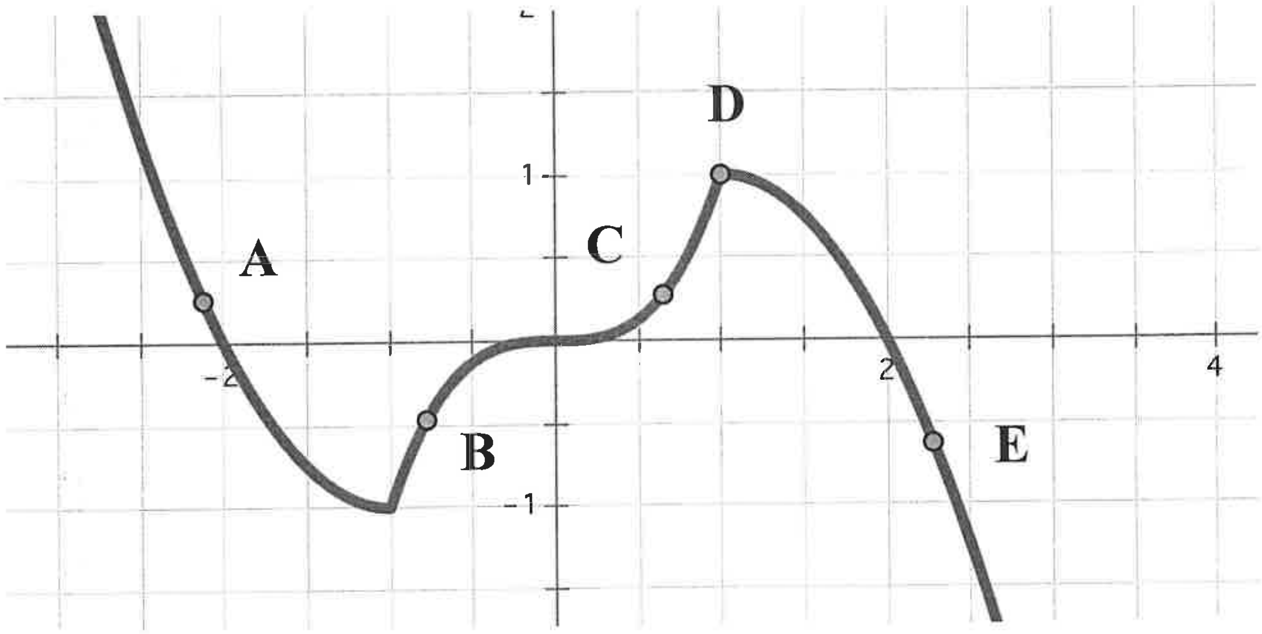
Honors PreCalculus '17-18
Advanced Curve Sketching

Name: Solution Key

score________

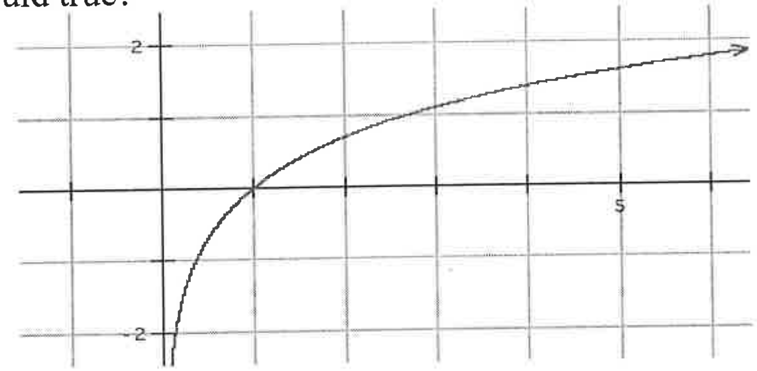


1. The graph of the function $f(x)$ is shown above. At which point on the graph of $f(x)$ is $f'(x) > 0$ and $f''(x) < 0$?

