Honors PreCalculus '21-22
Trig Identities Group Test
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For Problem 1-6, use:

(20, -21) is on the terminal side of C and $\sec Q = -\dfrac{13}{12}$ in QIII

$\cos C = \dfrac{20}{29}$ $\sin C = \dfrac{-21}{29}$ $\tan C = \dfrac{-21}{20}$

$\sin Q = \dfrac{-5}{13}$ $\cos Q = -\dfrac{12}{13}$ $\tan Q = \dfrac{5}{12}$

to find the exact values of:

1. sin (C+Q)

$= \sin C \cos Q + \cos C \sin Q$

$= \left(\frac{-21}{29}\right)\left(\frac{-12}{13}\right) + \left(\frac{20}{29}\right)\left(\frac{-5}{13}\right)$

$= \frac{152}{377}$

2. cos (C-Q)

$= \cos C \cos Q + \sin C \sin Q$

$= \frac{20}{29}\left(\frac{-12}{13}\right) + \left(\frac{-21}{29}\right)\left(\frac{-5}{13}\right)$

$= \frac{-135}{377}$

3. $\tan(C+Q)$

$= \dfrac{\tan C + \tan Q}{1 - \tan C \tan Q}$

$= \dfrac{\frac{-21}{20} + \frac{5}{12}}{1 - \left(\frac{-21}{20}\right)\left(\frac{5}{12}\right)} = \frac{-152}{240} \cdot \frac{240}{345}$

$= -\frac{152}{345}$

4. sec (2C) $= \dfrac{1}{\cos^2 C - \sin^2 C}$

$= \dfrac{1}{\left(\frac{20}{29}\right)^2 - \left(\frac{-21}{29}\right)^2} = \frac{841}{-41}$

5. cot (2C) $= \dfrac{1 - \left(-21/20\right)^2}{2\left(-21/20\right)} =$

$= \frac{-41}{400} \cdot \frac{10}{-21} = \frac{41}{840}$

6. $\csc 2C$ $= \dfrac{1}{2 \sin C \cos C}$

$= \dfrac{1}{2\left(\frac{-21}{29}\right)\left(\frac{20}{29}\right)}$

$= -\frac{841}{840}$

7. Prove: $\frac{\cos^4 x - \sin^4 x}{\cos^3 x - \sin^3 x} = + \frac{\cos x + \sin x}{1 + \sin x \cos x}$

$$\frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{(\cos x - \sin x)(\cos^2 x + \sin x \cos x + \sin^2 x)}$$

$$\frac{\cancel{(\cos x - \sin x)}(\cos x + \sin x)(1)}{\cancel{(\cos x - \sin x)}(1 + \sin x \cos x)}$$

9. Solve for $\theta \in [0°, 360°]$: $\cos\theta \sin\theta = \frac{\sqrt{3}}{4}$

$$2\sin\theta\cos\theta = \frac{\sqrt{3}}{2}$$

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$$2\theta = \begin{cases} 60 \pm 360n \\ 120 \pm 360n \end{cases}$$

$$\theta = \begin{cases} 30 \pm 180n \\ 60 \pm 180n \end{cases}$$

$$\theta = \{30, 210, 60, 240\}$$

8. Solve for $x \in [-360°, 360°]$:
$3 - 3\sin x - 2\cos^2 x = 0$
$(1 - \sin^2 x)$

$$2\sin^2 x - 3\sin x + 1 = 0$$

$$(2\sin x - 1)(\sin x - 1) = 0$$

$\sin x = 1/2$ $\sin x = 1$

$$x = \begin{cases} 30 \pm 360n \\ 150 \pm 360n \end{cases}$$ $x = 90 \pm 360n$

$$x \in \{+30, -330, 150, -210, 90, -270\}$$

10. Prove: $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$

$$\frac{\sin x}{1/\sin x} + \frac{\cos x}{1/\cos x}$$

$$\sin^2 x + \cos^2 x$$

$$1$$

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For Problem 1-6, use:

