PreCalculus Honors '20-21
Dr. Quattrin
Radical Functions Test
CALCULATOR ALLOWED
Round to 3 decimal places. Show all work.

Name: Solution Key

Score ___________

1. If $y=\sqrt{x^2+5}$, then $\frac{dy}{dx}=$ $\frac{1}{2}(x^2+5)^{-1/2}(2x)$

a) $\frac{1}{2\sqrt{x^2+5}}$

b) $\frac{1}{2\sqrt{2x}}$

c) $\frac{x}{\sqrt{x^2+5}}$

d) $\sqrt{2x}$

---

2. If $y=\sqrt{\frac{-4x}{x^2+1}}$, then $\frac{dy}{dx}=$ $\frac{1}{2}\left(\frac{-4x}{x^2+1}\right)^{-1/2}\left[\frac{(x^2+1)(-4)-(-4x)(2x)}{(x^2+1)^2}\right]$

$= \frac{(x^2+1)^{1/2}(4x^2-4)}{2(-4x)^{1/2}(x^2+1)^{4/2}}$

~~a)~~ $\sqrt{\frac{-2}{x}}$

~~b)~~ $-\frac{1}{2}\left(\frac{-4x}{x^2+1}\right)^{-1/2}$

c) $\frac{x^2-1}{(-x)^{1/2}(x^2+1)^{3/2}}$

d) $\frac{(-4x)^{1/2}(2x^2-2)}{(x^2+1)^{3/2}}$

~~e)~~ $\frac{4x^2-4}{x^2+1}$

---

3. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	8	8	2	3
8	2	-12	4	4

Given that $h(x)=f(f(x))$, $h'(2)=$ $f'(f(2)) \cdot f'(2)$

$= f'(4) \cdot f'(2) = 8\,(-2)$

a) -16 b) -12 c) -1 d) 2 e) 10

---

4. At what point on the curve $x^2-y^2+x=2$ is the tangent line vertical?

DENOM $=0$

a) $(1, 0)$ only
b) $(-2, 0)$ only
c) $(1, \sqrt{2})$ only
d) $(1, 0)$ and $(-2, 0)$
e) The tangent line is never vertical.

$2x - 2y\frac{dy}{dx} + 1 = 0$

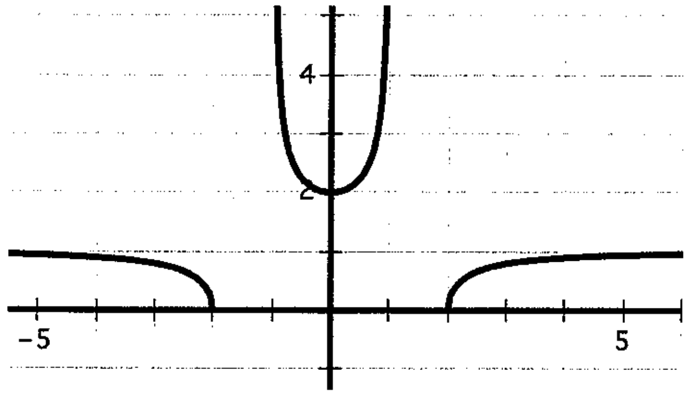
$\frac{dy}{dx} = \frac{-1-2x}{-2y}$

$y = 0 \rightarrow x^2 + x - 2 = 0$

$\cancel{x^2} + x$

---

5. Which of the following equations matches the graph below?



Zeros $(\pm 2, 0)$

VAs: ± 1

a) $y=\sqrt{\dfrac{x^2-4}{x^2-1}}$ b) $y=\sqrt{\dfrac{x^2-1}{x^2-4}}$ c) $y=\sqrt{\dfrac{4-x^2}{x^2-1}}$

d) $y=\sqrt{\dfrac{1-x^2}{x^2-4}}$ e) $y=\sqrt{\dfrac{x^2-4}{x^2}}$

6. What is the end behavior of $y = \sqrt{3x(14-x^2)}$?

a) Up on the left and down on the right

b) None on the left and none on the right

c) Up on the left and none on the right

d) Up on both ends

e) Down on the left and none on the right

---

7. The absolute minimum of $y = -\sqrt{9x - x^3}$ on $x \in [-4, 3]$ is

a) 4 b) 0 c) $2\sqrt{7}$ d) $-2\sqrt{7}$ e) ± 3

$$\frac{dy}{dx} = -\frac{1}{2}(9x - x^3)^{-1/2}(9 - 3x^2)$$

i) $\frac{dy}{dx} = 0 \rightarrow x^2 = 3 \rightarrow x = \pm\sqrt{3} \rightarrow$

