Honors PreCalculus
Dr. Quattrin
Limits and Derivatives Test
CALCULATOR ALLOWED
Round to 3 decimal places. Show all work.

Name: Solution Key

Score ___________

1. $\lim_{x\to9}\frac{(\sqrt{x-5}-2)(\sqrt{x-5}+2)}{(x-9)(\sqrt{x-5}+2)} = \lim_{x\to9}\frac{x-5-4}{(x-9)(\sqrt{x-5}+2)} = \lim_{x\to9}\frac{1}{\sqrt{x-5}+2}$

a) $\frac{1}{4}$ b) $-\frac{1}{4}$ c) 1 d) $4\sqrt{2}$ e) dne

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2. An equation of the line normal to the graph of $y=3x^4+5x^3-9x^2-6x+4$ at the point where $x=0$ is

$(0,4)$ $m=-6$

a) $6x+y=4$ b) $6x-y=-4$ c) $4x+4y=2$

d) $x-6y=-24$ e) $x+6y=24$

TAN $y-4=-6(x-0)$

NORMAL: $y-2=\frac{1}{6}x$

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3. If f is a differentiable function such that $f(3)=8$ and $f'(3)=5$, which of the following statements must be false?

(a) $\lim_{x\to3}f(x)=8$ (b) $\lim_{x\to3^+}f(x)=\lim_{x\to3^-}f(x)$ (c) $\lim_{x\to3}\frac{f(x)-8}{x-3}=5$

(d) $\lim_{h\to0}\frac{f(3+h)-5}{h}=8$ (e) $\lim_{x\to3}f'(x)=5$

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4. Find an equation of the tangent line to the curve $f(x)=4x^3-3x-1$ at the point in the first quadrant where $\frac{dy}{dx}=45$.

$f'=12x^2-3=45$

$12x^2=48$

$x^2=4$

$x=2$

$y=25$

(a) $y=25x-5$ (b) $y=45x+65$ (c) $y=45x-65$

(d) $y=65-45x$ (e) $y=65x-45$

$y-25=45(x-2)$

$y=45x-65$

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5. At what point on the graph of $y=-3x^2$ is the tangent parallel to the line $5x+2y=7$?

$\frac{dy}{dx}=-6x=-5/2 \to x=5/12$

(a) $\left(-\frac{5}{12},-\frac{75}{144}\right)$ (b) $\left(\frac{1}{15},-\frac{3}{225}\right)$ (c) $\left(\frac{5}{12},-\frac{75}{144}\right)$

(d) (5, 3) (e) None of these

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6. A particle moving in the xy-plane with its x-coordinate given by $x(t)=\frac{1}{4}t^4+\frac{2}{3}t^3-\frac{3}{2}t^2-1$ and its y-coordinate given by $y(t)=t^3-12t+1$. When is the particle moving up and left?

(a) $(-\infty,-3)$ (b) $(-3,-2)$ (c) $(-2,0)$

(d) $(0,2)$ (e) $(2,\infty)$

$x'=t^3+2t^2-3t$ $y'=3t^2-12$

v_x: $-$ 0 $+$ 0 $-$ 0 $+$

t: -3 0 1

v_y: $+$ 0 $-$ 0 $+$

t: -2 2

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1. Use the equation of the line tangent to $y = 6x^3 - 3x^2 + 5x - 4$ at $x = -1$ to approximate $f(-1.1)$

$y(-1) = -18$

$\frac{dy}{dx} = 18x^2 - 6x + 5$

$m = 29$

$y + 18 = 29(x+1)$

$f(-1.1) \approx y(-1.1) = 29(-1.1+1) - 18 = -20.9$

2. The motion of a particle is described by $x(t) = 3t^4 - 16t^3 - 30t^2 + 240t - 140$.

a) When the particle is stopped?
b) Which direction it is moving at $t = -1$?
c) Where is it when $t = -1$?
d) Find $a(-1)$.