(20, -21) is on the terminal side of C and
$\sec Q = -\frac{13}{12}$ in QIII
to find the exact values of:

1. sin (2Q) $= 2\sin Q \cos Q$

$= 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)$

$= \frac{120}{169}$

2. cos (2Q) $= \cos^2 Q - \sin^2 Q$

$= \left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{119}{169}$

3. $\tan(2Q)$ $= \frac{2\tan Q}{1-\tan^2 Q}$

$= \frac{2(5/12)}{1-(5/12)^2} = \frac{5/6}{1-25/144}$

$= \frac{5}{6} \cdot \left(\frac{144}{119}\right)$

$= \pm\frac{120}{119}$

4. sec (C+Q) $= \frac{1}{\cos C \cos Q \mp \sin C \sin Q}$

$= \frac{1}{\left(\frac{20}{29}\right)\left(\frac{-12}{13}\right) - \left(\frac{-21}{29}\right)\left(\frac{-5}{13}\right)}$

$= \frac{1}{\frac{-240 - 105}{377}} = \frac{-377}{345}$

5. cot (C-Q) $= \frac{1 + \tan C \tan Q}{\tan C - \tan Q}$

$= \frac{1 + \left(\frac{-21}{20}\right)\left(\frac{5}{12}\right)}{\frac{-21}{20} - \frac{5}{12}} = \frac{\frac{\cancel{225}\,-135}{240}}{\frac{-352}{240}}$

$= -\frac{\cancel{225}\,-135}{352}$

6. $\csc(Q+C)$ $= \frac{1}{\sin Q \cos C + \cos Q \sin C}$

$= \frac{1}{\left(\frac{-21}{29}\right)\left(\frac{-12}{13}\right) + \left(\frac{20}{29}\right)\left(\frac{-5}{13}\right)}$

$= \frac{377}{152}$

7. Prove:
$$\frac{1-\csc\alpha}{2\csc\alpha+1} = \frac{\sin^2\alpha+\cos^2\alpha-\csc^2\alpha}{2\cot^2\alpha+3\csc\alpha+3}$$

$$= \frac{1-\csc^2\alpha}{2(\csc^2\alpha-1)+3\csc\alpha+3}$$

$$= \frac{(1-\csc\alpha)(1+\csc\alpha)}{2\csc^2\alpha+3\csc\alpha+1}$$

$$= \frac{(1-\csc\alpha)\cancel{(1+\csc\alpha)}}{(2\csc\alpha+1)\cancel{(\csc\alpha+1)}}$$

8. Solve for $\theta\in[0^\circ,360^\circ]$: $\cos 2\theta=\cos\theta$

$$2\cos^2\theta-1-\cos\theta=0$$

$$(2\cos\theta+1)(\cos\theta-1)=0$$

$$\cos\theta=\frac{-1}{2} \qquad \cos\theta=1$$

$$\theta=\pm120\pm360n \qquad \theta=0\pm360n$$

$$\theta=\{0,\ 360^\circ,\ 120^\circ,\ 240^\circ\}$$

9. Solve for $x\in[0,\ 60^\circ]$:
$$\frac{\cos x\cos 20^\circ-\sin x\sin 20^\circ}{\sin x\cos 20^\circ+\cos x\sin 20^\circ}=\sqrt{3}$$

$$\frac{\cos(x+20)}{\sin(x+20)}=\sqrt{3}$$

$$\cot(x+20)=\sqrt{3}$$

$$\tan(x+20)=1/\sqrt{3}$$

$$x+20^\circ=\cancel{60}\ 30\pm180n$$

$$x=\cancel{20}\ 10\pm180n$$

$$x\in\{10\}$$

10. Prove: $\cot A+\tan A=2\csc 2A$

$$\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}=\frac{2}{\sin 2A}$$

$$\frac{\cos^2A+\sin^2A}{\sin A\cos A}=\frac{2}{2\sin A\cos A}$$

$$\frac{1}{\sin A\cos A}$$

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For Problem 1-6, use:

(20, -21) is on the terminal side of C and
$\sec Q = -\frac{13}{12}$ in QIII
to find the exact values of:

