Honors PreCalculus
Dr. Quattrin
Limits and Derivatives Test
CALCULATOR ALLOWED
Round to 3 decimal places. Show all work.

Name: Solution Key

Score 20

1. Find the instantaneous rate of change of $f(x)=x^2-\frac{1}{2x}$ at $x=-1$.

$f' = 2x+\frac{1}{2}x^{-2}$ $\quad f'(-1) = -2+\frac{1}{2}$

a) 0 b) $-\frac{3}{2}$ c) $\frac{3}{2}$ d) $-\frac{5}{2}$ e) $\frac{5}{2}$

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2. If $f(x)=x^2+3x+2$, then $\lim_{x\to 4}\frac{f(x)-f(4)}{x-4}=$ $\left.\frac{dy}{dx}\right|_{x=4} = 2(4)+3$

a) 0 b) $-\frac{3}{2}$ c) 11 d) 30 e) None of these

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3. Suppose f is a differentiable function such that $f(-1)=2$ and $f'(-1)=\frac{1}{2}$. Using the line tangent to the graph of $f(x)$ at $x=-1$, find the approximation of $f(1.1)$

$y-2=\frac{1}{2}(x+1)$

a) −3.05 b) −1.95 c) .95 d) 1.95 e) 3.05

$f(1.1) \approx \frac{1}{2}(1.1+1)+2=$

$m_{TAN} = \frac{-2}{x^2} = -2$

4. Find an equation of the normal line to the curve $f(x)=\frac{2}{x}$ at $x=1$.

$m_{NORM} = \frac{1}{2}$

(a) $y=\frac{1}{2}x+\frac{3}{2}$ (b) $y=-\frac{1}{2}x$ (c) $y=\frac{1}{2}x+2$

(d) $y=-\frac{1}{2}x+2$ (e) $y=2x+5$

$f(1)=2$

$y-2=\frac{1}{2}(x-1)$

$y=\frac{1}{2}x+3/2$

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5. Let $f(x)=x-\frac{1}{x}$. Find $f''(x)$.

$f'(x)=1+\frac{1}{x^2}$ $f''=-2x^{-3}$

a) $1+\frac{1}{x^2}$ b) $1-\frac{1}{x^2}$ c) $\frac{2}{x^3}$

d) $-\frac{2}{x^3}$ e) Does not exist

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6. A particle moving in a straight line such that $x(t)=2+7t-t^2$. When is the particle at rest?

$v(t)=7-2t=0$

$t=7/2$

a) $t=1$ b) $t=2$ c) $t=\frac{7}{2}$

d) $t=4$ e) $t=5$

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1. Use the equation of the line tangent to $y = 6x^3 - 3x^2 + 5x - 4$ at $x = -1$ to approximate $f(-1.1)$

$y(-1) = -18$ $\quad \frac{dy}{dx} = 18x^2 - 6x + 5$

$m = 29$

$y + 18 = 29(x+1)$

$f(-1.1) \approx 29(-1.1+1) - 18 = -20.9$

2. The motion of a particle is described by $x(t) = -3t^3 + 7t^2 - t + 1$.

a) When the particle is stopped?
b) Which direction it is moving at $t = 7$?
c) Where is it when $v(t) = 0$?
d) Find $a(7)$.

$v = -9t^2 + 14t - 1 = 0$

$t = \frac{-14 \pm \sqrt{14^2 - 36}}{-18}$

$a = -18t + 14$

a) $t = .075, 1.481$

b) $v(7) = -344 \therefore$ Left

c) $x(.075) = .963$

$x(1.481) = 5.127$

d) $a(7) = -112$

3. A particle's position $\langle x(t), y(t)\rangle$ at time t is described by $\langle t^3 + t^2 - 2t + 1, -35t + 2\rangle$.

a) Find the speed at $t = 3$.

$v = \langle 3t^2 + 2t - 2, -35\rangle$

$S = \sqrt{(3(3)^2 + 2(3) - 2)^2 + (-35)^2} = 46.755$

b) When, if ever, is the particle stopped? Prove it.

$v_y = -35$ ∴ IT IS ALWAYS MOVING

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5. Set up, but do not solve, the limit definition of the derivative for
$y=\sqrt[3]{x^6}+\frac{4}{x^4}-2\sqrt[5]{x^9}-4^2$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{\left((x+h)^{6/3}+4(x+h)^{-4}-2(x+h)^{9/5}-4^2\right)-\left(x^2+4x^{-4}-2x^{9/5}-4^2\right)}{h}$$

6. Use the Power Rule to find:

a) $\frac{d}{dx}\left[-12x^4+7x^3+\pi x^2+42x\right]=-48x^3+21x^2+2\pi x+42$

b) $D_x\left[\sqrt[3]{x^6}+\frac{4}{x^4}-2\sqrt[5]{x^9}-4^2\right]=2x-16x^{-5}-\frac{18}{5}x^{+4/5}$

7. Evaluate the following limits:

a) $\lim\limits_{x\to 4}\dfrac{2x^3-128}{x^4-256}$ =

$$\begin{array}{r|rrrr} 4 & 2 & 0 & 0 & -128 \\ & & 8 & 32 & 128 \\ \hline & 2 & 8 & 32 & \end{array}$$

$$= \lim_{x\to 4}\frac{\cancel{(x-4)}(2x^2+8x+32)}{\cancel{(x-4)}(x^3+4x^2+16x+64)}$$

$$\begin{array}{r|rrrrr} 4 & 1 & 0 & 0 & 0 & \cancel{256}\ -256 \\ & & 4 & 16 & 64 & 256 \\ \hline & 1 & 4 & 16 & 64 & \end{array}$$

$$= \frac{32+32+32}{64+64+64+64} = \frac{96}{256} = \frac{3}{8}$$

b) $\lim\limits_{x\to 2}\dfrac{x^4-5x^3-7x^2+3x+1}{x^3+7x-22}$ =

$$\begin{array}{r|rrrrr} 2 & 1 & -5 & -7 & 3 & 1 \\ & & 2 & -6 & -26 & -46 \\ \hline & 1 & -3 & -13 & -23 & \end{array}$$

$$\cancel{\lim_{x\to 2}} = \frac{-45}{0}$$

= DNE

EC) $\lim\limits_{x\to 2}\dfrac{(\sqrt{x^2+1}-\sqrt{5})(\sqrt{x^2+1}+\sqrt{5})}{(x^2-3x+2)(\sqrt{x^2+1}+\sqrt{5})}$

$$= \lim_{x\to 2}\frac{x^2+1-5}{(x-1)(x-2)(\sqrt{x^2+1}+\sqrt{5})}$$

$$= \lim_{x\to 2}\frac{\cancel{(x-2)}(x+2)}{(x-1)\cancel{(x-2)}(\sqrt{x^2+1}+\sqrt{5})} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$$