

Honors PreCalculus '17-18
Fall Take Home Midterm
Show all work. Round to 3 decimals.

Name SOLUTION KEY
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1. If $\csc B = -\frac{5}{3}$ in QIV, find the other five exact trig values and angle B:

$$\sin B = -\frac{3}{5}$$

$$\csc B = -\frac{5}{3}$$

$$y = -3$$

$$r = 5$$

$$\cos B = \frac{4}{5}$$

$$\sec B = \frac{5}{4}$$

$$x = 4$$

$$\tan B = -\frac{3}{4}$$

$$\cot B = -\frac{4}{3}$$

$$B = -0.644 \pm 2\pi$$

2. $(-12, -7)$ is on the terminal side of A. Find the six exact trig values and angle A:

$$A: \quad r = \sqrt{193}$$

$$\sin A = -\frac{7}{\sqrt{193}}$$

$$\csc A = -\frac{\sqrt{193}}{7}$$

$$\cos A = -\frac{12}{\sqrt{193}}$$

$$\sec A = -\frac{\sqrt{193}}{12}$$

$$\tan A = \frac{7}{12}$$

$$\cot A = \frac{12}{7}$$

$$A = -2.614 \pm 2\pi$$

(or $3.670 \pm 2\pi$)

3. Using the exact values found in #1 and #2, find the exact values of:

a) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$= \frac{-7}{\sqrt{193}} \left(\frac{4}{5}\right) - \left(\frac{-12}{\sqrt{193}}\right) \left(\frac{-3}{5}\right) = \frac{-64}{5\sqrt{193}}$$

b) $\sec(A+B) = \frac{1}{\cos A \cos B - \sin A \sin B}$

$$= \frac{1}{\left(\frac{-12}{\sqrt{193}}\right) \left(\frac{4}{5}\right) - \left(\frac{-7}{\sqrt{193}}\right) \left(\frac{-3}{5}\right)}$$

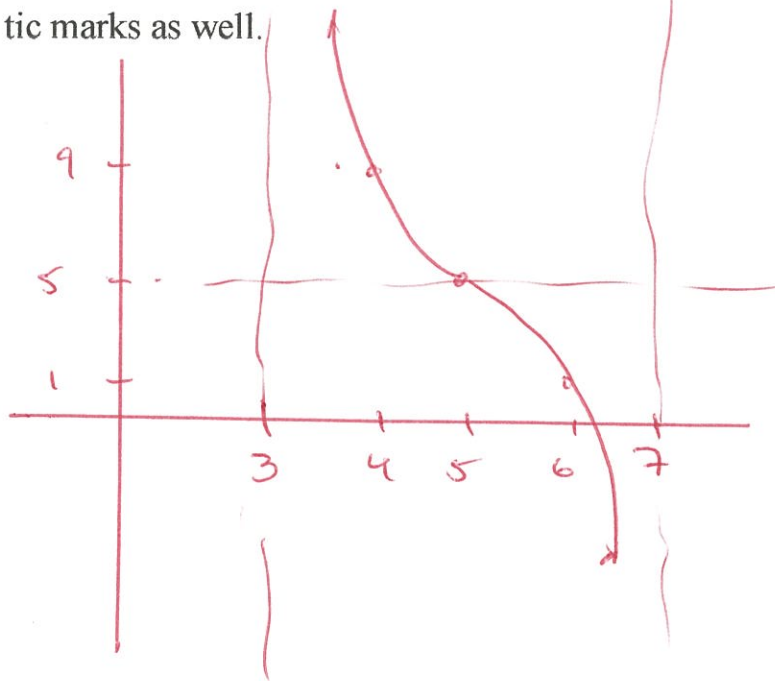
$$= \frac{5\sqrt{139}}{-69}$$

c) $\tan(2B) = \frac{2 \tan B}{1 - \tan^2 B} = \frac{2 \left(-\frac{3}{4}\right)}{1 - \left(-\frac{3}{4}\right)^2}$

