt{3}$$

$$\tan\left(x + \frac{\pi}{12}\right) = \frac{1}{\sqrt{3}}$$

$$x + \frac{\pi}{12} = \frac{\pi}{6} \pm \pi n$$

$$x = \frac{\pi}{12} \pm \pi n$$

3. A ring with radius = a carries a uniform charge = Q. The electric field intensity E at any point x along the axis of the ring is given by

$$E(x)=\frac{Qx}{\left(x^2+a^2\right)^{3/2}}.$$

At what point is the electric field intensity the greatest?

$$E'(x)=\frac{(x^2+a^2)^{3/2}Q-Qx\left(\frac{3}{\cancel{2}}(x^2+a^2)^{1/2}(\cancel{2}x)\right)}{(x^2+a^2)^3}=0$$

$$Q(x^2+a^2)^{1/2}\left[x^2+a^2-3x^2\right]=0$$

$$-2x^2+a^2=0$$

$$x^2=\frac{a^2}{2}$$

$$x=\pm\frac{a}{\sqrt{2}}$$

E'	$-$	0	$+$	0	$-$
x		$-\frac{a}{\sqrt{2}}$		$\frac{a}{\sqrt{2}}$	
		MIN		MAX	

$$\boxed{x=\frac{a}{\sqrt{2}}}$$

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to find the exact values of:

1. Csc (E-C) $= \dfrac{1}{\sin E \cos C - \cos E \sin C}$

$= \dfrac{1}{\left(\frac{15}{17}\right)\left(\frac{-4}{5}\right) - \left(\frac{8}{17}\right)\left(\frac{-3}{5}\right)}$

$= \dfrac{85}{-36}$

2. Sin 2E $= 2\sin E \cos E$

$= 2\left(\frac{15}{17}\right)\left(\frac{8}{17}\right) = \frac{240}{289}$

3. Cot 2Q $= \dfrac{1-\tan^2 Q}{2\tan Q}$

$= \dfrac{1-\left(\frac{5}{12}\right)^2}{2(5/12)} = \dfrac{119}{120}$

4. Sin (E+Q) $= \sin E \cos Q + \cos E \sin Q$

$= \left(\frac{15}{17}\right)\left(\frac{-12}{13}\right) + \left(\frac{8}{17}\right)\left(\frac{-5}{13}\right)$

$= \dfrac{-220}{221}$

5. Cos 2Q $= \cos^2 Q - \sin^2 Q$

$= \left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2$

$= \dfrac{119}{169}$

6. Tan (Q-E) $= \dfrac{\tan Q - \tan E}{1+\tan Q \tan E}$

$= \dfrac{\frac{5}{12} - \frac{15}{8}}{1+\left(\frac{5}{12}\right)\left(\frac{15}{8}\right)} = \dfrac{-140}{96} \cdot \dfrac{96}{171} = \boxed{\dfrac{-140}{171}}$

7. Prove: $\sin^2\theta\tan\frac{\theta}{2}=\sin\theta-\sin\theta\cos\theta$

$\sin^2 2w \frac{\sin w}{\cos w} = \sin 2w(1-\cos 2w)$

$= 2\sin w \cancel{\cos w} \frac{\sin w}{\cancel{\cos w}}$

$2\sin^2 w \quad = 2\sin w\cos w(2\sin^2 w)$

9. Prove: $\frac{\sec\theta}{\cot\theta+\tan\theta}=\sin\theta$

$$\frac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$$

$$\frac{1}{\cos\theta}\cdot\frac{\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta}$$

$\sin\theta$

8. Solve for $x\in[-2\pi, 2\pi]$:
$3-3\sin x-2\cos^2 x=0$

$(1-\sin^2 x)$

~~$(\cos x \quad)(2\cos x \quad)$~~

$2\sin^2 x-3\sin x+1$

$(2\sin x-1)(\sin x-1)$

$\sin x=1/2 \qquad \sin x=1$

$x=\begin{cases}\frac{\pi}{6}\pm 2\pi n\\ \frac{5\pi}{6}\pm 2\pi n\end{cases} \qquad x=\frac{\pi}{2}\pm 2\pi n$

$x=\frac{\pi}{6}, \frac{5\pi}{6}, -\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{2}, \frac{-3\pi}{2}$

10. Solve for x:
$(\cos^2 x-\sin^2 x)^2-(2\sin x\cos x)^2=\frac{1}{2}$

$(\cos 2x)^2-(\sin 2x)^2$

$\cos 4x = 1/2$

$4x=\pm\frac{\pi}{3}\pm 2\pi n$

$x=\pm\frac{\pi}{12}\pm\frac{\pi}{2}n$