

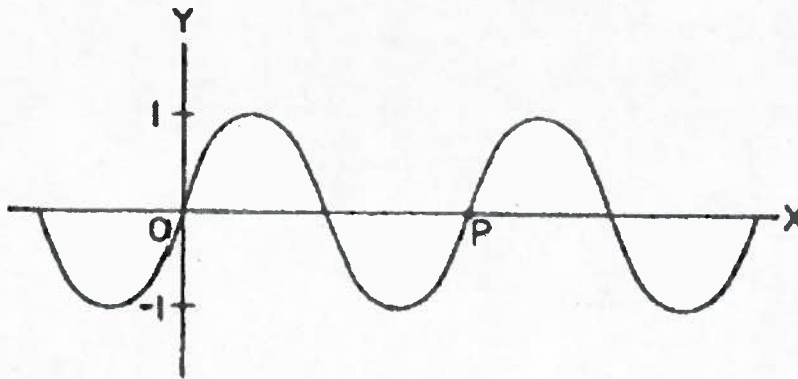
Precalculus  
Sinusoidal Functions v2  
CALCULATOR ALLOWED

Name Solution Key

1. On the graph of  $y = \cos x$ , as  $x$  increases on  $x \in \left[-\frac{1}{4}, \frac{1}{4}\right]$ , the function  $y$

- (a) decreases      (b) is constant      (c) increases  
(d) decreases, then increases      (e) increases, then decreases

2. This is the graph of  $y = \sin 3x$ .



What is the  $x$ -value of  $P$ ?

- (a)  $\frac{\pi}{3}$       (b)  $\frac{2\pi}{3}$       (c)  $2\pi$       (d)  $3\pi$       (e)  $6\pi$

3. Given  $g(x) = 3 + 2 \sin \left[ \frac{\pi}{4}(x+1) \right]$ , which of the following statements is true?

- I. The amplitude of  $g(x)$  is 2.  
II. The period of  $g(x)$  is 8.  
III. The phase shift is  $-1$ .

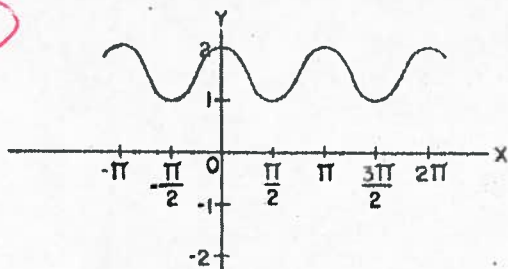
- (a) I only      (b) II only      (c) I and II only  
(d) II and III only      (e) I, II and III

4. What is the smallest positive value where  $y = 3 - 2\sec\left[\frac{\pi}{8}(x-1)\right]$  has a vertical asymptote?

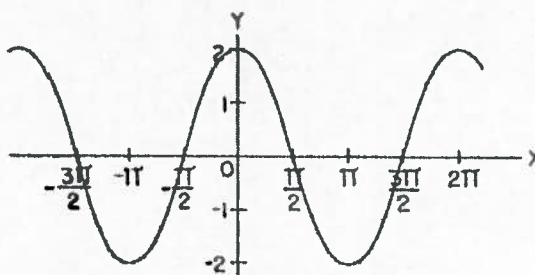
- (a) 1   (b) 5   (c) 9   (d) 13   (e) 17