1. sin (2C) $= 2\sin C\cos C$

$= 2\left(\frac{-21}{29}\right)\left(\frac{20}{29}\right)$

$= -\frac{840}{841}$

2. cos (2C)

$= \cos^2 C - \sin^2 C$

$= \left(\frac{20}{29}\right)^2 - \left(\frac{-21}{29}\right)^2$

$= \frac{-41}{841}$

3. $\tan(2C)$ $= \frac{2\tan C}{1-\tan^2 C}$

$= \frac{2(-21/20)}{1-(-21/20)^2} = \frac{-21}{10} \cdot \frac{400}{-41}$

$= \frac{840}{41}$

4. sec (Q-C) $= \frac{1}{\cos Q\cos C + \sin Q\sin C}$

$= \frac{1}{\left(\frac{-12}{13}\right)\left(\frac{20}{29}\right) + \left(\frac{\cancel{-5}}{13}\right)\left(\frac{-21}{29}\right)}$

$= \frac{377}{-135}$

5. cot (Q-C) $= \frac{1+\cancel{\cot}\tan Q\,\cancel{\cot}\tan C}{\cancel{\cot}\tan Q - \cancel{\cot}\tan C}$

$= \frac{1+\left(\frac{5}{12}\right)\left(\frac{-21}{20}\right)}{\frac{5}{12}-\left(\frac{-21}{20}\right)} = \frac{\frac{-135}{240}}{\frac{352}{240}}$

$= +\frac{135}{352}$

6. $\csc(Q-C)$ $= \frac{1}{\sin Q\cos C - \cos Q\sin C}$

$= \frac{1}{\left(\frac{-5}{13}\right)\left(\frac{20}{29}\right) - \left(\frac{-21}{29}\right)\left(\frac{-12}{29}\right)}$

$= \frac{377}{-352}$

7. Prove: $\dfrac{\sec^2\lambda+3\csc\lambda\sin\lambda-5}{\tan^2\lambda-4\tan\lambda+3}=\dfrac{1+\tan\lambda}{\tan\lambda-3}$

$$\frac{\tan^2\lambda+1+3-5}{\tan^2\lambda-4\tan\lambda+3}$$

$$\frac{\tan^2\lambda-1}{(\tan\lambda-3)(\tan\lambda-1)}$$

$$\frac{\cancel{(\tan\lambda-1)}(\tan\lambda+1)}{(\tan\lambda-3)\cancel{(\tan\lambda-1)}}$$

9. Solve for A: $\cos^2(2A)-\sin^2(2A)=\dfrac{\sqrt{3}}{2}$

$$\cos 2(2A)=\frac{\sqrt{3}}{2}$$

$$\cos^{-1}\left[\cos 4A=\frac{\sqrt{3}}{2}\right]$$

$$4A=\begin{cases}\pi/6 \pm 2\pi n\\ -\pi/6 \pm 2\pi n\end{cases}$$

$$A=\begin{cases}\pi/24 \pm \frac{\pi}{2}n\\ -\pi/24 \pm \frac{\pi}{2}n\end{cases}$$

8. Solve exactly for $x\in[-120, 120^\circ]$:
$\sec^2(3x)+\tan(3x)=1$

$$\tan^2 3x+1+\tan 3x=1$$

$$\tan^2 3x+\tan 3x=0$$

$$\tan 3x(\tan 3x+1)=0$$

$\tan 3x=0$ OR $\tan 3x=-1$

$3x=0^\circ\pm180^\circ n$ $\quad$ $3x=-45^\circ\pm180^\circ n$

$x=0^\circ\pm60^\circ n$ $\quad$ $x=-15^\circ\pm60^\circ n$

$$x\in\{-15, 0, 60, 120, -120, -60, 45, -75\}$$

10. Prove: $\dfrac{2}{1+\cos x}=2\csc^2 x-2\cot x\csc x$

$$=\frac{2}{\sin^2 x}-\frac{2\cos x}{\sin^2 x}$$

$$=\frac{2-2\cos x}{1-\cos^2 x}$$

$$=\frac{2\cancel{(1-\cos x)}}{\cancel{(1-\cos x)}(1+\cos x)}$$

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For Problem 1-6, use:

(20, -21) is on the terminal side of C and
$\sec Q = -\frac{13}{12}$ in QIII
to find the exact values of:

1. sin (C-Q) $= \sin C \cos Q - \cos C \sin Q$

$= \left(\frac{-21}{29}\right)\left(\frac{-12}{13}\right) - \left(\frac{20}{29}\right)\left(\frac{-5}{13}\right)$

$= \frac{352}{377}$

2. cos (Q-C) $= \cos Q \cos C + \sin Q \sin C$

$= \left(\frac{-12}{13}\right)\left(\frac{20}{29}\right) + \left(\frac{-5}{13}\right)\left(\frac{-21}{29}\right)$

$= \frac{-135}{377}$

3. $\tan(C-Q) = \frac{\tan C - \tan Q}{1 + \tan C \tan Q}$

$= \frac{-21/20 - 5/12}{1 + \left(\frac{-21}{20}\right)\left(\frac{5}{12}\right)} = \frac{-352/240}{135/240}$

$= \frac{-352}{135}$

4. sec (2Q) $= \frac{1}{\cos^2 Q - \sin^2 Q}$

$= \frac{1}{\left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2}$

$= \frac{169}{119}$

5. cot (2Q) $= \frac{1 - \tan^2 Q}{2 \tan Q}$

$= \frac{1 - (5/12)^2}{2(5/12)} = \frac{119/144}{5/6}$

$= \frac{119}{120}$

6. $\csc(2Q) = \frac{1}{2 \sin Q \cos Q}$

$= \frac{1}{2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)}$

$= \frac{169}{120}$

7. Prove: $1-\frac{5}{2}\csc\theta=\frac{6\sin^2\theta-19\sin\theta+10}{6\sin^2\theta-4\sin\theta}$

$$=\frac{(2\sin\theta-5)\cancel{(3\sin\theta-2)}}{2\sin\theta\cancel{(3\sin\theta-2)}}$$

$$=\frac{2\sin\theta}{2\sin\theta}-\frac{5}{2\sin\theta}$$

$$=1-\frac{5}{2}\csc\theta$$

8. Solve for $\theta\in[-180°, 180°]$:

$\sin\theta\cos 20°+\cos\theta\sin 20°=\frac{1}{2}$

$$\sin(\theta+20°)=\frac{1}{2}$$

$$\theta+20=\begin{cases}30 \pm 360n\\ 150 \pm 360n\end{cases}$$

$$\theta=\begin{cases}10 \pm 360n\\ 130 \pm 360n\end{cases}$$

$$\theta=\{10°, 130°\}$$

9. Solve for $x\in[0, \pi)$:

$\csc\left(4x-\frac{\pi}{3}\right)=2+2\csc\left(4x-\frac{\pi}{3}\right)$

$$\csc\left(4x-\frac{\pi}{3}\right)=-2$$

$$\sin\left(4x-\frac{\pi}{3}\right)=-\frac{1}{2}$$

$$4x-\frac{\pi}{3}=\begin{cases}-\pi/6 \pm 2\pi n\\ 7\pi/6 \pm 2\pi n\end{cases}$$

$$4x=\begin{cases}\pi/6 \pm 2\pi n\\ 3\pi/2 \pm 2\pi n\end{cases}$$

$$x=\begin{cases}\pi/24 \pm \pi/2\, n\\ 3\pi/8 \pm \frac{\pi}{2}n\end{cases}$$

$$x=\left\{\frac{\pi}{24}, \frac{5\pi}{24}, \frac{3\pi}{8}, \frac{7\pi}{8}\right\}$$

10. Prove: $\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=\cos x+\sin x$

$$\sqrt{2}\left[\cos x\cos\frac{\pi}{4}+\sin x\sin\frac{\pi}{4}\right]$$

$$\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right]$$

$$\cos x+\sin x$$