a) A [✓ b) B] c) C d) D e) E

INCREASING & CONCAVE DOWN

2. The graph of a twice differentiable function f is shown below. Which of the following could true?



$f(1)=0$

$f'(1)>0$

$f''(1)<0$

[✓ a)] $f''(1)<f(1)<f'(1)$

b) $f(1)<f''(1)<f'(1)$

c) $f(1)<f'(1)<f''(1)$

d) $f'(1)<f''(1)<f(1)$

e) $f'(1)<f(1)<f''(1)$

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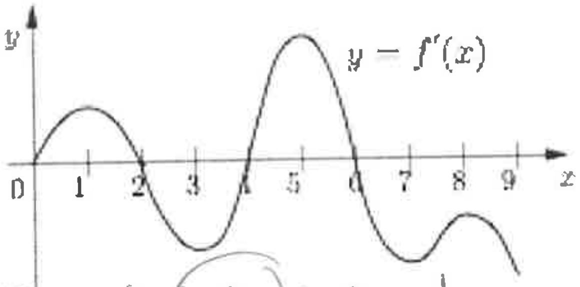
3. The graph of the derivative $f'(x)$ on the interval $[0, 9]$ is shown below.



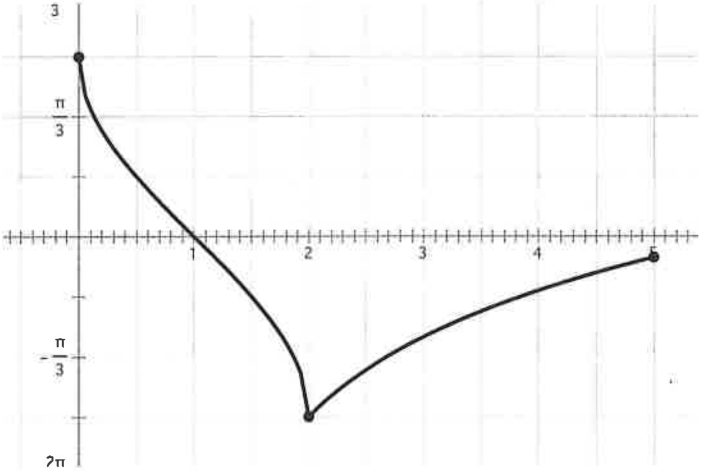
$-3f'$ $-$ 0 $+$ 0 $-$ 0 $+$

0 2 4 6

If $g'(x)=-3f'(x)$, how many maxima will $g(x)$ have on the interval $[0, 9]$?

a) None b) One [✓ c)] Two ~~d)~~ Three e) Four

4. This is the graph of $f'(x)$, the derivative of $f(x)$.



Which of the following sign patterns are hidden with the graph.

I.
$$\begin{array}{lccc} F'(x) & + & 0 & - \\ \hline x & & 1 & \end{array}$$

II.
$$\begin{array}{lccc} F''(x) & - & dne & + \\ \hline x & & 2 & \end{array}$$

III.
$$\begin{array}{lccccc} F''(x) & + & 0 & - & dne & - \\ \hline x & & 1 & & 2 & \end{array}$$

a) I only b) II only [✓ c) I and II only]

d) II and III only e) I, II, and III

5. A particle is moving along the x-axis in such a way that its velocity at time $t > 0$ is given by $v(t) = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

a) 1 b) $e^{1/2}$ [✓ c) e] d) $e^{3/2}$

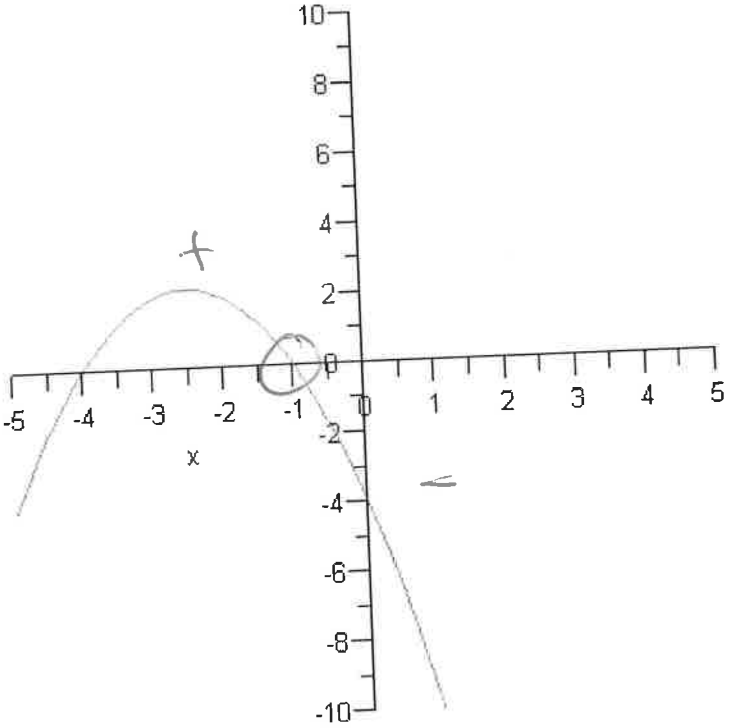
e) There is no maximum value of v.