5. Which of the following is the graph of  $y = \frac{3}{2} + \frac{1}{2}\cos 2x$ ?

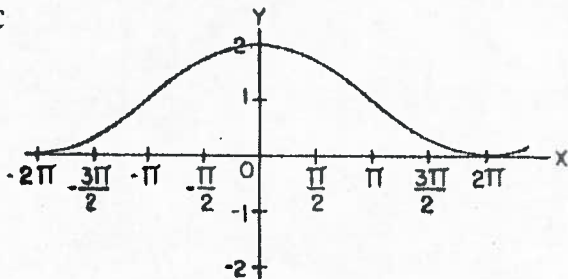
A



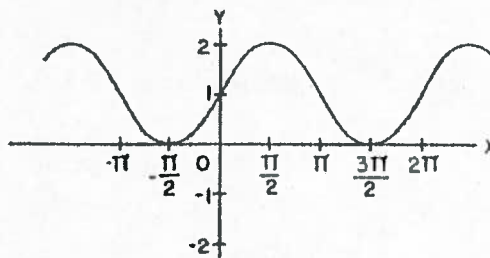
B



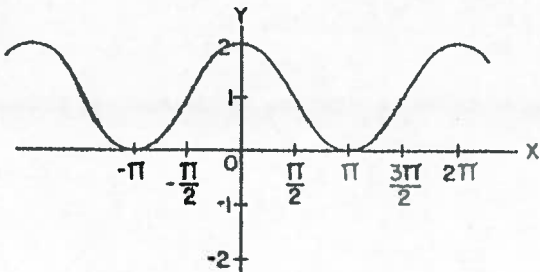
C



D



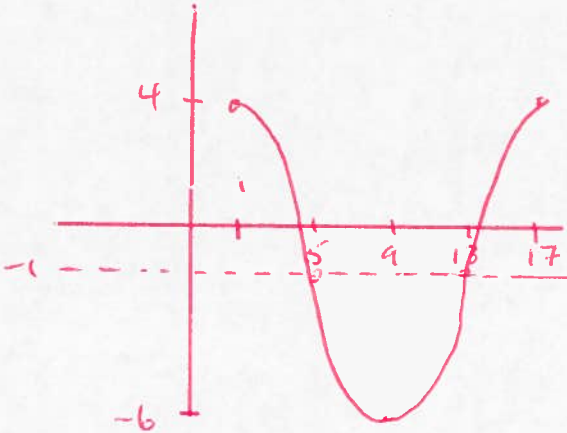
E



Show all work; round non-integer values to the nearest thousandth. List traits for ALL sketches. Sketch carefully and show relevant coordinate points as needed. 10 points each.

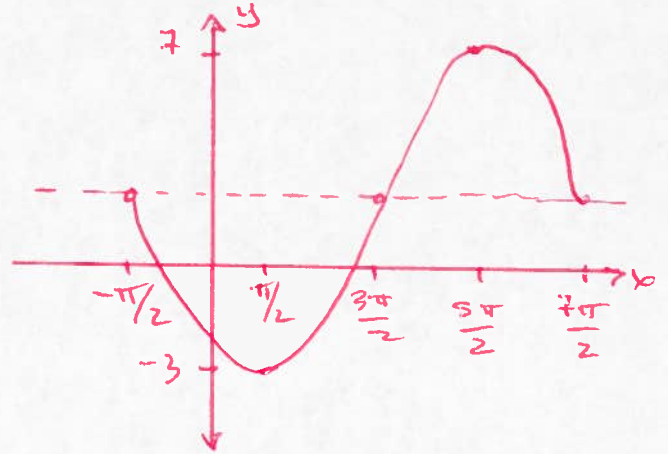
6. Sketch the primary cycle of

$$y = -1 + 5 \cos\left[\frac{\pi}{8}(x-1)\right].$$

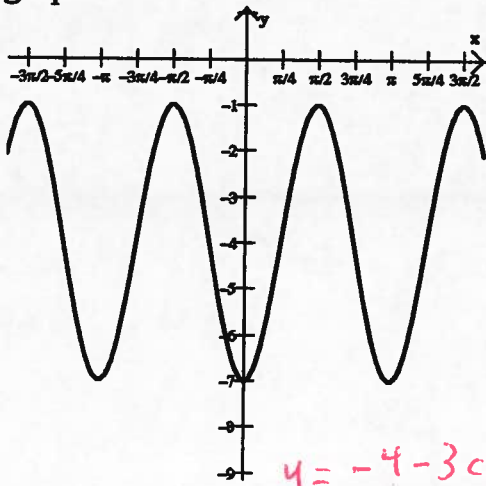


7. Sketch one cycle of

$$y = 2 - 5 \sin\left[\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right].$$