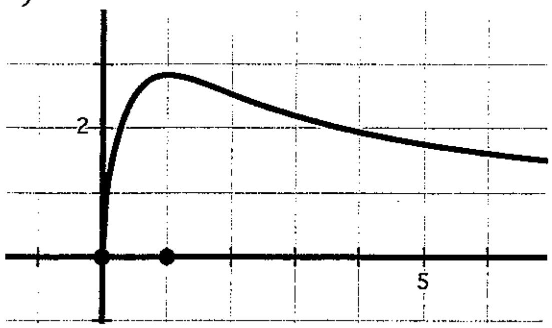
ii) $\frac{dy}{dx}$ DNE $\rightarrow x = 0, \pm 3 \rightarrow y = 0$

---

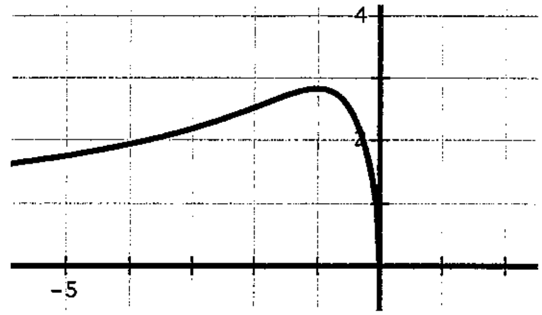
iii) ENDS: $x = -4, 3$ $\quad x = -4 \rightarrow y = -2\sqrt{7}$

8. Which of the following graphs match the equation $y=\sqrt{\dfrac{x^2}{x^2-4}}$?

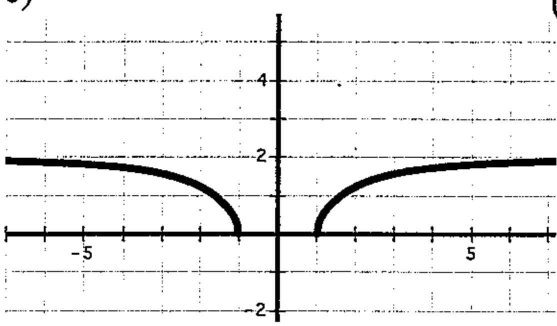
a)



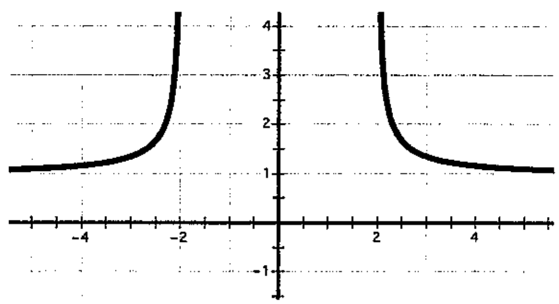
b)



c)



d)



---

9. The MythBusters explode a demolition charge in midair such that the volume of the expanding sphere of the blast is increasing at a rate of 12 cubic feet per second. When the volume of the sphere is 36π, how fast, in square feet per second, is the surface area increasing?

a) 8 b) $\dfrac{1}{3\pi}$ c) $\dfrac{8}{\pi}$ d) 8π e) 24π

$$V = \frac{4}{3}\pi r^3 = 36\pi \Rightarrow r = 3$$

$$12 = \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \qquad A = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{1}{3\pi} = \frac{dr}{dt} \qquad \frac{dA}{dt} = \cancel{12}\,8\pi r \frac{dr}{dt} = \cancel{18}\,8\pi(3)\left(\frac{1}{3\pi}\right) =$$

PreCalculus Honors '20-21 Name: Solution Key

Dr. Quattrin

Radical Functions Test -- CALCULATOR ALLOWED

Round to 3 decimal places. Score ________

Show all work.