$V' = \frac{1-\ln t}{t^2} = 0$

$t = e, 0$

0 + 0 −

0 e



6. Above is shown the graph of $f'(x)$. Give a value of x where f has a local maximum.

a) −4 [✓ b) −1] c) $-\frac{5}{2}$ d) 1 e) no value of x

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7. This problem involves finding the absolute maximum and absolute minimum of the function $f(x) = x^3 - 3x + 4$ restricted to the closed interval $x \in [0, 2]$. Which of the following statements is correct?

a) $f(x)$ has both an absolute maximum and absolute minimum at the end points.

b) $f(x)$ has both an absolute maximum and absolute minimum at interior points.

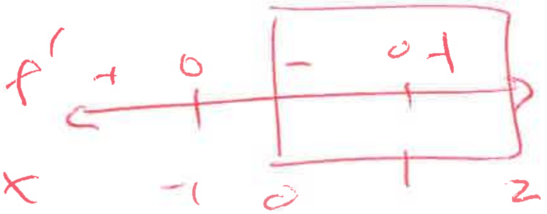
[✓ c)] $f(x)$ has both an absolute maximum at an end point and an absolute minimum at an interior point.

~~d)~~ $f(x)$ has both an absolute maximum at an interior point and an absolute minimum at an end point.

e) None of the above

$f' = 3x^2 - 3$

$x = \pm 1$



MIN @ $x = 1$

MAX @ $x = 0, 2$

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Round to 3 decimal places.
Show all work.

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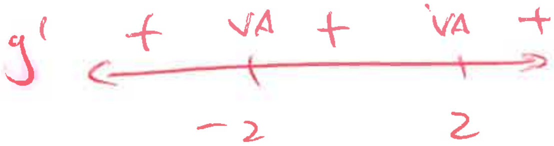
1. Given $y = \frac{-x}{x^2-4}$, find the sign pattern for $\frac{dy}{dx}$ and determine if the critical values are at a maximum, minimum, or neither.

$$\frac{dy}{dx} = \frac{(x^2-4)(-1)-(-x)(2x)}{(x^2-4)^2} = \frac{x^2+4}{(x^2-4)^2}$$



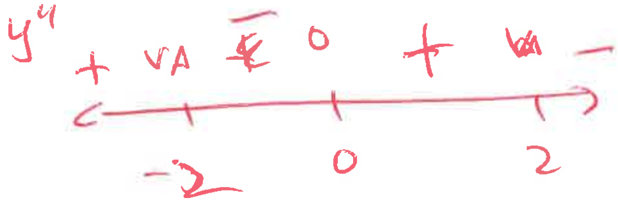
No Ext.

2. Given $y = \frac{-x}{x^2-4}$, find the sign pattern for $\frac{d^2y}{dx^2}$ and name the points of Inflection.

$$\frac{d^2y}{dx^2} = \frac{(x^2-4)^2(2x) - (x^2+4)2(x^2-4)^1(2x)}{(x^2-4)^4} = \frac{(x^2-4)(2x) - 4x(x^2+4)}{(x^2-4)^3}$$

$$= \frac{2x^3-8x-4x^3-16x}{(x^2-4)^3}$$

$$= \frac{-2x^3-24x}{(x^2-4)^3}$$



POI (0, 0)

3. Given $y=\left(\frac{1}{5}e^{-x}\right)\sqrt{9-x^2}$, find the sign pattern for $\frac{dy}{dx}$ and determine if the critical values are at a maximum, minimum, or neither.