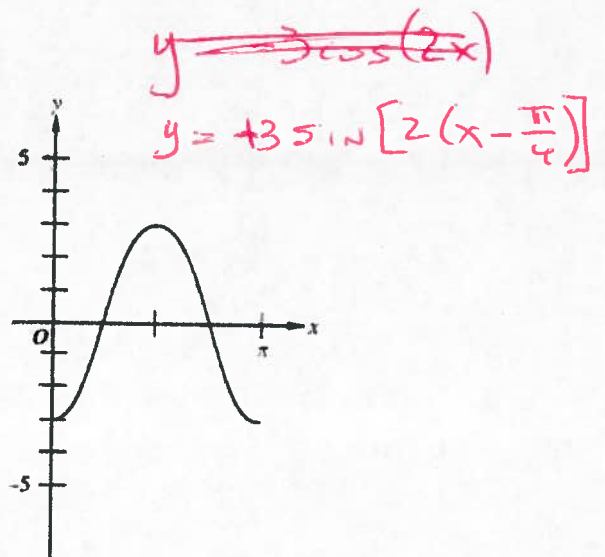


8. Find a cosine equation for this graph:



$$y = -4 - 3 \cos(2x)$$

9. Find a sine equation for this graph:



$$y = 3.5 \sin\left[2\left(x - \frac{\pi}{4}\right)\right]$$

10. If  $H(x) = -1 + 4 \cos\left[\frac{\pi}{3}(x-11)\right]$ , find the first four negative values of  $x$  where  $H(x) = 0$ .

$$0 = -1 + 4 \cos\left[\frac{\pi}{3}(x-11)\right]$$

$$\frac{1}{4} = \cos\left[\frac{\pi}{3}(x-11)\right]$$

$$\pm 1.318 \pm 2\pi n = \frac{\pi}{3}(x-11)$$

$$\pm 1.259 \pm 6n = x-11$$

$$x = \begin{cases} 12.259 \pm 6n \\ 9.741 \pm 6n \end{cases}$$

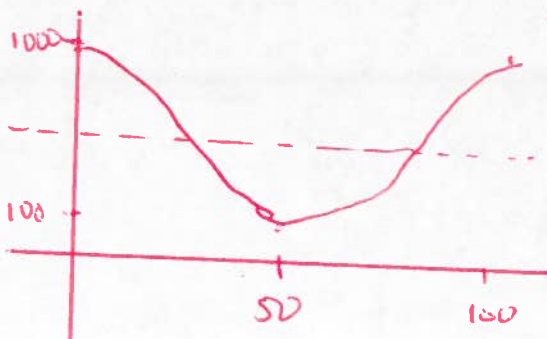
$$x = \{-2.259, -5.741, -8.259, -11.741\}$$

11. A space ship is in an elliptical orbit around Earth. At time  $t=0$ , it is at its apogee (highest point) which is  $d = 1000$  km. It reaches its perigee of  $d = 100$  km 50 minutes later.

a) Assuming  $d$  varies sinusoidally with time  $t$ , sketch a graph of one cycle and find the equation.

b) Isolate  $t$  as a function of  $d$ .

c) If the ship is only in communication range when it is below 700 km, for how many minutes is the ship out of range?



$$d) d = 550 + 450 \cos\left(\frac{\pi}{50} t\right)$$

$$b) t = \frac{50}{\pi} \cos^{-1}\left(\frac{d-550}{450}\right)$$

$$c) 700 = 550 + 450 \cos\left(\frac{\pi}{50} t\right)$$

$$\frac{1}{3} = \cos\left(\frac{\pi}{50} t\right)$$

$$\pm 1.231 \pm 2\pi n = \frac{\pi}{50} t$$

$$\pm 19.591 \pm 100n = t$$

∴ OUT OF RANGE FOR  $t \in [0, 19.591] \cup [80.409, 100]$

TOTAL OF 39.182 MINUTES