1. Find the zeros and Domain of $y=\sqrt{x^3-7x^2+8x+16}$ on $x\in[-1,6]$. Show the algebraic work to support the zeros.

Zeros: $(-1,0)\,(4,0)$

Domain: $x \in [-1, 6]$

$$\begin{array}{r|rrrr} -1 & 1 & -7 & 8 & 16 \\ & & -1 & 8 & -16 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

$(x+1)(x-4)^2$

y^2: $-\ 0\ +\ 0\ +$ at -1, 4

2. Find the Extreme Points of $y=\sqrt{x^3-7x^2+8x+16}$ on $x\in[-1,6]$. Show the algebraic work to support the critical values.

Extreme Points: $(2/3, 4.383)$
$(-1,0)\ (4,0)$
$(6, 5.292)$

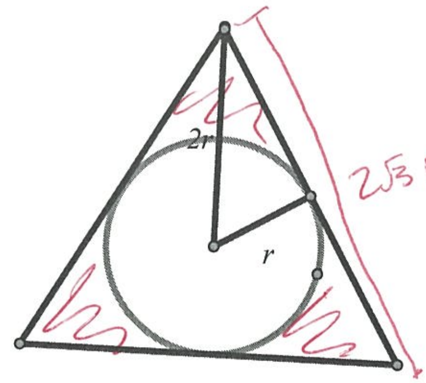
$$\frac{dy}{dx}=\frac{3x^2-14x+8}{2(x^3-7x^2+8x+16)^{1/2}}$$

i) $\frac{dy}{dx}=0 \rightarrow x=\frac{16\pm\sqrt{16^2-4(3)(8)}}{6}=\frac{16\pm\sqrt{160}}{6}=2/3, 4$

ii) $\frac{dy}{dx}$ DNE $\rightarrow x=-1, 4$

iii) $-1, 6,$

5. A circle is inscribed in an equilateral triangle. The circle's circumference is expanding at 6π in/sec and the triangle maintains the contact of its side with the circle.



The area of an equilateral triangle is equal to half its apothem a times the perimeter p, and, in this case, the apothem is equal to the radius of the circle. How fast the area outside the circle but inside the triangle is expanding when the area of the circle is $64\pi\ in$. [NB. The small right triangle is a 30-60-90 triangle.]

$C = 2\pi r$

$\frac{dC}{dt} = 6\pi = 2\pi \frac{dr}{dt}$

$3 = \frac{dr}{dt}$

$A_0 = 64\pi \rightarrow r = 8$

$A_{SHADE} = \frac{1}{2}(6\sqrt{3}r)(r) - \pi r^2$

$\frac{d}{dt}\left[A = (3\sqrt{3} - \pi) r^2\right]$

$\frac{dA}{dt} = (3\sqrt{3} - \pi)(2r)\frac{dr}{dt}$

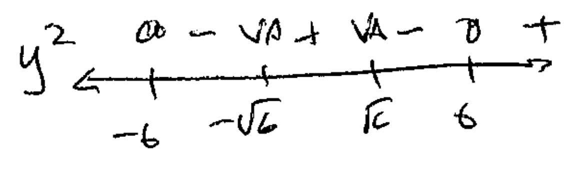
$= (3\sqrt{3} - \pi)\, 16\, (3)$

$= 98.619 \qquad = 144\sqrt{3} - 48\pi$

3. Find the zeros, VAs, and domain of $y=\sqrt{\dfrac{x^2-36}{x^2-6}}$. Show the Algebra that supports your answer.

Zeros: $(\pm 6, 0)$ VAs: $x = \pm\sqrt{6}$

Domain: $x \in (-\infty, -6] \cup (-\sqrt{6}, \sqrt{6}) \cup [6, \infty)$



4. Find the Extreme Points of $y=\sqrt{\dfrac{x^2-36}{x^2-6}}$. Show the Algebra and derivative that supports your answer.