$$y' = \boxed{\frac{1}{5}e^{-x}}\,\cancel{\tfrac{1}{2}}(9-x^2)^{-1/2}(-\cancel{2}x) + (9-x^2)^{1/2}\left(-\boxed{\frac{1}{5}e^{-x}}\right)$$

$$= \frac{1}{5}e^{-x}\left[\frac{-x}{(9-x^2)^{1/2}} + \frac{(9-x^2)^{1/2}}{(9-x^2)^{1/2}}(9-x^2)^{1/2}\right] = \frac{(+x^2-x-9)}{5e^{x}\,(9-x^2)^{1/2}}$$

$$x = \frac{1 \pm \sqrt{37}}{2} = \begin{cases} \cancel{+3.541} \\ -2.541 \end{cases}$$

y' END + 0 − END

-3 -2.541 3

MIN MAX MIN

$(-2.541, 4.048)$

4. Given $y=\left(\frac{1}{5}e^{-x}\right)\sqrt{9-x^2}$, find the sign pattern for $\frac{d^2y}{dx^2}$ and name the points of Inflection.

$$\frac{d^2y}{dx^2} = \overset{u}{\frac{1}{5}e^{-x}}\,\overbrace{(x^2-x-9)\left(-\frac{1}{2}(9-x^2)^{-3/2}(-2x)\right)}^{Dv} + \overset{v}{(9-x^2)^{-1/2}}\left[\overbrace{\frac{1}{5}e^{-x}(2x-1) + -\frac{1}{5}e^{-x}(x^2-x-9)}^{Du}\right]$$

$$= \frac{1}{5}e^{-x}\left[\frac{(x^2-x-9)x}{(9-x^2)^{3/2}} + \frac{-x^2+3x+8}{(9-x^2)^{1/2}}\right] = 0$$

$$x = \begin{cases} (-1.581, 2.478) \\ (2.772, .014) \end{cases}$$

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NO CALCULATOR ALLOWED
Show all work.