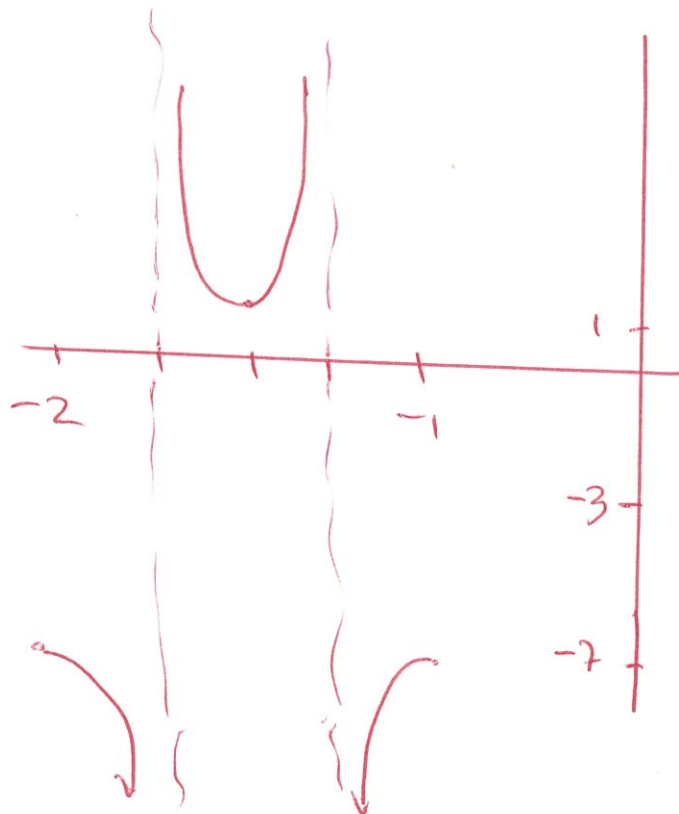
$$= \frac{-3/2}{1 - 9/16} = \frac{-3}{2} \cdot \frac{16}{7}$$

$$= \frac{-24}{7}$$

4. Sketch a graph of the function $y = 5 + 4 \cot\left[\frac{\pi}{4}(x-3)\right]$. State k, A, B, and h. Label the 5 tic marks as well.



5. Sketch a graph of the function $y = -3 - 4 \sec[2\pi(x+2)]$. State k, A, B, and h. Label the 5 tic marks as well.



6. Find the exact value

$$\begin{aligned} \text{a) } \tan^2 \frac{\pi}{3} - \sec^2 \frac{2\pi}{3} &= (\sqrt{3})^2 - (-2)^2 \\ &= 3 - 4 \\ &= -1 \end{aligned}$$

$$\begin{aligned} \text{b) } \csc \frac{2\pi}{3} \tan \frac{4\pi}{3} \cos \frac{11\pi}{6} + \sin \frac{4\pi}{3} \\ &= \left(\frac{2}{\sqrt{3}}\right) (\sqrt{3}) \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{-\sqrt{3}}{2}\right) \\ &= \sqrt{3} + \left(\frac{-\sqrt{3}}{2}\right) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \text{c) } \cos \frac{5\pi}{6} \cos \frac{2\pi}{3} + \sin \frac{5\pi}{6} \sin \frac{2\pi}{3} \\ &= \left(\frac{-\sqrt{3}}{2}\right) \left(\frac{-1}{2}\right) + \left(\frac{1}{2}\right) \left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

7. A patient in a hospital is experiencing fluctuations in his blood pressure. His pressure seems to be varying sinusoidal with time. Over a period of 24 hours, it varies from 90 to 150 mmHg, and it cycles twice through this 24-hour period. At 9am the pressure is 120 mmHg and on its way up.

a) Sketch a graph of the pressure $P(t)$ as a function of time.



b) Write a sinusoidal equation that describes $P(t)$ as a function of time t .