Extreme Points: $(0, \sqrt{6})$, $(\pm 6, 0)$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{x^2-36}{x^2-6}\right)^{-1/2}\left[\frac{(x^2-6)(2x)-(x^2-36)(2x)}{(x^2-36)^{2}}\right]$$

$$= \frac{(x^2-6)^{1/2}[60x]}{2(x^2-36)^{1/2}(x^2-6)^2}$$

$$= \frac{30x}{(x^2-36)^{1/2}(x^2-6)^{3/2}}$$

6. A particular velocity function is given by the equation

$v(t)=\sqrt{\left[\left(\frac{C(t)}{7}+4t\right)^{5/7}-1\right]}$. What is the equation for the acceleration?

$$= \left(\left[\frac{c(t)}{7}+4t\right]\right)^{5/7}-1\Big)^{1/2}$$

$$v' = \frac{1}{2}\left(\left(\frac{c(t)}{7}+4t\right)^{5/7}-1\right)^{-1/2}\left(\frac{5}{7}\left(\frac{c(t)}{7}+4t\right)^{-2/7}\left(\frac{c'(t)}{7}+4\right)\right)$$

PreCalculus Honors '20-21 Name:______________________

Dr. Quattrin

Radical Functions Test – NO CALCULATOR ALLOWED

Show all work.

7. Find the traits and **sketch** $y=\sqrt{x^3-7x^2+8x+16}$ on $x\in[-1, 6]$.

Domain: $x \in [-1, 6]$

Range: $y \in [0, 5.292]$

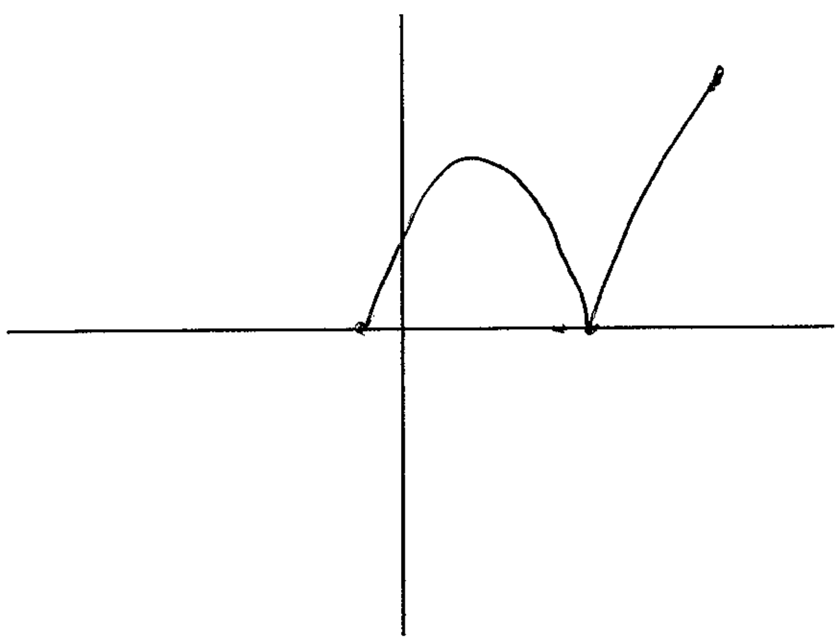
Y – Int: $(0, 4)$

Zeros: $(-1, 0)$ $(4, 0)$

Extreme Points: SEE #2

End Behavior (Left): NONE

End Behavior (Right): NONE



8. List the traits and **sketch** of $y=\sqrt{\frac{x^2-36}{x^2-6}}$.

Domain: $x \in \mathbb{R} \neq 3$

Range: $y \in [0,1) \cup [\sqrt{6}, \infty)$

Y – Int: $(0, \sqrt{6})$

VAs: $x = \pm\sqrt{6}$

Zeros: $(\pm 6, 0)$

Extreme Points: $(0, \sqrt{6})$, $(\pm 6, 0)$

End Behavior (Left): $y = 1$

End Behavior (Right): $y = 1$