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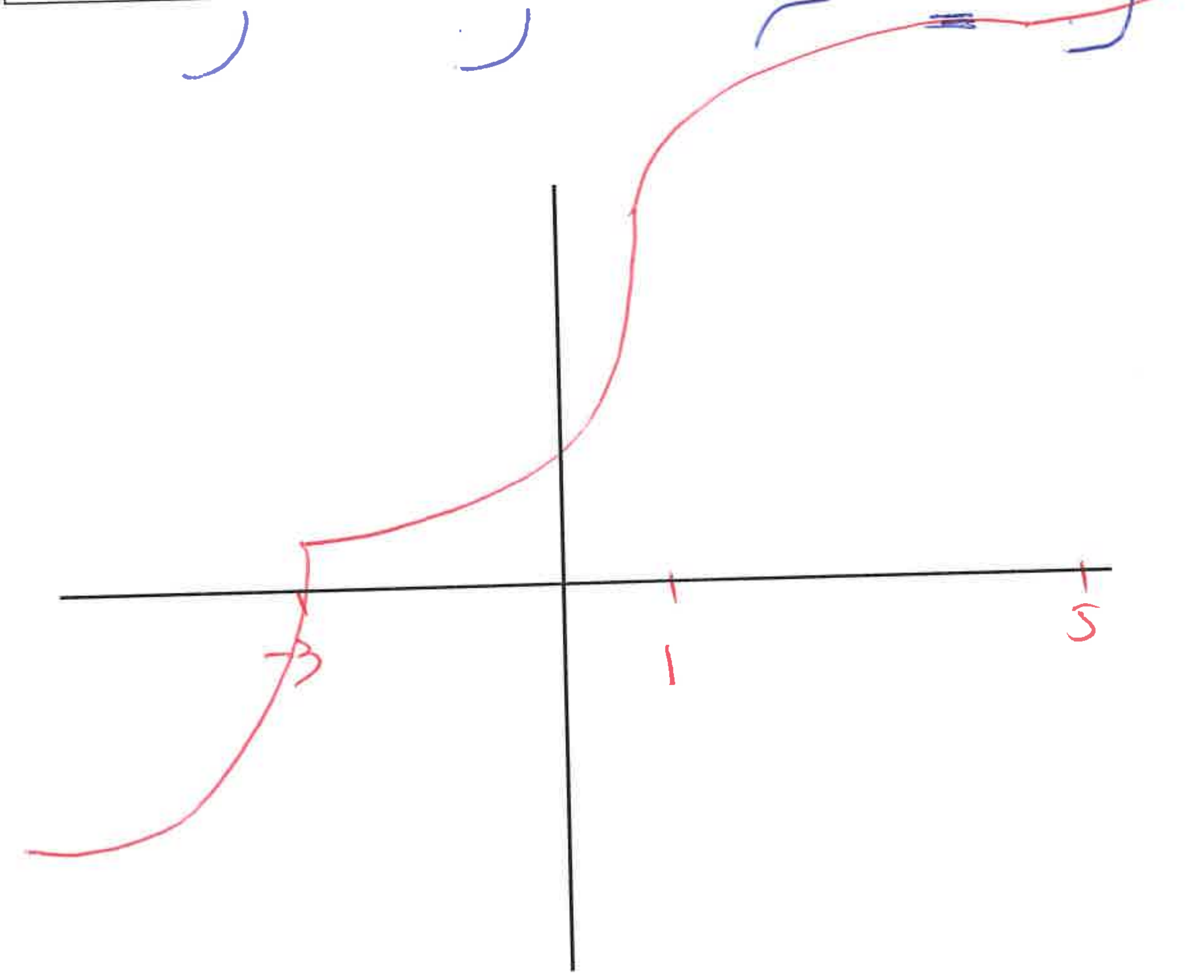
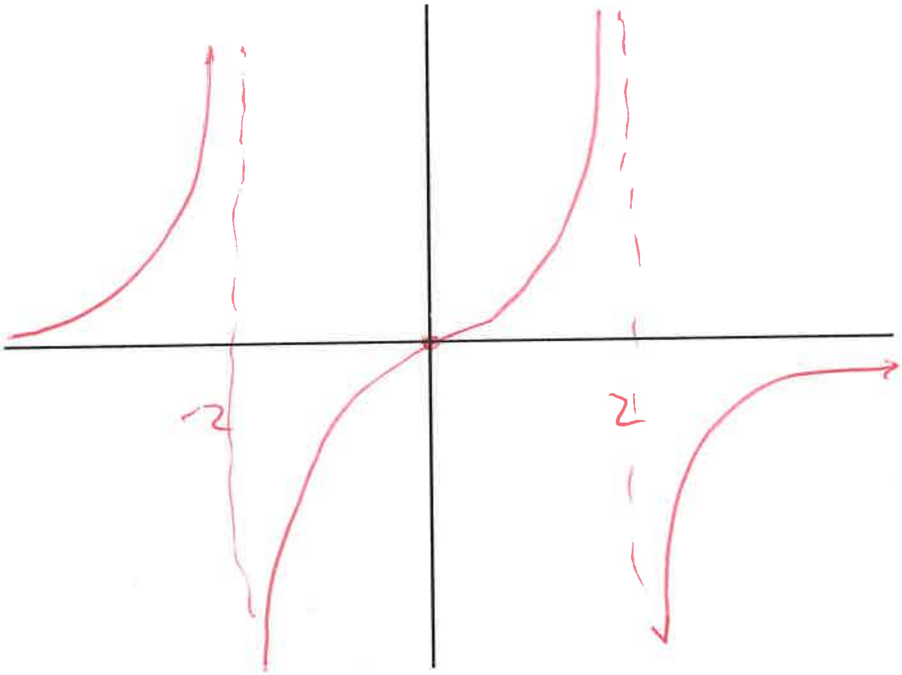
5. Sketch the graph of a continuous function with the following information:

	$x<-3$	$x=-3$	$-3<x<1$	$x=1$	$1<x<5$	$x=5$	$5<x$
$f'(x)$	Positive	DNE	Positive	DNE	Positive	0	Positive
$f''(x)$	Positive	DNE	Positive	DNE	Negative	0	Positive



6. Set up a Key Trait table and sketch $y=\frac{-x}{x^2-4}$

x		-2		0		2	
y'	+	DNE	+	+	+	DNE	+
y''	+	DNE	−	0	+	DNE	−
Conclusion		VA		POI		VA	



7. Set up a Key Trait table and sketch $y=\left(\frac{1}{5}e^{-x}\right)\sqrt{9-x^2}$

x	-3		-2.541 ~~-1.578~~		-1.518		3
y'	DNE	+	0	–	–	–	DNE
y''	DNE	–	–	–	0	+	DNE
Con	END PT	(	MAX	)	POI	(	END PT