$$P(t) = 120 + 30 \cos \frac{\pi}{6}(t - 12)$$

$$\text{or } P(t) = 120 + 30 \sin \frac{\pi}{6}(t - 9)$$

c) What would you expect the pressure to be at 1:15pm?

$$1:15 \Rightarrow t = 13.25$$

$$P(13.25) = 143.8$$

d) What are the first three times that $P(t) = 130$?

$$130 = 120^{+30} \cos \frac{\pi}{6} (t-12)$$

$$\frac{1}{3} = \cos \frac{\pi}{6} (t-12)$$

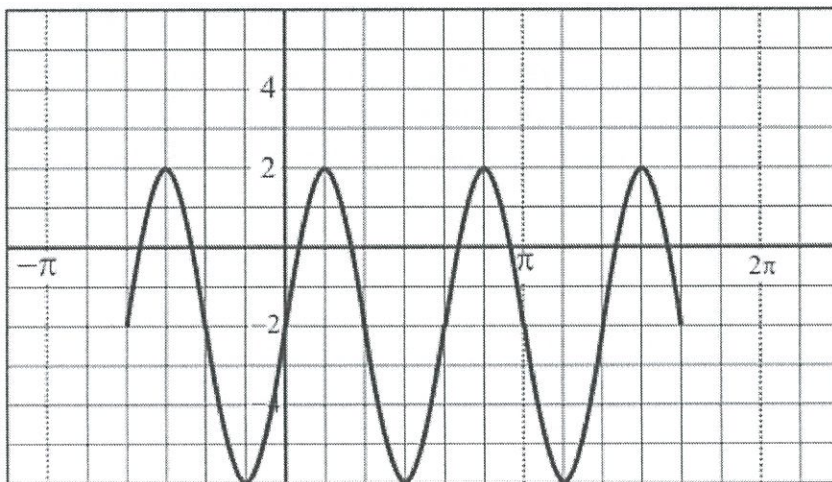
$$\left. \begin{array}{l} 1.231 \pm 2\pi n \\ -1.231 \pm 2\pi n \end{array} \right\} = \frac{\pi}{6} (t-12)$$

$$\left. \begin{array}{l} 2.351 \pm 12n \\ -2.351 \pm 12n \end{array} \right\} = t-12$$

$$t = \begin{cases} 14.351 \pm 12n \\ 9.649 \pm 12n \end{cases}$$

$$t = 2.351, 9.649, 14.351$$

8. Find one cosine function and one sine function for the graph below.



$$y = -2 + 4 \sin 3x$$

$$y = -2 + 4 \cos \left[3 \left(x - \frac{\pi}{6} \right) \right]$$

9. Prove: $\frac{\sin x}{1-\cos x} + \frac{1-\cos x}{\sin x} = 2\csc x$

$$\frac{\sin^2 x}{\sin x(1-\cos x)} + \frac{(1-\cos x)^2}{\sin x(1-\cos x)}$$

$$\frac{\sin^2 x + 1 - 2\cos x + \cos^2 x}{\sin x(1-\cos x)} = 1$$

$$\frac{2 - 2\cos x}{\sin x(1-\cos x)}$$

$$\frac{2(1-\cos x)}{\sin x(1-\cos x)}$$

$$\frac{2}{\sin x} = 2\csc x$$

10. Solve $2\tan^2 x - 3\sec x = 0$ for $x \in [-\pi, 3\pi)$

$$2(\sec^2 x - 1) - 3\sec x = 0$$

$$2\sec^2 x - 3\sec x - 2 = 0$$

$$(2\sec x + 1)(\sec x - 2) = 0$$

$$\sec x = -\frac{1}{2}$$

NO SOL

$$\sec x = 2$$

$$\cos x = \frac{1}{2}$$

$$x = \pm \frac{\pi}{3} + 2\pi n$$

$$x \in \left\{ -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3} \right\}$$