$V = 12t^3 - 48t^2 \mp 60t + 240$

$a(t) = 36t^2 - 96t + 60$

a) = $t = \pm\sqrt{5}, 4$)

b) $v(-1) = 240$ ∴ RIGHT

c) $x(-1) = -391$

d) $a(-1) = \cancel{}\, 72$

V

0	0	0
$-\sqrt{5}$	$\sqrt{5}$	4

$V = t^3 - 4t^2 \mp 5t + 20$

$t^2(t-4) \mp 5(t-4)$

$(t^2-5)(t-4)$

3. A particle's position $\langle x(t), y(t)\rangle$ at time t is described by $\langle t^3-6t^2+9t+1, -t^2+6t+2\rangle$. $v(t) = \langle 3t^2 - 12t + 9, -2t+6\rangle$

a) Find the speed at $t=3$. $v(3) = (0, 0)$

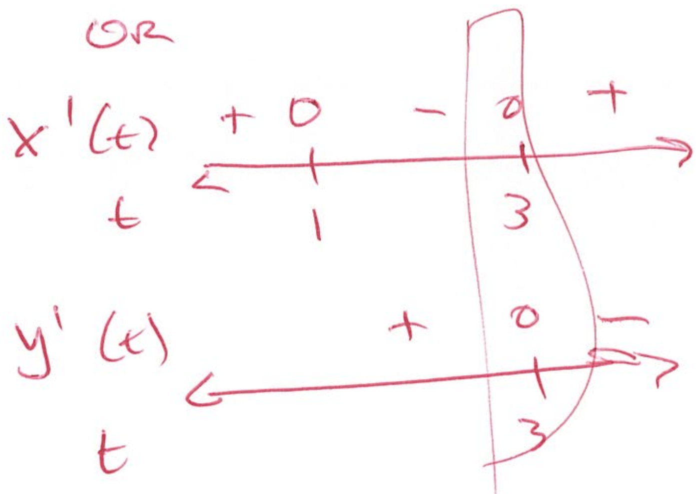
$s = \sqrt{0^2+0^2}$

$= 0$

b) When, if ever, is the particle stopped? Prove it.

$t = 3$ BECAUSE SPEED $= 0$

OR



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5. Set up, but do not solve, the limit definition of the derivative for
$y = \sqrt[4]{x^5} + \frac{4}{x^3} - 2\sqrt[7]{x^9} - \pi^3$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\left(\sqrt[4]{(x+h)^5} + \frac{4}{(x+h)^3} - 2\sqrt[7]{(x+h)^9} - \pi^3\right) - \left(\sqrt[4]{x^5} + \frac{4}{x^3} - 2\sqrt[7]{x^9} - \pi^3\right)}{h}$$

6. Use the Power Rule to find:

a) $\frac{d}{dx}\left[7x^4 - x^3 + \pi + 42x\right] = 28x^3 - 3x^2 + 42$

b) $D_x\left[\sqrt[4]{x^5} + \frac{4}{x^3} - 2\sqrt[7]{x^9} - \pi^3\right] = \frac{5}{4}x^{1/4} - \frac{12}{x^4} - \frac{18}{7}x^{2/7}$

7. Evaluate the following limits:

(a) $\lim\limits_{x \to -1/2} \dfrac{8x^3+1}{4x^2-8x-5}$

$= \lim\limits_{x \to -1/2} \dfrac{\cancel{(2x+1)}(4x^2-2x+1)}{\cancel{(2x+1)}(2x-5)}$

$= \dfrac{4(-1/2)^2-2(-1/2)+1}{2(-1/2)-5} = \dfrac{3}{-6} = -\dfrac{1}{2}$

(b) $\lim\limits_{x \to -3} \sqrt{\dfrac{x^4-81}{x^3+2x^2-4x-3}}$

$= \lim\limits_{x \to -3} \sqrt{\dfrac{(x^2-9)(x^2+9)}{(x+3)(x^2-x-1)}}$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -4 & -3 \\ & & -3 & 3 & 3 \\ \hline & 1 & -1 & -1 & 0 \end{array}$$

$= \lim\limits_{x \to -3} \sqrt{\dfrac{(x-3)\cancel{(x+3)}(x^2+9)}{\cancel{(x+3)}(x^2-x-1)}}$

$= \sqrt{\dfrac{-6(18)}{11}} = DNE$